Quasiperiodic Events in an Earthquake Model

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We introduce a modification of the Olami-Feder-Christensen earthquake model [Phys. Rev. Lett. **68**, 1244 (1992)] in order to improve the resemblence with the Burridge-Knopoff mechanical model and with possible laboratory experiments. A constant and finite force continually drives the system, resulting in instantaneous relaxations. Dynamical disorder is added to the thresholds following a narrow distribution. We find quasiperiodic behavior in the avalanche time series with a period proportional to the degree of dissipation of the system. Periodicity is not as robust as criticality when the threshold force distribution widens, or when an increasing noise is introduced in the values of the dissipation.

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The concept of self-organized criticality (SOC) [1], introduced by Bak, Tang and Wiesenfeld in 1987, was an attempt to explain the appearance of scale invariance in nature. In their sandpile model both the random, slow addition of "blocks" on a two dimensional lattice and a simple, local, and conservative rule drive the system into a critical state where power law distributed avalanches maintain a steady regime far from equilibrium. There is no correlation between the avalanches, and eventually they reach the boundaries of the lattice liberating the excess of energy. Five years later Olami, Feder, and Christensen (OFC) made an important contribution to the SOC ideas by mapping the Burridge-Knopoff spring-block model [2] into a nonconservative cellular automata [3], simulating the earthquake's behavior and introducing dissipation in the family of SOC systems. The fact that avalanches are uncorrelated in the sandpile model has been used as an argument to propose that it is not possible to predict real earthquakes [4]. However, foreshocks, aftershocks, and clustering properties [5] indicate the existence of correlation between different events. Many seismologists believe that large earthquakes are quasiperiodic [6,7], but periodic behavior has appeared in theoretical models only as a special or as a trivial solution [8–10], or as a result of a phase locking due to periodic boundary conditions (BCs) [11–13], or synchronized regions [14] in cellular automata.

The spring-block model consists in a two dimensional array of blocks on a flat surface. Each block is connected with its four nearest neighbors, and in the vertical direction, to a driving plate which moves horizontally at velocity v. The connections are made by springs, and when the force acting on a block overcomes the static friction with the surface, the block slips. Then a redistribution of forces takes place in the neighbors that eventually trigger new displacements. In the OFC model the force on a block is stored in a site of a squared lattice, and the static friction threshold has the same value for all blocks. Starting from a random distribution of forces, the site closest to the threshold is found and the *exact* force necessary to provoke a slip in this block is added to every site of the grid. This infinitely accurate tuning is only possible in the mechanical model if the displacement of the plate is infinitely slow ($v \rightarrow 0$), considering the fact that the real time resolution is finite. When a site reaches the threshold it is set to zero, and a fraction α of its force is redistributed to its neighbors. If $\alpha = 1/4$ the system is conservative. If one of the neighbors reaches the threshold the process is repeated until all the sites have their values below the threshold. The number of slips is defined as the avalanche size. Then again, the site closest to the threshold is found and a new avalanche is triggered. The avalanche distributions follow power laws and by varying the degree of dissipation α, the slope of the distribution can be tuned. Although the criticality of the system in the nonconservative regime ($\alpha < 1/4$) has been widely debated [15,16], the model results in a power law distribution of avalanches (for $\alpha \sim 0.2$) similar to the Gutenberg-Richter law [17] and also reproduces other characteristics of real earthquakes [18,19].

In the aim of improving resemblance with the mechanical model we have introduced two variations in the OFC model. (i) Thresholds are distributed randomly following a Gaussian distribution of standard deviation σ. When a block slips a new threshold is imposed to its site. This is closer to the actual block-surface friction problem and allows the system to start from a configuration of zero force in every block [20]. (ii) Instead of assuming infinitely accurate tuning, we add a quantum of force in each step, as in Ref. [21], but keeping the separation of time scales (relaxations are considered instantaneous) which is more realistic [22]. In the mechanical model this is equivalent to a finite velocity (considering finite time resolution). The dynamics becomes more complex because several sites can reach the threshold as the plate moves, so several clusters start to grow and eventually they can touch each other merging into a single one. The avalanche size is defined as the number of slips in each cluster.

In order to study the temporal series of avalanches we need to define a unit of time. Let us consider the case $\alpha = 0$, $\sigma = 0$ (isolated blocks with single threshold). When the force that drives the system reach a value F_m equal to the threshold, every site returns to its original value and a trivial periodicity rules the dynamics; this period is a natural unit. Therefore our unit of time is the real time T_m that a mechanical model spends to add a force F_m to the system.

We performed simulations taking a quantum of force $\delta F = 10^{-4}$ and threshold values around 1 for different σ values: 0, 0.001, and 0.01. Then an isolated block would need 10^4 steps to reach the threshold, so it would spend a time $\delta t/T_m = 10^{-4}$ in each step. All the simulations presented in this Letter took place with open BCs, but the same results were obtained in a system with free BCs as defined in Ref. [3].

Figure 1 shows the avalanche size distribution for different sizes of the lattice (more than 3×10^8 clusters for $L = 512$) in a system, with $\alpha = 0.2$ and $\sigma = 0.001$. Avalanche size is defined as the number of slips in each cluster. The distributions follow a power law with a slope equal to -1.91. The curves present cutoffs sensitive to the size of the system but they collapse (see inset) when the finite size scaling relation $P(s, L)L^{\beta} = f(sL^{-\nu})$ is applied with $\beta = 4.2$ and $\nu = 2.2$. Simulations for other α values with $L = 512$ indicate that the absolute value of the slope of the distributions increases as the dissipation decreases. Both the fact that ν is larger than 2 and the lack of universality are still in controversy [23,24]. The power law behavior of the avalanche distributions for small α values is more robust in this finite velocity model than in the original OFC one due to the fact that larger clusters have less probability in the OFC model. This is very consistent with Drossel [14] if we consider our "floating-point precision" equal to the step in which the driven force is increased. The avalanche size distributions (at least for $\alpha > 0.1$) do not suffer considerable variations when σ moves from 0 to 0.01. For $\sigma = 0.1$ the system is not critical anymore, but this is not a realistic value for the fluctuations of the friction force associated with an interface between a block and a flat surface.

The analysis of the autocorrelation function

$$C(t) = \frac{\sum (s(\tau)s(\tau + t)) - \langle s(\tau)\rangle^2}{\sum (s(\tau) - \langle s(\tau)\rangle)^2} \qquad (1)$$

where $s(t)$ corresponds to the avalanche time series (Figs. 2 and 3) displays a strong correlation between avalanches. Some of the peaks for $\sigma = 0.001$ are shown in Fig. 2. The position and height of all the peaks for different σ appear in Fig. 3. The position of the peaks indicates that for every analyzed σ the system has a quasiperiodic behavior with a period proportional to the degree of dissipation: $T = T_m(1 - 4\alpha)$.

In general, the height of the peaks decreases as the dissipation decreases; and for $\alpha = 0.225$ the only system where a sign of periodicity can be identified is for $\sigma = 0.001$. Nevertheless, the peak is just slightly above the background noise (see Fig. 2). For the conservative case, as in the "sandpile" model, avalanches are uncorrelated. Periodicity in the system is not as robust as criticality when σ varies: peaks are almost delta functions for $\sigma = 0$ (see curve in gray in Fig. 2) but their width increases dramatically when σ increases. The height of the peaks shows a monotonous variation with α for $\sigma = 0$, but for larger σ there are local variations not well understood yet. Although the curves for smaller σ values cross each other, the peaks for larger σ are extremely wide, noisy, and have a very small height, indicating that periodicity vanishes when noise is added to the system.

We have characterized the avalanches in our model into small, medium or large in the following way: In the avalanche size distribution, in a log-log plot, we take the linear

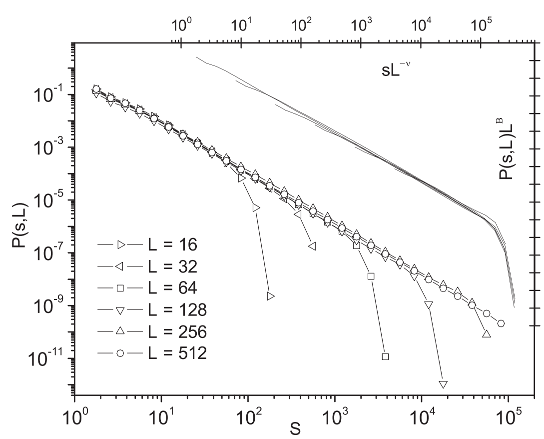

FIG. 1. Avalanche size distributions for the number of slips in each cluster, with $\alpha = 0.2$ and $\sigma = 0.001$. The slope is -1.91. Inset: collapse of all the curves under the scaling relation $P(s, L)L^{\beta} = f(sL^{-\nu})$, with $\beta = 4.2$ and $\nu = 2.2$.

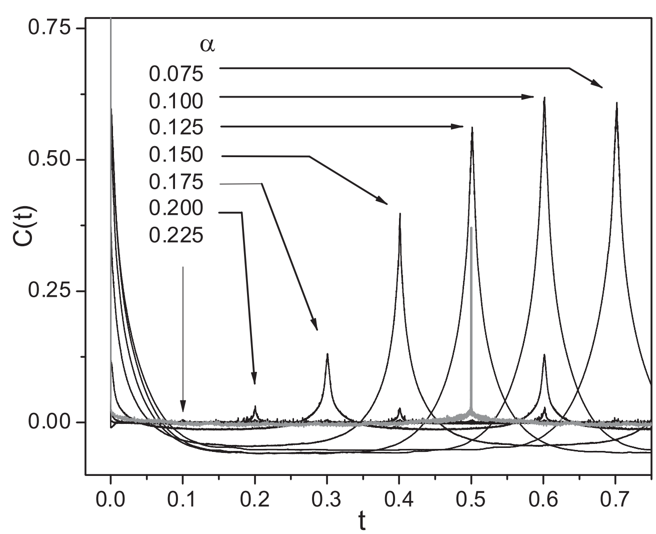

FIG. 2. Autocorrelation function for the avalanche time series [Eq. (1)] for a system with $\sigma = 0.001$ and $L = 128$. Notice in gray the same function for a system with $L = 128$, $\alpha = 0.125$, but with $\sigma = 0$.

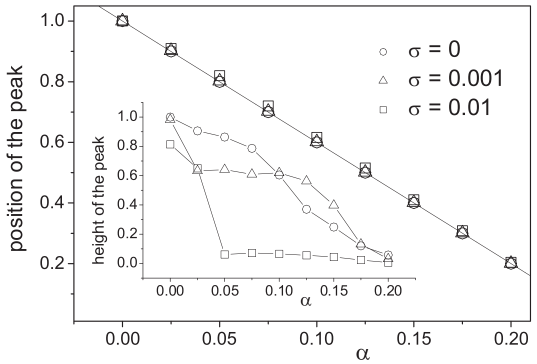

FIG. 3. Position of the peaks for the correlation function of the avalanche time series [Eq. (1)] for different σ. They follow the equation $T = Tm(1 - 4\alpha)$. Inset: Height of the peaks.

zone; in the case of $L = 128$ it spans from 1 to 6×10^3 (see Fig. 1). Then this interval is divided in three zones logarithmically equispaced. As a result, avalanches smaller than 18 are considered small, those lying between 19 and 330 are medium, and those greater than 330 are large. The average number of avalanches larger than zero around a large one (normalized to the total number of large avalanches) for a system with $L = 128$ is displayed in Fig. 4. It is clear that large avalanches (the important target concerning prediction) are clustered in time following a quasiperiodic regime depending of α. This kind of periodicity have been observed previously [11–14] but mainly periodic BCs are used. It appears as a consequence of a phase locking with single avalanches, and the introduction of open BCs should affect it; nevertheless we are observing both criticality and periodicity coexisting together. Figure 5 shows that (a) indeed there is a lot of activity at the lattice's border with many small avalanches at the center, but when we consider the size of the events (b) the distribution becomes flat in the inner part with (c) avalanches that in average grow in the direction of the center of the lattice; (d) 18.6% of the large avalanches

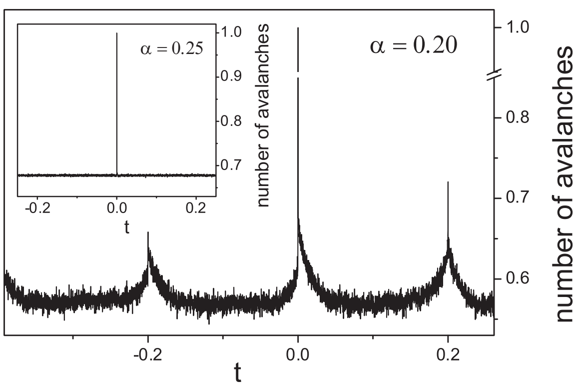

FIG. 4. Average number of avalanches in the vicinity of a large avalanche, normalized to the total number of large avalanches, in a system with $L = 128$, $\sigma = 0$, and $\alpha = 0.2$. Inset: in a system with $L = 128$, $\sigma = 0$, and $\alpha = 0.25$ (conservative case).

have trigger points in the four columns (rows) closest to the border, but beyond 10 columns (rows) away from the border the distribution is flat and 73% of the large avalanches have their trigger points there. This indicates that periodicity is not a result of a phase-locked solution with trivial (size one) avalanches in areas at the inner part of the system that are eventually destroyed (and created) due to large events triggered at the borders of the lattice [14], but can appear in a more complex scenario.

In order to construct an avalanche time series for the OFC model we need to define a scale δt to measure the time. In the finite velocity model it is natural to choose $\delta t/T_m = \delta F$; we will use the same value for the OFC one (the qualitative result is independent of δt). If more than one event took place in the same δt, the value of the avalanche size in this interval is equal to the sum of the avalanche size in all those events. The power spectrum for the avalanche time series for the OFC model with $L = 128$ and $\alpha = 0.15$ appears in Fig. 6. It displays a sharp peak at a frequency equal to 2.5 corresponding to a period equal to 0.4. The power spectrum for a finite velocity model with the same L, α, and $\sigma = 0$ shows no qualitative differences with the OFC one, but the intensity of the peaks are larger. This shows that periodicity is not confined to this particular finite velocity model but it is a general characteristic of the OFC one. Nevertheless, in this case, as $v \to 0$, the unit of time $T_m \to \infty$. The correlation function method did not bring satisfactory results due to poor statistics for small values of α (the peaks appear for the respective periods, but the background noise shows large fluctuations for small α values).

Because of the relation between dissipation and periodicity in the model we performed a few simulations where the value of α is not constant but randomly distributed

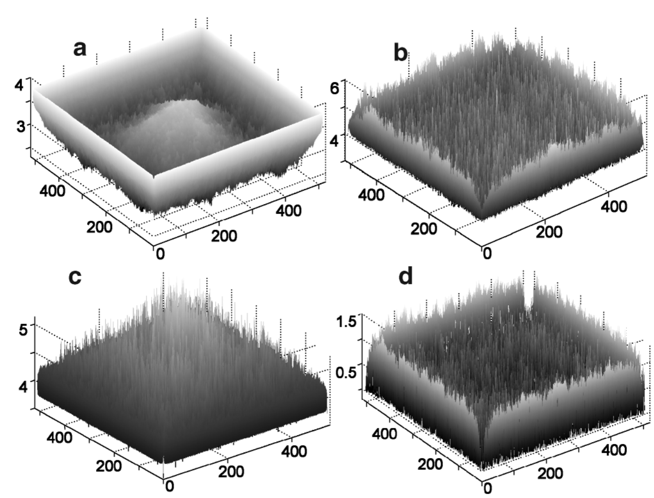

FIG. 5. Spatial distribution (in log scale) of (a) Trigger points of the clusters, (b) Trigger points $\times$ avalanche size, (c) Centers of mass of the clusters $\times$ avalanche size, (d) Trigger points of the clusters but only for large avalanches; in a system with $L = 512$, $\alpha = 0.2$, and $\sigma = 0.001$.

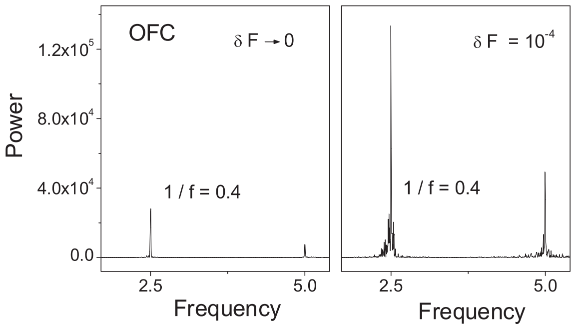

FIG. 6. Power spectrum for the avalanche time series for the OFC model (left) and for our model (right). Both with $L = 128$, $\alpha = 0.15$, and $\sigma = 0$.

following a Gaussian centered in α, with standard deviations σ_α equal to 0.005, 0.01, and 0.02. When a block slips α is changed. This dynamical disorder in α is less realistic than the quenched one [25], but the qualitative result when it is increased is the same: the rupture of the criticality. For $\alpha = 0.2$ and the σ_α values being 0.005 and 0.01, the height of the peak in the correlation function decreases more than 90% and the period widens up to 5%. The avalanche size distributions do not suffer considerable variations. Similar results were obtained for $\alpha = 0.15$. For $\sigma_\alpha = 0.02$ criticality disappears for both values of α. This corroborates that periodicity is more fragile than criticality when noise is added to the system.

Quasiperiodic signals in earthquake time series have been used in an attempt to predict the next main shock, but generally with unsuccessful results [6,7]. This situation, in combination with poor statistics and the lack of a theory that explains periodicity, has created doubts about the real existence of those series of quasiperiodic events [26]. Gao *et al.* [27] found an annual periodicity following the 1992 Landers earthquake in California, but they suggest seasonal differences in water extraction rates, rainfall and barometric pressure as the cause of it. Considering the periodicity found in this simple mapping of the block-spring model into a nonconservative cellular automata, we can speculate that the earthquake's natural behavior is a quasiperiodic state and that the variations or absence of periodicity is due to changes in the dissipative regime and/or in the relative velocity of the plates and/or in the amount of energy that can be stored in a given zone between two tectonic plates (related to our threshold that rules the unit of time).

In conclusion, we have introduced two variations in the OFC model improving resemblance with the spring-block one, and bridging the gap between the model and possible experiments. We found a nontrivial quasiperiodic behavior in the system with a period proportional to the degree of dissipation. For small variations in the thresholds or in the degree of dissipation, periodicity tends to vanish, while the system remains showing avalanche size distributions that follow power laws.

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