Charge-Fluctuation-Induced Dephasing of Exchange-Coupled Spin Qubits

Xuedong Hu[1] and S. Das Sarma[2]

[1]*Department of Physics, University at Buffalo, The State University of New York, 239 Fronczak Hall, Buffalo, New York 14260-1500, USA*

[2]*Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20740-4111, USA*

(Received 26 July 2005; revised manuscript received 30 September 2005; published 13 March 2006)

Exchange-coupled spin qubits in semiconductor nanostructures are shown to be vulnerable to dephasing caused by charge noise invariably present in the semiconductor environment. This decoherence of exchange gate by environmental charge fluctuations arises from the fundamental Coulombic nature of the Heisenberg coupling and presents a serious challenge to the scalability of the widely studied exchange gate solid state spin quantum computer architectures. We estimate dephasing times for coupled spin qubits in a wide range (from 1 ns up to >1 μs) depending on the exchange coupling strength and its sensitivity to charge fluctuations.

DOI: 10.1103/PhysRevLett.96.100501 PACS numbers: 03.67.Lx, 73.63.Kv, 85.35.Gv, 05.40.Ca

A central issue in quantum information processing is quantum coherence, i.e., how long a quantum state survives without decay allowing robust quantum computation. The other central issue is scalability, i.e., whether a practical *macroscopic* quantum computer can be built by suitably scaling up individual *microscopic* qubits. The perverse dichotomy in quantum computation has been that architectures that can be scaled up fairly easily (e.g., solid state systems) suffer from serious environmental decoherence problems, whereas architectures based on isolated trapped atoms and ions, which have excellent coherence, are typically not easily scalable. In this context, the proposed spin quantum computer (QC) architectures [1–5] in semiconductor nanostructures look particularly promising, since electron spin usually has long coherence time as the relativistic nature of spin produces weak direct environmental coupling, and semiconductors, at least as a matter of principle, allow for relatively easy scaling up. Motivated by pioneering early suggestions [1–3], there has been impressive recent experimental advance in the study of spin qubits in gated GaAs quantum-dot systems [6–9], an architecture widely regarded as one of the most promising solid state QC architectures.

One of the most significant advantages of a spin qubit is its relative isolation from its environment, leading to exceedingly long relaxation and dephasing times in systems such as isolated donor electron and nuclear spins in Si:P and GaAs quantum dots [10,11]. Specifically, electron spin relaxation due to spin-orbit interaction and coupling to phonons is significantly reduced at low temperatures and for localized spins [12]. The only important decoherence channel left is the nuclear spin hyperfine coupling-induced electron spin spectral diffusion, so that the spin decoherence time for an isolated electron could range from tens of microseconds in GaAs quantum dots up to hundreds of milliseconds in Si:P donor electrons [13].

New decoherence channels could open up when qubits are manipulated and/or coupled. An advantage of spin qubits in solids is the availability of exchange interaction, which originates from Coulomb interaction and the Pauli exclusion principle. Even though spins are magnetic and magnetic interactions are weak, two-spin operations can actually be very fast (as short as 100 ps), because the underlying exchange interaction is electrostatic, which is strong. However, therein lies a new decoherence channel (compared to single spins) for the spin qubits: When spin coupling is needed and exchange interaction is turned on using external gates, charge fluctuations in the environment (an important source of decoherence for charge qubits in semiconductor [14] and superconducting [15] nanostructures) could lead to gate errors and dephasing [16].

In this Letter, we quantify how charge fluctuations affect exchange gates for spin qubits in a double quantum dot. In particular, we calculate the modification of exchange coupling in the presence of barrier variation and double dot level detuning (both of which could arise from charge fluctuations in the environment), evaluate errors in exchange gates, and calculate dephasing rates for logical spin qubits encoded in the double dot singlet-triplet states.

Gated quantum dots are defined electrostatically from a two-dimensional electron gas. A movement of a trapped charge close by changes the confining potential mainly in two ways: rise or fall of the barrier between the dots (thus change in tunneling rates) and detuning of the orbital levels in the two dots [17], as illustrated in Fig. 1. When the central barrier between the double dot rises or falls, the dot potential minima also shift slightly farther from or closer to each other.

With this qualitative understanding of how charge fluctuations affect quantum-dot confinement, we evaluate the exchange coupling of two electrons in a gate-defined unbiased double quantum dot in the presence of charge fluctuations. We use a two-dimensional quartic potential [16,18]: $V(x, y) = \frac{1}{2} m\omega^2[(x^2 - L^2)^2/L^2 + y^2]$. The central barrier height for this potential is $V_B = m\omega^2 L^2/2$,

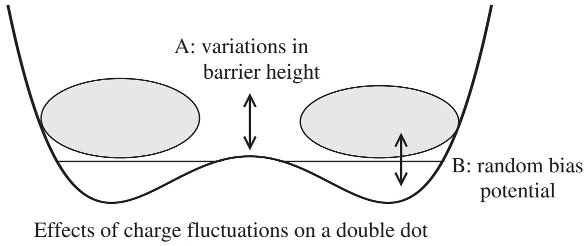

FIG. 1. Charge fluctuations invariably present in the semiconductor environment affect a double quantum dot mainly by causing variations of barrier potential height (A) and producing a random bias potential between the two dots (B).

directly related to the interdot distance [19]. We use GaAs parameters in this calculation, with $m = 0.067m_0$, where m_0 is the bare electron mass. To calculate the exchange splitting for two electrons in this double dot potential, we employ the Heitler-London (HL) approximation [20], so that the exchange splitting J between the two-electron singlet and unpolarized triplet states can be expressed in terms of a few easy-to-calculate matrix elements:

$$\begin{aligned} J = {} & \frac{2S^2}{1-S^4}[\langle\psi_L|V - V_L|\psi_L\rangle + \langle\psi_R|V - V_R|\psi_R\rangle] \\ & + \frac{2S^2}{1-S^4}\langle\psi_L(1)\psi_R(2)|\frac{e^2}{\epsilon r_{12}}|\psi_L(1)\psi_R(2)\rangle \\ & - \frac{2S}{1-S^4}[\langle\psi_L|V - V_R|\psi_R\rangle + \langle\psi_R|V - V_L|\psi_L\rangle] \\ & - \frac{2}{1-S^4}\langle\psi_L(1)\psi_R(2)|\frac{e^2}{\epsilon r_{12}}|\psi_L(2)\psi_R(1)\rangle. \end{aligned} \quad (1)$$

Here V_L and V_R are harmonic potential wells that share the same location as the left and right potential minima, respectively, of given potential V, ψ_L or ψ_R is the ground state if only V_L or V_R, respectively, is present, indices 1 and 2 refer to the two electrons, S is the overlap integral between the single dot orbitals, and r_{12} is the interelectron distance. The first and third terms in Eq. (1) refer to single particle contributions to J, the second term originates from the direct repulsion between the two electrons, and the last term is the exchange contribution. The HL approximation works well for higher confinement energies ($\hbar\omega \gtrsim 3$ meV, corresponding to smaller dots), when the on-site Coulomb interaction is sufficiently large, and for larger interdot distances.

In Fig. 2, we plot the exchange splitting and the barrier-height-V_B dependence of the exchange for a series of configurations of double dots. For strongly coupled dots, exchange splitting up to 1 meV can be obtained, where dJ/dV_B can be larger than 1, rendering the coupled spin qubits as sensitive to charge noise as a charge qubit. Notice that, at the large exchange limit, a more sophisticated method is required for reliable evaluation of the exchange splitting [4,16], though the modifications are generally within 1 order of magnitude so that the qualitative feature of our current results remains. Charge fluctuations could also introduce a small bias ΔV between the originally unbiased double dot, which leads to a second-order correction $[\Delta J \propto (\Delta V)^2]$. For relatively small charge fluctuations, we neglect this bias effect on exchange since it is a higher-order effect.

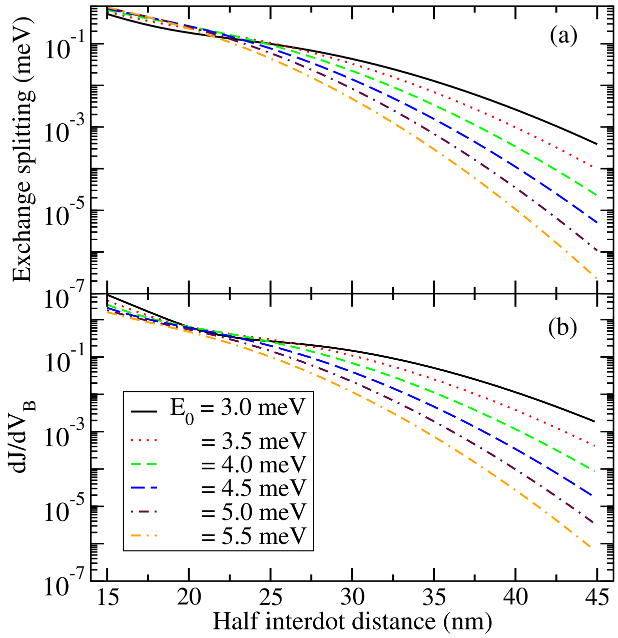

FIG. 2 (color online). Panel (a) presents exchange splitting as a function of interdot distance (directly related to the interdot barrier height) for various dot sizes (directly related to the single dot confinement energy E_0) with the quartic double dot potential, while panel (b) presents dJ/dV_B (derivative of J over the interdot barrier potential) as a function of interdot distance. Panel (b) gives a quantitative estimate of the sensitivity of exchange coupling to background charge fluctuations.

In some experimental situations, the double dot is strongly biased so that one of the doubly occupied two-electron singlet states is the ground state [7,9]. In this case, the exchange splitting between the two-dot two-electron singlet and triplet states could be dominated by the tunnel coupling between the two- and one-dot singlet states (while the triplet cannot tunnel due to spin blockade [21]) and takes on the value $J \approx |t|^2/E_b$, where t is the tunnel coupling between the two singlet states and E_b their energy difference, dominated by the applied bias potential between the two dots. Charge fluctuations affect both t and E_b, so that $dJ = [(2J/t)(dt/dV_B)]dV_B - (J/E_b)dE_b$ (assuming dV_B and dE_b represent different components of charge noise), where V_B is the barrier potential that determines tunnel coupling t. If dJ is dominated by dE_b, $dJ = -(J/E_b)dE_b$. For $J \sim 1$ μeV and $E_b \sim 100$ μeV, $|dJ/dE_b| \sim 0.01$, similar to a weakly coupled unbiased situation. (Here dJ/dE_b, as opposed to dJ/dV_B, controls the charge fluctuation effect on the exchange gate.) Our estimates here are consistent with the experimentally measured exchange dependence on bias voltage presented in

Ref. [9], where dJ/dE_b ranges between 0.01 and 10^{-4} and falls in the same range as our theoretical results in Fig. 2.

Having analyzed how charge fluctuations affect the exchange splitting of a double dot, we now study how spin quantum computing could be influenced. Charge fluctuations can affect coupled spin qubits in two different ways. If exchange interaction is turned on briefly to perform two-qubit operations, a switching event of a nearby charge trap leads to a gate error. If the exchange interaction is constantly on and the singlet and the unpolarized triplet states are the logical qubit states, background charge fluctuation causes pure dephasing between the two states. Below, we analyze these two situations separately.

A switching event during an exchange gate inevitably leads to a gate error. Consider a SWAP gate, where an exchange pulse corresponding to $\int J dt/\hbar = \pi$ leads to a swap of the states of two spins. If instead $\int J dt/\hbar = \pi + \delta$, the result of the exchange pulse is

$$\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}\overset{\text{SWAP}}{\longrightarrow}\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}+\frac{\alpha_1\beta_2-\alpha_2\beta_1}{\sqrt{2}}(1-e^{i\delta})|S\rangle. \tag{2}$$

Here $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ is the two-spin singlet state. Thus, the two spins would remain entangled after the SWAP operation. This error is linearly proportional to δ when $\delta \ll \pi$, and its prefactor $\alpha_1\beta_2 - \alpha_2\beta_1$ does not vanish unless the two single spin states are identical.

If we have three spins and intend to swap the state of the first to the third, errors accumulate linearly, with residual entanglement between qubits 1 and 3, and 2 and 3. The total error should grow with the number of spins N linearly even if errors in each swap are random, since different errors are not directly additive as they represent different unwanted entanglement after the swaps. One can estimate errors in more complicated operations such as a controlled-NOT gate [22], where the feature of linear increase of error with the number of qubits persists.

If two-spin singlet and unpolarized triplet states are used as logical qubit states and the exchange coupling is kept on, background charge fluctuations cause dephasing between the two states. Since charge fluctuation is also a major source of decoherence for double dot charge qubits [14], we can extract the necessary information needed to calculate spin decoherence rate from the measured charge relaxation rates in the GaAs quantum-dot system. Similar to what is well established in the decoherence properties of Cooper pair boxes [15], the relaxation rate at the degeneracy point of a double dot single-electron charge qubit (where the double dot ground and first excited states are split by $2|t|$, where t is the tunneling strength between the two dots. The Hamiltonian for such a charge qubit can be written as $H = |t|\sigma_z + V\sigma_x$, where V is the potential bias between the two dots) is given by a simple expression [15] $\Gamma_1 = (\pi/2\hbar^2)S_V(\omega = 2|t|/\hbar)$, where $S_V(\omega) = (1/2\pi)\times \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle V(\tau)V(0)\rangle$ is the charge fluctuation (in terms of the bias gate potential fluctuation) correlation in the environment. From the functional dependence of $S_V(\omega)$ on ω (such as $1/f$ noise, which has only one parameter and is used here), we could use the knowledge of Γ_1 to determine $S_V(\omega)$ and then use it to calculate the time (τ)-dependent phase diffusion $\Delta\phi_c$ for a charge qubit [so that a factor $\exp(-\Delta\phi)$ appears in the off-diagonal density matrix element of the two-level system formed from the double dot] when it is far away from the degeneracy point, where the effective Hamiltonian becomes $H = V\sigma_z$:

$$\Delta\phi_c(\tau) = \frac{1}{2\hbar^2}\int_{\omega_0}^{+\infty} d\omega S_V(\omega)\left(\frac{\sin\omega\tau/2}{\omega/2}\right)^2. \tag{3}$$

Here the integral has a low frequency cutoff that is generally taken as the inverse of the measurement time [15]. In the case of two-spin singlet and unpolarized triplet states, the effective two-level Hamiltonian can be written as $H = J\sigma_z$ (there is no σ_x term here as spin symmetry prevents direct relaxation between the two states), so that the two-spin dephasing is given by [15]

$$\Delta\phi_s(\tau) = \frac{1}{2\hbar^2}\int_{\omega_0}^{+\infty} d\omega S_J(\omega)\left(\frac{\sin\omega\tau/2}{\omega/2}\right)^2 \cong \left(\frac{dJ}{dV}\right)^2\Delta\phi_c(\tau), \tag{4}$$

where $(dJ/dV)^2 = (\partial J/\partial V_B)^2 + (\partial J/\partial E_b)^2$. Accordingly, two-spin dephasing should be sensitively dependent on the barrier and bias dependence of the exchange splitting $(dJ/dV)^2$. Equation (4) is valid when $S_{V_B}(\omega)$ for the interdot barrier height and $S_{V=E_b}(\omega)$ for the interdot bias are identical (recall that they originate from the same source of charge fluctuations and, therefore, should have similar behavior).

Figure 3 shows the charge qubit phase diffusion. Two-spin phase diffusion can be obtained according to Eq. (4). The phase diffusion grows as almost a quadratic function of time (actually $t^2 \ln t$). For charge relaxation time on the order of 10 ns [14], dephasing time for a biased charge qubit is on the order of 1 ns (lower horizontal line in Fig. 3). Two-spin dephasing time sensitively depends on dJ/dV, which in turn varies widely (see Fig. 2 and Ref. [9]). For example, if $dJ/dV = 1$ (when the double dot is tightly coupled), the two-spin dephasing time is as short as the charge dephasing time. On the other hand, if $dJ/dV = 0.01$, which is a reasonable estimate (see Fig. 2), two-spin dephasing time would be on the order of 10 times the charge relaxation time $T_1 \sim 10$ ns, leading to $T_2 \sim 0.1\ \mu$s. If dJ/dV could be reduced to 0.001, for small J [9], or using schemes that can minimize dJ/dV through device design, in the spirit of the pseudodigital design (to combat gate voltage fluctuations) of Ref. [23], in which J reaches maximum while dJ/dV is close to zero, the two-spin dephasing time would be $\sim 1\ \mu$s, on the same order as the single spin decoherence time due to nuclear spin-

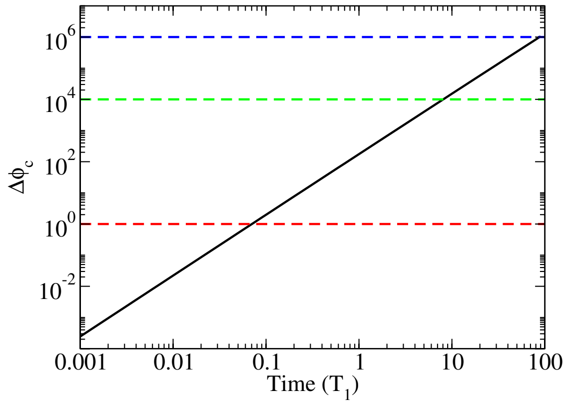

FIG. 3 (color online). Phase diffusion of a biased double dot charge qubit with a cutoff frequency of 1 Hz. Dephasing of a two-spin qubit can be determined from this figure as well, since it is linearly proportional to the charge qubit phase diffusion, as illustrated by Eq. (4). The lower horizontal line indicates that $\Delta\phi_c \sim 1$ corresponds to $t \sim T_1/10$; the middle (upper) horizontal line indicates that $\Delta\phi_c \sim 10^4$ (10^6) corresponds to $t \sim 10T_1$ ($100T_1$). In other words, if $dJ/dV \sim 0.001$, $\Delta\phi_s \sim 1$ would correspond to a time of $100T_1$, where charge relaxation time T_1 is on the order of 10 ns, so that spin dephasing time ~ 1 μs.

induced spectral diffusion in GaAs [13]. In general, extremely small values of dJ/dV are needed to essentially eliminate the charge-fluctuation-induced spin decoherence.

Charge-fluctuation-induced spin qubit dephasing is obviously not just limited to GaAs quantum dots. Whenever charge degrees of freedom (e.g., electrostatic gates) are used to boost the speed of a quantum computing scheme, dephasing effect from charge noise in the underlying structure could arise. For instance, charge noise could have negative effects on trapped ions when semiconductor microtraps are used, so that charge fluctuations in the semiconductor environment could lead to decoherence in the ionic states. Similarly, various proposed solid state quantum computing schemes using only exchange gate architectures are potentially susceptible to charge noise decoherence.

In conclusion, although single spin coherence is unaffected by charge noise, background charge fluctuations could lead to significant gate errors and/or decoherence in semiconductor-based electron spin qubits through interqubit exchange coupling. Our results show that charge noise could be the most important decoherence channel for exchange-coupled spin qubits, and further development in device design and fabrication is needed to reduce the sensitivity of exchange coupling to charge fluctuations. In particular, the linear scaling of charge-fluctuation-induced spin dephasing of the exchange gate architecture with the number of gates is a rather serious problem that could limit the scalability of exchange-based spin quantum computation. Our finding of the charge-fluctuation-induced spin qubit dephasing time being in a wide range from nanoseconds to microseconds points to the need to optimize the double dot design in order to desensitize exchange coupling J to the environmental charge noise.

We thank J. R. Petta and C. M. Marcus for useful discussions. This work was supported in part by LPS, NSA, ARO, and NSF.

[1] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[2] B. E. Kane, Nature (London) **393**, 133 (1998).
[3] D. P. DiVincenzo *et al.*, Nature (London) **408**, 339 (2000).
[4] X. Hu and S. Das Sarma, Phys. Rev. A **61**, 062301 (2000).
[5] X. Hu and S. Das Sarma, Phys. Status Solidi B **238**, 360 (2003); X. Hu, cond-mat/0411012; S. Das Sarma *et al.*, Solid State Commun. **133**, 737 (2005).
[6] J. M. Elzerman *et al.*, Nature (London) **430**, 431 (2004).
[7] A. C. Johnson *et al.*, Nature (London) **435**, 925 (2005); F. Koppens *et al.*, Science **309**, 1346 (2005).
[8] T. Hatano, M. Stopa, and S. Tarucha, Science **309**, 268 (2005).
[9] J. R. Petta *et al.*, Science **309**, 2180 (2005).
[10] G. Feher, Phys. Rev. **114**, 1219 (1959); D. K. Wilson and G. Feher, *ibid.* **124**, 1068 (1961).
[11] T. Fujisawa *et al.*, Nature (London) **419**, 278 (2002); R. Hanson *et al.*, Phys. Rev. Lett. **91**, 196802 (2003); **94**, 196802 (2005).
[12] V. N. Golovach, A. Khaetskii, and D. Loss, Phys. Rev. Lett. **93**, 016601 (2004).
[13] R. de Sousa and S. Das Sarma, Phys. Rev. B **67**, 033301 (2003); **68**, 115322 (2003).
[14] T. Hayashi *et al.*, Phys. Rev. Lett. **91**, 226804 (2003); J. Petta *et al.*, *ibid.* **93**, 186802 (2004).
[15] O. Astafiev *et al.*, Phys. Rev. Lett. **93**, 267007 (2004).
[16] G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999).
[17] S. W. Jung, T. Fujisawa, Y. Hirayama, and Y. H. Jeong, Appl. Phys. Lett. **85**, 768 (2004); M. Pioro-Ladriére *et al.*, Phys. Rev. B **72**, 115331 (2005).
[18] The growth direction for the gated quantum dots, with much higher confinement energy, is regarded as immune to charge fluctuations and, therefore, inactive for our considerations.
[19] A local barrier potential that does not affect electron orbitals could lead to quantitative changes to our results but not the range of values for the exchange splitting J.
[20] For well separated quantum dots with small overlap, Heitler-London approximation, though simple, works quite well. See C. Herring, in *Magnetism*, edited by G. T. Rado and H. Suhl (Academic, New York, 1966), Vol. IIB.
[21] K. Ono *et al.*, Science **297**, 1313 (2002).
[22] M. Thorwart and P. Hänggi, Phys. Rev. A **65**, 012309 (2002).
[23] M. Friesen, R. Joynt, and M. A. Eriksson, Appl. Phys. Lett. **81**, 4619 (2002).