Observation of Collective Modes of Ultracold Plasmas

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(Received 16 November 2005; published 15 March 2006)

Applying a radio-frequency electric field to an expanding ultracold neutral plasma leads to the observation of as many as six peaks in the emission of electrons from the plasma. These are identified as collective modes of the plasma and are in qualitative agreement with a model of Tonks-Dattner resonances, electron sound waves propagating in a finite-sized, inhomogeneous plasma. Such modes may provide an accurate method to determine the time-dependent electron temperature.

DOI: 10.1103/PhysRevLett.96.105003 PACS numbers: 52.35.Dm, 32.80.Pj, 52.35.Fp, 52.70.Ds

Ultracold neutral plasmas, created by the photoionization of laser-cooled atoms [1], have properties determined by the initial atomic density and excitation fraction and have an initial electron energy determined by the tuning of the photoionization laser with respect to the ionization limit. Such ultracold plasmas (UCPs) are rather novel plasma systems, not only because of their low temperatures, but also because they are unconfined, inhomogeneous, and freely expanding into vacuum.

To date, measurements have been confined to global properties of the system, such as electron temperature [2], ion temperature [3,4], expansion velocities [5], and three-body recombination rates [6]. Although there is plenty of evidence supporting the plasma character of such systems from these measurements and associated theory, much of plasma physics can be viewed as the study of collective modes, which is what makes plasma such a rich and diverse physical system.

In this work, we present observations of collective modes of UCPs, excited by the application of rf electric fields, and detected through the enhanced emission of electrons during the plasma expansion. We observe as many as six mode frequencies and track the modes through the changing plasma density during expansion. We tentatively identify these modes as Tonks-Dattner resonances [7,8], the modes of an inhomogeneous finite-sized plasma obeying the Bohm-Gross dispersion relationship [9]. For small values of the wave vector k, the mode frequency approaches the cold plasma frequency, while at larger k the modes can be viewed as electron sound waves with a linear dispersion relationship. We find good agreement with calculations of these Tonks-Dattner resonances, although we have to specify an outer edge of the plasma, which enters into the boundary conditions for the calculation. To simplify the model, we also assume a time-independent electron temperature, ignoring the very early time change in temperature observed in [2,10]. The unconfined nature of the UCP is rather different than the confined systems where these resonances have been previously observed.

Our creation of the ultracold plasmas is similar to previous work [1]. A magneto-optic trap (MOT) is used to collect $\sim 5 \times 10^6$ metastable xenon atoms and to cool them to approximately 20 μK. The spatial density distribution is roughly Gaussian with a rms radius $\sigma_0 \sim 280$ μm and a peak density of about $n_0 = 2 \times 10^9$ cm^{-3}. The plasma is then produced using a stepwise two-photon excitation process (882 nm + 514 nm, 10 ns pulse), ionizing up to 30% of the MOT population. We control the ionization fraction with the intensity of the photoionizing laser, while the initial electron energy ΔE is controlled by the frequency of the 514-nm photon.

The ionized cloud rapidly loses a small number of electrons, resulting in a slightly attractive potential for the remaining electrons, and it quickly reaches a quasineutral plasma state. It then expands with an asymptotic velocity typically in the 50–100 m/s range caused by the outward electron pressure [5]. Approximately 1.5 cm above and below the plasma are wire mesh grids which are dc-biased to provide a small electric field ($\sim$50 mV/cm) so that electrons leaving the plasma are guided to a microchannel plate detector. The electron signal is recorded as a function of time. As can be seen in the lowest panel of Fig. 1, the signal consists of a prompt peak, followed by a region of little electron loss. This is followed by a long $\sim$200 μs loss of electrons, interpreted as the decay of the plasma as electrons evaporate out of the potential well, which is getting shallower due to expansion. The upper panels of Fig. 1 show the recorded signal when an rf voltage is applied to one of the grids; the rf signal is coupled into the chamber through a non-impedance-matched feedthrough, and it is thus difficult to estimate the exact size of the applied field. We typically supply -45 to -5 dBm at the feedthrough, which results in oscillating fields of order 0.4–50 mV/cm (higher powers are necessary at higher frequencies). Clearly visible in the data are multiple peaks induced by the rf fields, with positions in time (i.e., different plasma densities) depending on the frequency. We considered that the later peaks might be a response of the plasma to the excitation at earlier times ("ringing"). To test this, instead of cw rf excitation, we applied the rf with a 1 μs-duration pulse and stepped through the plasma evolution with 0.5 μs steps. The signals obtained in this manner were combined, resulting in curves nearly identical to those of Fig. 1. This shows that the late time peaks

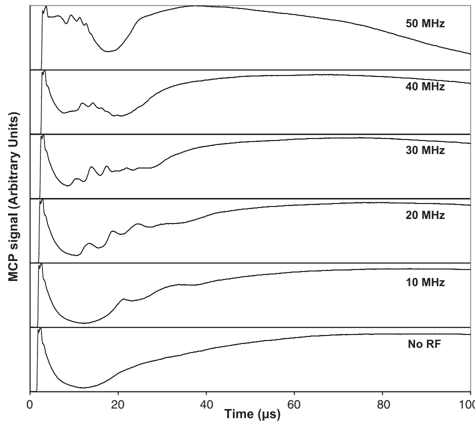

FIG. 1. Typical electron emissions from an expanding ultracold plasma. rf electric fields with the noted frequencies are applied to the expanding plasma, yielding distinct peaks that depend on frequency. The applied rf power is adjusted to make the peaks visible. Note the distortion of the curves at times immediately after the prompt peak for higher rf frequencies, due to the higher rf power required to observe resonance peaks at those frequencies.

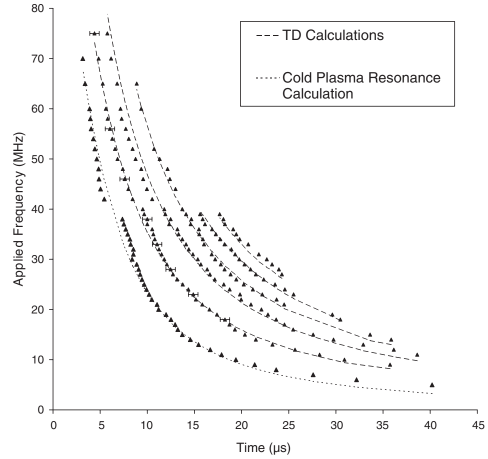

FIG. 2. Mode frequencies vs time. The data have errors at all points approximately the same as the representative error bars. Dashed lines are Tonks-Dattner theory fits with $T = 19$ K, $n_0 = 1.9 \times 10^9$ cm^{-3}, and expansion velocity $v = 65$ m/s. The dotted line is a cold plasma theory fit with $n_0 = 1.9 \times 10^9$ cm^{-3}, $v = 65$ m/s, and multiplicative factor of 0.24 scaling the peak plasma frequency.

correspond to the excitation of the plasma at that time and are not a delayed response from earlier excitation.

The times at which the resonance peaks occur are related to the frequency of the rf field applied to the system. We varied the frequency from 5 MHz to 80 MHz and determined the time at which each peak occurred by subtracting the background (no rf curve) and fitting a Gaussian to each peak. In this way we generated the family of frequency versus time curves shown in Fig. 2. We observe that higher frequencies require higher rf power to make visible features. This higher rf power tends to distort the background (signifying some nonresonant or highly damped heating process), which makes the background subtraction process less effective (see Fig. 1 for frequencies >30 MHz). By using the lowest rf power possible that still permitted observations of these modes, we obtained data sets with roughly uniform uncertainties at all times.

The families of curves such as those in Fig. 2 have limited observed dependence on parameters that can be controlled in our apparatus. The curves in Fig. 2 were obtained for an initial electron energy of 100 K, although they are quite similar for other initial energies. This may be explained by the tendency of these UCPs to equilibrate to ~25 K in a few μs due to competition between adiabatic cooling and three-body recombination-induced heating [2]. The curves shift if the number of particles in the plasma is changed or if the initial radius of the plasma is changed; changes that would lower the density cause the frequency versus time curves to shift to the left, as is to be expected if one were to assume the plasma frequency behaves as $\omega_p \propto n_e(t)^{1/2} \propto t^{-3/2}$, where $\omega_p^2(r) = n_e(r)e^2/\epsilon_0 m_e$.

We varied the density by varying the excitation probability, which also serves to vary the neutrality of the plasma. We found that the largest number of peaks are found at the highest densities and neutralities. This explains the lack of such features in the earlier work [5]. Note the "bulge" at around 35 MHz in some of the curves of Fig. 2. By reducing the rf power to the lowest level at which the peaks could still be observed, the bulge was significantly reduced, suggesting that it is the result of high rf power distorting the electron emission signals. However, at such low rf power only the first one or two peaks could be resolved in the signal; for the purposes of this work the rf power was set at the lowest value at which we could still observe at least three peaks. The full effects of increased rf amplitude on the plasma behavior have yet to be fully understood.

In Ref. [5] the application of an rf electric field resulted in the observation of a single peak. This signal was interpreted as the excitation of the fundamental plasma resonance at the plasma frequency. A simple model assumed that the peak of the response corresponded to the time when the average plasma frequency (assuming a Gaussian density distribution) equaled the rf frequency [$\omega = 0.6\omega_p(0)$, where $\omega_p^2(r) = n(r)e^2/\epsilon_0 m_e$]. Since $\omega_p \propto n_e^{1/2}$, this was used to extract the time-dependent plasma density and expansion velocities. The extracted asymptotic expansion velocities matched well with a simple hydrodynamic model for initial energies above 100 K, lending confidence to this interpretation (at lower initial energies three-body recombination provides an additional source of energy). Subsequent theoretical work [11] used cold plasma theory with a Gaussian density profile and found the peak response would actually correspond to a plasma frequency ranging from $\omega = 0.22\omega_p(0)$ to $\omega = 0.38\omega_p(0)$, but would not change the conclusions about the expansion velocity [12]. In recent work [2] the expansion velocity for large initial energy (>400 K) is used as a way to calibrate the density. Nonetheless, aside from an overall scaling factor, this response was consistent with a measure of the relative density as a function of time when compared to expansion modeling.

The interpretation of the single peak as the fundamental plasma resonance was predicated on the existence of only a single peak. If we assume only dipole modes are excited, cold plasma theory (which neglects pressure gradient terms) predicts only the single resonance. Calculations of higher order ($l > 1$) modes for a Gaussian density profile [13] show they lie at lower frequencies, inconsistent with the observed frequencies. In addition, our rf electric fields are rather uniform across the plasma, which should make coupling to higher order modes less likely.

Another possible explanation for the peaks is the presence of ion acoustic waves in the plasma. Ion acoustic waves have a velocity, $v_{\mathrm{iaw}} = \sqrt{kT_e/m_i}$ where T_e is the electron temperature and m_i is the ion mass. This is exactly the expansion velocity of the plasma (at least for $T > 100$ K), so one might expect "frozen" acoustic waves as the plasma expands, as was observed in simulations [10]. Since the electrons are so light and mobile, their density will follow the ion density. If we consider the simple model of peaks being due to regions with many electrons at specific densities, even a strongly modulated electron density does not yield peaks, but just a small distortion of the response. To further test the ion acoustic wave hypothesis, we created plasmas with various strong density modulations (such as imaging a wire through the center of the exciting beam). The resultant peaks were qualitatively unchanged, suggesting that ion acoustic waves are not a source of the peaks.

Given the lack of an adequate explanation from cold plasma theory, we must look toward phenomena dependent on temperature, even though our plasmas are quite cold. When the electron pressure term is included in the plasma fluid equations, this leads to the Bohm-Gross dispersion relationship for plasma waves [9], which we write in a local density approximation as:

$$\omega^2 = \omega_p^2(r) + \frac{3k_B T_e}{m_e} k^2(r), \tag{1}$$

where $k(r)$ is the local wave number and $\omega_p(r)$ is the local plasma frequency.

This describes a wave with frequency ω_p as $k \to 0$ (the cold plasma result), and a linearly dispersive wave for high k (an electron sound wave). We can estimate whether this may be relevant to ultracold plasmas by using the size of the plasma to estimate a relevant $k = 2\pi/\sigma$. For typical UCP sizes and temperatures, the temperature-dependent second term in the dispersion relation is approximately the same size as the first: we need to consider the effects of temperature.

Resonances in confined plasmas governed by the Bohm-Gross dispersion relation were first reported by Tonks [7] and were later studied in detail by Dattner [8], and are commonly referred to as Tonks-Dattner (TD) resonances. Qualitatively, in those regions where the electron density is low enough [$\omega_p(r) < \omega$], the plasma wave propagates [$k(r)$ is real], and in those regions where the electron density is high [$\omega_p(r) > \omega$], the plasma wave is evanescent [$k(r)$ is imaginary]. The longitudinal plasma waves reflect at the cutoff radius r_c where $\omega_p(r_c) = \omega$ and form radial longitudinal standing plasma waves between the plasma wall and the cutoff radius r_c. A determination of the resonant frequencies can be found using "pressure theory," which makes use of the moments of the Vlasov equation truncated by the adiabatic scalar pressure approximation together with the quasistatic approximation of Poisson's equation [14]. These equations result in a fourth-order ODE, and solutions must be obtained numerically. By making further approximations of purely electrostatic radial waves in the plasma and neglecting any coupling to the electromagnetic waves, the pressure theory yields the following equation [15] for the density fluctuation $\delta n(r)$

$$\nabla^2 \delta n(r) + k^2(r) \delta n(r) = 0 \tag{2}$$

with the boundary conditions $\delta n(0) = 0$ and $\nabla \delta n(r_w) = 0$ [or $j_r(r_w) = 0$], where r_w is the wall radius. Unlike previous work, our unconfined plasma has no wall, and we will have to choose an appropriate value for r_w (see below). The eigenvalues may be determined by using the WKB method [16]. In spherical coordinates, the resulting equations are

$$\tan\left[\int_{r_c}^{r_w} k(r)dr\right] = \frac{r_w}{2}[k(r_w)], \qquad \omega_p(0) < \omega \tag{3}$$

$$\int_{r_c}^{r_w} k(r)dr = \left(p + \frac{1}{4}\right)\pi, \qquad p = 1, 2, 3, \ldots, \qquad \omega_p(0) > \omega, \tag{4}$$

where $\omega_p(0)$ corresponds to the peak density. Since $k^2(r) = [\omega^2 - \omega_p^2(r)]/(3k_B T_e/m_e)$, Eqs. (3) and (4) can be numerically solved to provide the resonant frequency ω.

For our ultracold neutral plasma, we assume the initial electron density profile is a spherically symmetric Gaussian distribution because the initial atom distribution is Gaussian and the laser excitation profile is approximately uniform across the cloud. We also assume self-similar expansion, with a time-dependent rms radius $\sigma = \sqrt{\sigma_0^2 + (vt)^2}$, where v is the asymptotic expansion velocity [simulations [17] show the plasma has linearly increasing internal fields, which generates self-similar expansion].

Since our ultracold neutral plasma is not confined, it freely expands into vacuum. Previous calculations of Tonks-Dattner resonances have been done for cylindrical plasmas with defined walls, making the choice of r_w unambiguous. There is no physical wall for our system, so we choose the "wall" r_w to be 3σ, as this limit of integration for Eqs. (3) and (4) is reasonable in that it includes a large portion of the plasma and gives results consistent with our estimated densities and with cold plasma theory. We note that at the 3σ point, the local Debye length $\lambda_D(r)$ is on the order of the size of the plasma σ, where $\lambda_D(r) = \sqrt{\epsilon_0 k_B T_0 / e^2 n(r)}$.

Using the time-dependent Gaussian density profile and our choice of r_w, we numerically integrate Eqs. (3) and (4), with the results shown in Fig. 2. We assume (in accordance with previous work) that the lowest frequency mode is not given by the TD theory ($p = 0$ is not a valid solution), but is given by cold plasma theory. We use a fit to the cold plasma curve and find an expansion velocity of 65 m/s. We adjust T_e and n_0 to fit the generated TD curves to the data (to all data curves except the lowest) and find $T_e = 19$ K and $n_0 = 1.9 \times 10^9$ cm^{-3}, which are well within the expected range of temperature and density for our system. Using this n_0 found by fitting the TD calculations to the data, we fit $\omega_p(0)$ to the cold plasma data curve of Fig. 2, finding that a scale factor of 0.24 is required, which is consistent with Ref. [11], although not necessarily consistent with previous measurements [5].

For these Tonks-Dattner resonances, given a choice of r_w, the temperatures can be well determined; however, since the mode frequencies are dependent on r_w, we do not view this method as a quantitative way to determine T. Changing r_w by 10% results in a 20% change in the fitted temperature and as much as a factor of 2 change in the fitted density. This highlights one of the unusual features of expanding UCPs—the lack of a defined boundary condition. Theoretical work that can definitively calculate these electron sound waves in such a geometry is needed, which will then allow the observation of these modes to be an accurate method for temperature determination.

For this work, we fit the calculated curves to the data using a time-independent temperature. Previous electron temperature measurements [2] saw some early time dependence, but were limited to a small range in the expansion time where the method was applicable ($t < 10$ μs). The good agreement between the observed modes and the TD theory lends strong credence to the interpretation of these modes as TD resonances and also implies that the time dependence of the temperature at long times is relatively weak. By using the TD curves with additional theoretical input, it should be possible to dispense with our constant temperature approximation and fit temperatures at each time to our data sets. Other temperature measurements [3,4] image ions in the plasma to determine the ion temperatures, but they are typically much different than the electron temperatures. Ion imaging can be used to determine the expansion velocity of the plasma and thus, indirectly, the time-integrated electron temperature; using the TD resonances may provide a useful alternative technique for determining the time-dependent electron temperature of expanding UCPs.

Further areas of investigation include studying the amplitude of these modes and their dependence on system parameters. This will require a better knowledge of the actual rf electric field in our chamber. In addition, the theory presented above only determines positions and says nothing about coupling strengths (in fact, it assumes weak coupling). We clearly observe more modes for larger plasmas, which is not addressed in the theory. Such electron sound waves should exhibit Landau damping, so a study of their widths may yield information about damping with the UCPs. The application of a uniform magnetic field should lead to the appearance of new modes.

In summary, we have observed a series of collective modes of an ultracold plasma excited with uniform rf electric fields. We interpret these modes as electron sound wave resonances in the finite plasma, known as Tonks-Dattner resonances. We calculate the mode frequencies with WKB theory and find good agreement, using a physically reasonable assumption about the outer boundary condition for the waves. If a rigorous theory of electron sound waves in an unconfined geometry can be developed, these modes will offer a direct and precise way to determine the time-dependent electron temperature of ultracold plasmas over a large region of their expansion.

This work was partially supported by the National Science Foundation PHY-0245023.

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