Non-Gaussian Low-Frequency Noise as a Source of Qubit Decoherence

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We study decoherence in a qubit with the distance between the two levels affected by random flips of bistable fluctuators. For the case of a single fluctuator we evaluate explicitly an exact expression for the phase-memory decay in the echo experiment with a resonant ac excitation. The echo signal as a function of time shows a sequence of plateaus. The position and the height of the plateaus can be used to extract the fluctuator switching rate γ and its coupling strength v. At small times the logarithm of the echo signal is $\propto t^3$. The plateaus disappear when the decoherence is induced by many fluctuators. In this case the echo signal depends on the distribution of the fluctuators parameters. According to our analysis, the results significantly deviate from those obtained in the Gaussian model as soon as $v \gtrsim \gamma$.

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Introduction.—Quantum dynamics of two-level systems has recently attracted special attention in connection with ideas of quantum information processing. The central problem regarding operation of qubits and logical gates is maintaining the phase coherence in the presence of a noisy environment [1]. At low temperatures the noise is dominated by discrete sources; it is caused by random charge exchange between localized states and electrodes in the Josephson [2] or semiconductor double quantum-dot qubits [3]. The charge fluctuations are often modeled by a set of harmonic oscillators with certain frequency spectrum [4,5]. In these "spin-boson" models the qubit decoherence is determined solely by the *pair correlation function* of random forces, $S_\chi(f)$, that implicitly assumes the noise to be Gaussian [6]. This assumption, however, does not hold in most practical systems where $S_\chi(f) \propto 1/f$ and the processes have extremely broad distribution of the relaxation times [7].

To understand the role of the non-Gaussian statistics, we follow [8] and model the environment by a set of two-state systems (fluctuators) that randomly switch between their states. Their nonequilibrium dynamics can then be taken into account explicitly as was done in the analysis of coherent quantum transport in the presence of $1/f$ noise [9]. Recent application of a similar approach to qubits demonstrated new features in the decoherence that are not reproduced in the Gaussian approximation [10]. Quantum aspects of non-Markovian kinetics were addressed in [11].

In the present Letter we extend the work [10] in two directions. First, we evaluate explicitly the phase-memory decay in the *echo* experiment. We find a pronounced non-Gaussian behavior and explain plateaus observed in the time dependence of echo signal [2]. Second, we consider the case where the interaction strengths between the qubit and fluctuators are broadly distributed. This distribution strongly modifies the time dependence and smears away the plateaus. We suggest a recipe for extracting the fluctuators' parameters from the measured echo signal.

It is worth noting that a broad distribution of fluctuators' switching rates and the coupling strengths makes the problem similar to the conventional models of the *spectral diffusion* in spin systems, structural glasses, and molecules embedded in a condensed phase [12–17].

Model.—We assume that the qubit is a two-level system (TLS) surrounded by fluctuators—systems with two locally stable states. Possible candidates for such fluctuators in solid state devices are charge traps, see Fig. 1, or structural dynamic defects. An occupied trap together with

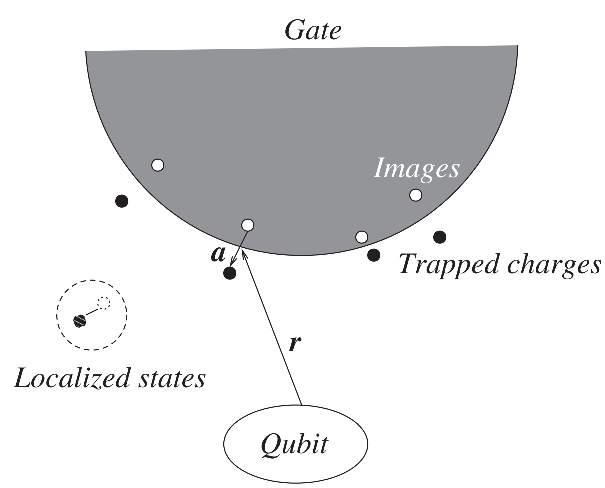

FIG. 1. Schematic distribution of charged traps located near the gate surface and producing oppositely charged images.

its charge image produces a dipole electric field fluctuating in time due to hops between the trap and the gate and acting upon the qubit. If the defects are not charged, they can behave as elastic dipoles producing time-dependent strains and interacting with the qubit via the deformational potential.

A qubit coupled to the environment will be modeled by the Hamiltonian $\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_q + \tilde{\mathcal{H}}_{\text{man}} + \tilde{\mathcal{H}}_{qF} + \tilde{\mathcal{H}}_F$, where $\tilde{\mathcal{H}}_q$ and $\tilde{\mathcal{H}}_F$ describe the qubit and the fluctuators separately. $\tilde{\mathcal{H}}_q$ is the Hamiltonian of the qubit pseudospin $\boldsymbol{\sigma}$ in a static "magnetic field," $\mathbf{B} = [B_x, B_z]$. Here B_z characterizes the splitting of the energies of the two states, and B_x describes their tunneling coupling. This Hamiltonian can be diagonalized by rotation in the pseudospin space with new z axis parallel to $\mathbf{B}$. The rotated Hamiltonian, $\mathcal{H}_q$, is then $\mathcal{H}_q = -(B/2)\sigma_z$. $\tilde{\mathcal{H}}_F$ can be diagonalized by a similar rotation in the fluctuator's pseudospin space and then split into three parts, $\mathcal{H}_F = \mathcal{H}_F^{(0)} + \mathcal{H}_{F-\text{env}} + \mathcal{H}_{\text{env}}$. The first part is just a Hamiltonian of an isolated two-level tunneling system, $\mathcal{H}_F^{(0)} = \sum_i (E_i/2)\tau_z^i$, where the Pauli matrices $\boldsymbol{\tau}^{(i)}$ correspond to ith fluctuator. The spacing between the two levels, E_i, is formed by the diagonal splitting, Δ_i, and the tunneling overlap integral, Λ_i, as

$$E_i = \sqrt{\Delta_i^2 + \Lambda_i^2} \equiv \Lambda_i/\sin\theta_i. \tag{1}$$

The interaction, $\mathcal{H}_{qF}$, between the qubit and the fluctuators is specified as [cf. with Ref. [13]]

$$\mathcal{H}_{qF} = \sum_i v_i \sigma_z \tau_z^i, \qquad v_i = u(r_i)\cos\theta_i. \tag{2}$$

Here we assumed for simplicity that the coupling strength v_i is determined only by θ_i defined by Eq. (1) and the distance r_i between the qubit and the ith fluctuator.

The interaction between the fluctuators and the environment manifests itself through time-dependent random fields applied to the qubit. The frequencies of these fields being much smaller than the temperature T and qubit splitting B, the fields can be treated *classically*: $\hat{\tau}_z^{(i)} \to \xi_i(t)$. Accordingly, $\mathcal{H}_{qF}$ is the Hamiltonian of the qubit pseudospin in a random, time-dependent magnetic field $\mathcal{X}(t)$ formed by independent contributions of surrounding fluctuators:

$$\mathcal{H}_{qF} = \mathcal{X}(t)\sigma_z, \qquad \mathcal{X}(t) = \sum_i v_i \xi_i(t). \tag{3}$$

The random functions $\xi_i(t)$ characterize the fluctuators' state: $\xi_i(t)$ instantly switches between $\pm 1/2$ at random times (random Poissonian process). Modeling the environment as a system of equilibrium bosons (phonons—electron-hole pairs), the switching rates, γ_i, can be calculated in second order perturbation theory in the fluctuator environment coupling [18,19],

$$\gamma_i = \gamma_0(T)\sin^2\theta_i. \tag{4}$$

γ_0 is thus the *maximal* fluctuator switching rate at a given temperature, T. For simplicity we assume here that the fluctuator spends on average an equal time in each state. Although justified only for $E_i \ll T$, this assumption produces correct temperature dependences [16].

The qubit is manipulated by applied ac "magnetic field," $\mathbf{F}(t) \parallel \mathbf{x}$, with frequency close to B, so that $\mathcal{H}_{\text{man}} = (1/2)F(t)\sigma_x$. The echolike manipulation allows substantial suppression of the decoherence comparing to the "free-induction" signal decay. We assume that the echo π pulse is applied at time τ and the signal detected at 2τ. For details on the echo procedure, see Ref. [2].

The external pulses are usually short enough for both relaxation and spectral diffusion during each of the pulses to be neglected. The echo decay is known to be proportional to the "phase-memory functional" [20]

$$\psi = \langle e^{i\varphi_\tau}\rangle_{\xi_i}, \qquad \varphi_\tau \equiv \int_0^{2\tau} \beta(t',\tau)\mathcal{X}(t')dt', \tag{5}$$

where $\beta(t',\tau) \equiv \text{signum}(\tau - t')$. The average is calculated over the realizations of the random processes $\xi_i(t)$ and random initial states of fluctuators. This averaging reflects the experimental procedure where the observable signal is an accumulated result of numerous repetitions of the same sequence of inputs. Equation (5) is obtained by analysis of the qubit's density matrix under the perturbation $\mathcal{H}_{\text{man}}$.

Single fluctuator.—Let us start analyzing Eq. (5) with the case of a qubit interacting with a single fluctuator. In the spirit of Ref. [16] we have obtained the exact solution for the echo signal [21],

$$\psi = \frac{e^{-2\gamma\tau}}{2\mu^2}\left[(1+\mu)e^{2\mu\gamma\tau} + (1-\mu)e^{-2\mu\gamma\tau} - \frac{v^2}{2\gamma^2}\right], \tag{6}$$

where $\mu = \sqrt{1-(v/2\gamma)^2}$. This result is essentially non-Gaussian. Indeed, assuming that the phase, φ_τ, obeys the Gaussian statistics one would get instead of Eq. (6) $\psi_G = e^{-\langle\varphi_\tau^2\rangle/2}$ with

$$\langle\varphi_\tau^2\rangle/2 = (v/4\gamma)^2(4\gamma\tau - 3 + 4e^{-2\gamma\tau} - e^{-4\gamma\tau}). \tag{7}$$

Comparing the two expressions we notice that the Gaussian result (7) is the weak coupling limit, $v \ll 2\gamma$, of the exact solution (6). However, in the strong coupling case, when $v \gtrsim 2\gamma$, the exact solution strongly differs from Eq. (7). In Fig. 2 both functions are plotted for different values of the ratio $v/2\gamma$.

One can see that at $v \geq 2\gamma$ the Gaussian assumption strongly underestimates the phase-memory functional for $\tau > 2\pi/v$. Similar conclusion for the free-induction signal has been recently obtained in Ref. [10]. The reason for the failure of the Gaussian approximation in the strong coupling case is similar to the well-known motional narrowing of spectral lines [12]. Indeed, if $v \geq 2\gamma$, then each fluctuator *splits* the qubit's levels rather than broadens them. The

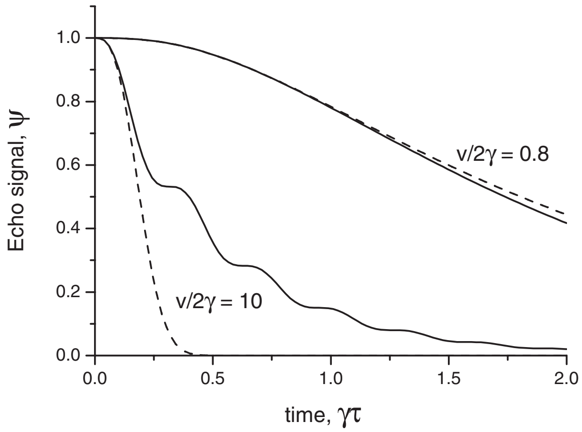

FIG. 2. Echo signal for different values of the ratio $v/2\gamma$ (shown near the curves), Eq. (6). Dashed lines—calculations along the Gaussian assumption, Eq. (7).

qubit just experiences rare hops between these states; the splitting is of the order of v and the typical hopping rate is γ. In contrast, at $v \ll \gamma$ the splitting between the levels is smeared, the typical decay rate of the echo signal being $\sim v^2/\gamma$. This limit is reproduced within the Gaussian assumption. At $\gamma\tau \ll 1$, the phase-memory functional behaves as

$$\psi \approx 1 - \gamma v^2 \tau^3/3, \tag{8}$$

regardless of the value of the ratio v/γ. This result naturally holds also in the Gaussian approximation. At $\gamma\tau \gg 1$ we find

$$-\ln\psi \approx \begin{cases} 2\gamma\tau, & v > 2\gamma; \\ v^2\tau/4\gamma, & v \ll 2\gamma. \end{cases}$$

At $v > 2\gamma$ there appear steps in the time dependence of the echo signal shown in Fig. 2. This was experimentally observed in Ref. [2] (see Fig. 3 there). At $v \gg \gamma, \sqrt{\gamma/\tau}$ Eq. (6) acquires a simple form

$$\psi = e^{-2\gamma\tau}[1 + (2\gamma/v)\sin v\tau]. \tag{9}$$

According to Eq. (9), the plateaulike features ($d\psi/d\tau \approx 0$) occur at $v\tau \approx 2k\pi$ and their heights $\psi \approx e^{-4\pi k\gamma/v}$ exponentially decay with the number k. These plateaus for $\gamma \ll v$ can be understood as follows. The probability for the fluctuator to flip during the time $2\pi/v$ is very small, hence it either does not flip at all or flips only once. If it flips during the first half of the beating period, $t < \pi/v$, the phase of the functional (5) at $2\tau = 2\pi/v$ evolves from 0 to π. If it flips during the second half of the period, $\pi/v < t < 2\pi/v$, the phase evolves from π to 2π. After averaging the two contributions will cancel each other, which implies that at τ close to $2\pi/v$ the signal ψ is almost insensitive to small variations of τ.

Measuring experimentally the position and the height of the first plateau, one can determine both the fluctuator coupling strength v, and its switching rate γ. For example, the echo signal measured in Ref. [2] shows a plateaulike feature at $\tau = 3.5$ ns at the height $\psi = 0.3$, which yields $v \approx 285$ MHz, and $\gamma \approx 27$ MHz. If the fluctuator is a charge trap near the gates producing a dipole electric field, see Fig. 1, its coupling strength is $v = e^2(\mathbf{a}\cdot\mathbf{r})/r^3$. Estimating the actual gate-qubit distance $r \approx 0.5$ μm, we obtain a reasonable estimate for the tunneling distance between the charge trap and the gate, $a \sim 20$ Å.

Many fluctuators.—What if the qubit is coupled to many fluctuators, which are not correlated: $\langle \xi_i(t)\xi_j(t')\rangle \propto \delta_{ij}$? The phase-memory functional is then a product of the partial functionals due to individual fluctuators, $\Psi = \prod_i \psi^{(i)}$. Following the Holtsmark approach [12,16,21] we approximate Ψ as

$$\Psi \approx \exp[-\sum_i (1 - \psi^{(i)})]$$

$$= \exp[-\int d\gamma dv \mathcal{P}(\gamma, v)(1 - \psi^{(i)})].$$

Our approach [21] provides accurate description of the decoherence by fluctuators with particular locations as long as the number of active fluctuators is large.

We will show now that summation over many fluctuators with a broad distribution of v_i and γ_i may significantly change the time dependence of the echo signal. Since the tunneling splitting, Λ, depends exponentially on the distance in real space between the positions of the two-state fluctuator, it is reasonable to assume that the distribution function $\mathcal{P}(\Delta, \Lambda) = \eta/\Lambda$. In terms of E and θ it implies $\mathcal{P}(E, \theta) = \eta/\sin\theta$. Here η is proportional to the number of fluctuators with $E \lesssim T$, which are not frozen at a given temperature. Hence, $\eta \propto T$. From (4) it follows that $\mathcal{P}(\gamma, v) \propto \gamma^{-1}$.

Because of existence of a finite maximal rate, γ_0, summation over fluctuators with different γ_i does not affect much the decoherence at small times. Similar to Eq. (8) we find that $-\ln\Psi \propto \tau^3$ for $\gamma_0\tau \ll 1$. This asymptotic behavior can, however, be modified due to a distribution of the coupling strengths v_i entering Eq. (2). Let us assume that $u(r) \propto r^{-2}$, which is the case if the fluctuators act as electric dipoles. A uniform distribution of the fluctuators over a two-dimensional gate surface corresponds to the distribution of the coupling constants $\mathcal{P}(u) \propto u^{-2}$, which coincides with the distribution of the coupling constants in glasses, where two-level systems interact via dipole-dipole interaction [13]. The phase-memory functional can be evaluated by integrating the echo signal ψ, (6), where γ and v are related to u and θ by Eqs. (2) and (4),

$$\Psi(\tau) = \exp\left[-u^* \int_0^\infty \frac{du}{u^2} \int_0^{\pi/2} \frac{[1 - \psi(u, \theta, \tau)]d\theta}{\sin\theta}\right].$$

Here u^* is the value of $u(r)$ taken at the average distance between the fluctuators having energy splitting $\lesssim T$. The integration over u can be extended down to zero since $(1 - \psi) \propto u^2$ at $u \to 0$. The combination $\mathcal{L} \equiv -(\gamma_0/u^*)\ln\Psi$ turns out to be a universal function of the product $\gamma_0\tau$. At small time $\mathcal{L} \approx (\pi/4)(\gamma_0\tau)^2$. The stronger time dependence compared to the single-fluctuator case, Eq. (8), is due to strongly coupled fluctuators with large v. In the general case, when $u(r) \propto r^{-b}$, and the fluctuators are uniformly distributed over an area of dimension d, one finds $\mathcal{L} \propto \tau^{1+d/b}$ for $\gamma_0\tau \ll 1$. Comparing the short-time experimental dependence with this prediction one can estimate the ratio d/b, i.e., extract information on the spatial distribution of fluctuators and mechanism of their interaction with the qubit. We observe that for any long-range interaction the dependence of $\ln\Psi(\tau)$ is superlinear. Consequently, one cannot introduce T_2^{-1} as a decrement of exponential decay of the echo signal, and the dephasing time can be estimated only from the requirement $-\ln\Psi(\tau) \approx 1$. At $\gamma_0\tau \gg 1$ one obtains $\mathcal{L} \propto \tau$ [21]. Thus, for long times the dephasing time T_2 can be introduced in the conventional way. One should also keep in mind that energy relaxation described by the characteristic time T_1 provides an additional exponential echo decay as $\propto e^{-2\tau/T_1}$; see [22]. This decay is usually small in the case of longitudinal noise that we discuss, but can be added if necessary.

In reality, the distribution over u should be cut off at some $u_{\max}$ corresponding to the minimal possible distance between the fluctuator and the gate. Our selection of the relevant fluctuators (see above) is valid provided that $u_{\max}/\gamma_0 \gg 1$.

It is instructive to compare the dephasing induced by the fluctuators with their contribution to the noise spectrum, $S(\omega) \propto \int d\gamma dv \mathcal{P}(\gamma, v)v^2/(\omega^2 + \gamma^2)$. The distribution $\mathcal{P}(\gamma, v) \propto \gamma^{-1}$ results in the $1/\omega$ spectrum. Thus many fluctuators with a broad distribution of relaxation rates produce $1/f$ noise. However, since the v dependence of the quantity $(1 - \psi)$ generally significantly differs from v^2 the phase-memory decay and $1/f$ noise are dominated by *different* groups of fluctuators; see [21] for details. Consequently, in the general case the fluctuator-induced decoherence *cannot* be expressed through the noise spectrum. This is in contrast to the statements that often appear in the literature. Non-Gaussian effects due to a single or many fluctuators may be responsible for the experimentally observed decoherence [2,23].

In conclusion, the presented results can be used to probe the sources of qubit decoherence by measuring time dependence of the echo signal. Plateaus in this dependence signal that a major contribution comes from a *single* fluctuator, the parameters of which can be extracted from the position and height of the plateaus using Eq. (9). A smooth time dependence of $\ln\Psi$ suggests a combined effect of many fluctuators. These conclusions are qualitatively correct also for the "free-induction" signal.

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