Acoustically Induced Transparency in Optically Dense Resonance Medium

Y. V. Radeonychev,[1] M. D. Tokman,[1] A. G. Litvak,[1] and Olga Kocharovskaya[1,2]

[1]*Institute of Applied Physics, Russian Academy of Science, 46 Ulyanov Street, 603950 Nizhny Novgorod, Russia*

[2]*Department of Physics, Texas A&M University, College Station, Texas 77843, USA*

(Received 10 September 2002; published 7 March 2006)

It is shown that mechanical vibration (acoustical oscillation) of a solid medium along the propagation of multifrequency laser radiation enables one to control the resonant absorption. There exists an optimal spectral structure of the incident field dependent on vibration amplitude as well as the number and intensity of the frequency components that provides the full resonant transparency. A mechanism of the transparency is discussed. Transparency of this kind is shown to appear also via adiabatic modulation of the atomic transition frequency by an external microwave field.

DOI: 10.1103/PhysRevLett.96.093602 PACS numbers: 42.50.−p, 42.62.−b, 42.65.−k, 42.79.Jq

Electromagnetically induced transparency (EIT) and other effects of quantum coherence have been widely studied ([1], and references there). Recent investigations showed that suppression of the resonant absorption is also possible in classical systems due to parametric interactions of radiation with collective oscillations of a medium [2]. In this Letter we discuss a hybrid case of the resonant transparency in a two-level *quantum* system induced via excitation of *collective* mechanical motion of atoms, namely, mechanical (acoustical) harmonic vibration. We show that transparency of this kind appears also due to adiabatic harmonic modulation of the atomic transition frequency via the Zeeman or Stark effects. Atomic vibration and modulation of the transition frequency play a role similar to the quantum coherence in EIT but do not require the third resonant level.

It is well known that atomic mechanical vibration leads to the Doppler modulation, which converts any monochromatic wave into a comblike spectrum in the reference frame of the vibrating system. This technique is used in Mossbauer gamma-ray spectroscopy to get multipeak absorption or fluorescence spectral contour of nuclei [3]. Reduction of the resonant absorption and frequency transformation of quasimonochromatic gamma radiation into a polychromatic one inside a thick target vibrated at ultrasonic frequency was studied theoretically and experimentally [4]. It was interpreted via the formation of intermediate excited state of a nuclei and multiple nuclear Raman scattering [4]. The effects similar to Mossbauer spectroscopy [3] were obtained in the optical range via Stark modulation of atomic energy levels by a microwave field in hydrogen plasma [5]. This is worth noting that acousto-optical modulation is the common technique to produce sidebands of coherent fields in optically transparent media. It should also be mentioned that the Raman technique enables one to generate multifrequency optical radiation up to extremely wide range and extremely short pulses [6]. In these two cases, contrary to the case under consideration, all the radiation components are far from optical resonance. Therefore the effect of induced transparency does not take place. In the well-known case of self-induced transparency, one deals with transient effect, whereas the present case refers to a steady-state regime.

In this Letter we study the destructive interference of resonance excitations of vibrating (or frequency modulated) two-level atoms and predict a number of effects: the possibility of the full resonant transparency for different vibration amplitude and spectral structures of an incident laser radiation, control of the resonant transparency by a nonresonant drive, and generation of a burstlike radiation.

Let us consider propagation of a multifrequency coherent radiation,

$$\mathbf{E} = \frac{1}{2}\sum_{L=-\infty}^{\infty} \vec{\mathcal{E}}_L e^{-i(\omega_L t - k_L z)} + \text{c.c.},$$
$$\omega_L = \omega_0 + L\Omega_D, \qquad k_L = n_L\omega_L/c_L \tag{1}$$

(where n_L and c_L are the refractive index and the phase velocity at the frequency ω_L) through a medium of two-level atoms. Let the frequency ω_0 be resonant to the frequency of atomic transition, and the frequency interval Ω_D much exceeds the transition linewidth. If atoms do not move, the only effect is resonant absorption of radiation at the frequency ω_0.

Now let atoms oscillate along the axis z such that their positions, z, are

$$z(t) = z_a + R\sin\Omega t. \tag{2}$$

Equation (2) implies that the respective acoustical wavelength is much larger than the length of a medium sample.

The moving atomic reference frame and the fixed laboratory one are different. We consider the density matrix equation for atomic coherence $\rho_{21}(z_a, t)$ in the laboratory reference frame taking this difference into account by the well-known way [7]. Then, within the rotating wave approximation, we find

$$\frac{\partial\rho_{21}}{\partial t} + v(t)\frac{\partial\rho_{21}}{\partial z} + i\omega_{21}\rho_{21} - in_{12}\sum_L \alpha_L e^{-i(\omega_L t - k_L z)} = -\gamma_{21}\rho_{21}, \tag{3}$$

where ω_{21} is the atomic frequency, n_{12} is the level popu-

lation difference, $\alpha_L = \vec{\mu}_{21}\vec{\mathcal{E}}_L/(2\hbar)$ is the Rabi frequency of the component $\vec{\mathcal{E}}_L$ ($\vec{\mu}_{21}$ is dipole moment of atomic transition), γ_{21} is the coherence relaxation rate, and

$$v(t) = \partial z/\partial t = R\Omega\cos\Omega t. \tag{4}$$

We consider the approximation $k_L = k$, which for the case $n_L = 1$, $c_L = c_0$ means that the length h_s of a medium sample is much smaller than the beat wavelength of any frequency components k_n and k_m under consideration:

$$h_s \ll \frac{\pi}{|k_n - k_m|}. \tag{5}$$

The integration of Eq. (3) under $k_L = k$ gives in the slowly varying amplitude approximation (valid for $\gamma_{21} \gg 1/T_L$, c_L/h_L, where T_L and h_L are temporal and spatial scales of amplitude changing)

$$\rho_{21} = in_{12}e^{ikz}\sum_{n,m=-\infty}^{\infty}\sum_L \varkappa_L^{(n)} J_n J_{n+m}\alpha_L e^{-it(\omega_L+m\Omega)}, \tag{6}$$

where $J_n = J_n(P)$ is the Bessel function of nth order, $P = kR$ is modulation index, and

$$\varkappa_L^{(n)} = \frac{\gamma_{21} - i(\omega_{21} - \omega_L + n\Omega)}{\gamma_{21}^2 + (\omega_{21} - \omega_L + n\Omega)^2}. \tag{7}$$

Equation (6) shows that radiation with *any* frequency ω_L has a comblike spectrum in the reference frame of vibrating atoms. If the frequency difference $\omega_L - \omega_0$ is a multiple to the frequency of atomic vibration ($\omega_L + m\Omega = \omega_0$), this radiation contributes to the polarization ρ_{21} at the resonance frequency ω_0 via corresponding spectral component having amplitude ($\varkappa_L^{(n)} J_n J_{n+m}\alpha_L$). In other words, the nonresonant field becomes partially resonant to the atomic transition after vibration is "turned on." Moreover, since the fields of different frequencies are mutually coherent, their resonant contributions interfere. To our knowledge, this kind of interference has not yet been discussed.

We further consider that frequency ω_0 is resonant to atomic transition ($\omega_{21} - \omega_0 \lesssim \gamma_{21}$) and that the frequency interval Ω_D coincides with the frequency of atomic vibration, $\Omega_D = \Omega$. In this case the expression (6) can be represented in the form of multifrequency response similar to (1),

$$\rho_{21} = \sum_{p=-\infty}^{\infty}\sigma_{21}^{(p)}e^{-i(\omega_p t - kz)}, \tag{8}$$

where

$$\sigma_{21}^{(p)} = in_{12}\varkappa_0 J_p\sum_{L=-\infty}^{\infty}(J_L + \Gamma_{pL})\alpha_L. \tag{9}$$

In Eq. (9) the first term in parentheses is responsible for the resonant absorption of all the field components, while the term Γ_{pL} takes into account the absorption at the tails of the homogeneously broadened transition line,

$$\Gamma_{pL} = \frac{1}{\varkappa_0 J_p}\sum_{k\neq 0}\varkappa_k J_{p+k}J_{L+k},$$
$$\varkappa_p = \frac{\gamma_{21} - i(\omega_{21} - \omega_0 + p\Omega)}{\gamma_{21}^2 + (\omega_{21} - \omega_0 + p\Omega)^2}, \qquad p = 0, k. \tag{10}$$

In the case where the frequency of atomic vibration Ω exceeds the transition linewidth γ_{21} ($\Omega > \gamma_{21}$) the expression (10) can be expanded as a series in powers of (γ_{21}/Ω),

$$\Gamma_{pL} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{J_p}\left(\frac{\gamma_{21}}{\Omega}\right)^{2n}\sum_{k\neq 0}\frac{J_{p+k}J_{L+k}}{k^{2n}}. \tag{11}$$

Substitution of (8) and (9) into the wave equations for the field components α_p (in the slowly varying amplitude approximation) gives a self-consistent homogeneous system:

$$\frac{\partial\alpha_p}{\partial z} + \frac{1}{c}\frac{\partial\alpha_p}{\partial t} = -\tilde{g}\omega_p J_p\sum_{L=-\infty}^{\infty}(J_L + \Gamma_{pL})\alpha_L, \tag{12}$$

where $\tilde{g} = \frac{2\pi N}{c\varepsilon\hbar}|\mu_{21}|^2 n_{12}\varkappa_0$, N is atom density, $c = c_0/\sqrt{\varepsilon}$ is light velocity in a medium, and ε is dielectric permittivity, $\omega_p = \omega_0 + p\Omega$. For the case of steady-state propagation ($\partial\alpha_p/\partial t = 0$), the search for the solution of the set (12) in the form $\alpha_p \sim \exp(-\lambda z)$ leads to an infinite set of homogeneous algebraic equations:

$$\sum_{L=-\infty, L\neq p}^{\infty}(J_L + \Gamma_{pL})\alpha_L + \left(J_p + \Gamma_{pp} - \frac{\lambda}{\tilde{g}\omega_p J_p}\right)\alpha_p = 0. \tag{13}$$

A general solution of (13) can be represented as a sum of a multifrequency strongly absorbed mode, $\mathbf{E}^{(0)}$, and weakly absorbed modes, $\mathbf{E}^{(m)}$,

$$\mathbf{E} = \mathbf{E}^{(0)}e^{-g_0 z} + \sum_{m=1}^{\infty}\mathbf{E}^{(m)}e^{-g_m z}, \tag{14}$$

where $\mathbf{E}^{(p)} = \frac{1}{2}\sum_{k=-\infty}^{\infty}\mathcal{E}_k^{(p)}e^{-i(\omega_k t - kz)} + \text{c.c.}$, $p = 0, m$, and $g_0 = \frac{2\pi N}{c\varepsilon\hbar}\omega_0|\mu_{21}|^2 n_{12}\varkappa_0$ is the well-known coefficient of the resonant absorption without vibration. It is worth noting that a similar appearance of strongly and weakly absorbed modes can occur in rather different physical conditions, for instance in the four-wave mixing effect [8]. For the case (11) one can write $g_m = g_0(\gamma_{21}/\Omega)^2 F_m$, where the functions F_m (not presented here due to their length) are expressed via Bessel functions of all frequency components. They are bounded, $0 < F_m < 1$.

Let us consider the case where the absorption line is narrow as compared to the vibration frequency, $\gamma_{21}/\Omega \ll 1$, such that $g_0 h_s(\gamma_{21}/\Omega)^2 \ll 1$. This implies $g_m h_s \ll 1$ for all $m > 0$, and this means induced transparency for all the modes $\sum_{m=1}^{\infty}\mathbf{E}^{(m)} = \mathbf{E}_T$. The solution (14) takes the form

$$\mathbf{E} = \mathbf{E}^{(0)}e^{-g_0 z} + \mathbf{E}_T. \tag{15}$$

For the incident radiation, $\mathbf{E}_{\text{in}} = \frac{1}{2}\sum_{L=-\infty}^{\infty}E_L e^{-i(\omega_L t - kz)} + \text{c.c.}$, of the form (1), where $\mathcal{E}_L(z=0) = E_L$, one obtains

$$\mathbf{E}^{(0)} = \frac{1}{2}\sum_{p=-\infty}^{\infty}\left\{J_p\frac{\omega_p}{\omega_0}\sum_{L=-\infty}^{\infty}J_L E_L\right\}e^{-i(\omega_p t-kz)} + \text{c.c.}, \tag{16}$$

$$\mathbf{E_T} = \frac{1}{2}\sum_{p=-\infty}^{\infty}\left\{E_p - J_p\frac{\omega_p}{\omega_0}\sum_{L=-\infty}^{\infty}J_L E_L\right\}e^{-i(\omega_p t-kz)} + \text{c.c.} = \mathbf{E}_{\text{in}} - \mathbf{E}^{(0)}. \tag{17}$$

The solution (15)–(17) shows that for any amplitude of the vibration and any initial energy distribution among the frequency components of the incident laser field (1) the field tends, due to resonant absorption, to a specific spectral structure (17) determined by the initial conditions and amplitude of vibration, which does not experience resonant absorption any more. We call this structure the ''transparent mode.'' It is formed as a result of superposition of the comblike spectra of different components $\mathcal{E}_L$ in the reference frame of vibrating system. The spectrum of each field component has a resonant part, $J_L\mathcal{E}_L$, that causes its absorption. Only such components survive, resonant parts of which are antiphased to $\mathcal{E}_0$. In transparent mode they compensate the resonant part of $\mathcal{E}_0$,

$$J_0\mathcal{E}_0(g_0 z \gg 1) = -\sum_{L=-\infty, L\neq 0}^{\infty} J_L\mathcal{E}_L, \tag{18}$$

such that the total amplitude of the resulting resonant part in the reference frame of vibrating atoms is equal to zero. In other words, the transparent mode exists due to destructive interference of its different spectral components at the frequency of atomic resonance.

It follows from (17) that if a multifrequency incident field $\mathbf{E}_{\text{in}} = \frac{1}{2}\sum_{L=-A}^{B} E_L e^{-i(\omega_L t-kz)} + \text{c.c.}$ (A and B are arbitrary integer numbers) has an optimal spectral structure, such that $E_0 = -\sum_{L\neq 0} J_L E_L/J_0$, it does not experience resonant absorption, $\mathbf{E}_{\text{in}} = \mathbf{E_T}$. It is worth noting that this optimal structure implies an arbitrary relation between amplitudes of the spectral components of the input field.

In particular case of a bichromatic input, $\mathbf{E}_{\text{in}} = \frac{1}{2}\times(E_0 e^{-i(\omega_0 t-kz)} + E_{-1}e^{-i(\omega_{-1}t-kz)}) + \text{c.c.}$, the nonresonant field cancels the resonant absorption as soon as $E_{-1} = \frac{J_0}{J_1}E_0$. In this case the input field $\mathbf{E}_{\text{in}}$ is the optimal spectral structure and experiences no changes. As follows from (17) if $E_{-1} < \frac{J_0}{J_1}E_0$, the resonant field is absorbed. An increase of E_{-1} leads to a decrease of the resonant absorption. In the case $E_{-1} > \frac{J_0}{J_1}E_0$, amplification of the resonant field takes place. This provides a method of control of the resonant absorption by the nonresonant drive via vibrationally induced transparency.

If the input field spectral structure is not optimal, amplification of some components as well as absorption of others take place. Besides, as seen from (17), the appearance of additional components $\omega_p = \omega_0 + p\Omega$ occurs. In particular, even in the case of only a nonresonant monochromatic input (E_{-1}, for instance) the resonant radiation, $\mathcal{E}_0 = \frac{1}{2}J_0 J_1 E_{-1}e^{-i(\omega_0 t-kz)} + \text{c.c.}$, is ''generated'' among all other frequencies. Indeed, as follows from (17), $\mathbf{E_T} = \frac{1}{2}E_{-1}(1 - \frac{\omega_{-1}}{\omega_0}J_1^2)e^{-i(\omega_{-1}t-kz)} + \frac{1}{2}\sum_{p\neq -1}\frac{\omega_p}{\omega_0}J_p J_1 E_{-1}\times e^{-i(\omega_p t-kz)} + \text{c.c.}$. Since the input is monochromatic, there is no interference but only absorption of the resonant component of the comblike spectrum in the atomic reference frame. In the laboratory reference frame, such a comblike spectrum with removed resonant component is equivalent to $\mathbf{E_T}$, which means (i) partial absorption of the nonresonant input and (ii) spectrum generation.

Let us consider absorption in vibrating medium in the case of the resonant monochromatic incident field, E_0. Similar to the above paragraph, according to (17), one has $\mathbf{E_T} = \frac{1}{2}E_0(1 - J_0^2)e^{-i(\omega_0 t-kz)} - \frac{1}{2}\sum_{p\neq 0}\frac{\omega_p}{\omega_0}\times J_p J_0 E_0 e^{-i(\omega_p t-kz)} + \text{c.c.}$. This means (i) the appearance of partial (by the factor $1 - J_0^2$) resonant transparency for any amplitude of the atomic vibration and (ii) generation of nonresonant satellites. Experimental study of this regime in nuclear resonant scattering of γ radiation and theoretical interpretation via multiple Raman scattering was done in [4]. It is interesting to note that in this case the field $\mathbf{E_T}$ can be written in the form of amplitude-frequency modulated wave,

$$\mathbf{E}_{\mathbf{T}} = \tfrac{1}{2}E_0 e^{-i(\omega_0 t-kz)}\{1 - J_0 e^{-iP\sin\Omega t}\} + \text{c.c.}, \tag{19}$$

where frequency modulation index $P = kR$. The intensity averaged over the period $2\pi/\omega_0$ normalized to the input intensity has the form

$$\frac{\langle I\rangle_\omega}{\langle I_{\text{in}}\rangle_\omega} = 1 + J_0^2(P) - 2J_0(P)\cos[P\sin(\Omega t)]. \tag{20}$$

When $P > P^* \approx 1.239$ [where $J_0(P^*) = 2\cos P^*$] it can periodically (with vibration period Ω) exceed the incident intensity. When $J_0(P)$ takes the maximal negative value, the $\langle I\rangle_\omega$ has a burstlike time dependence (Fig. 1). In addition, frequency modulation enables one to compress the field into short pulses [9]. At the same time, the intensity averaged over the period $2\pi/\Omega$ can never exceed the input intensity, $\langle I\rangle_\Omega/\langle I_{\text{in}}\rangle_\Omega = 1 - J_0^2(P)$, due to reso-

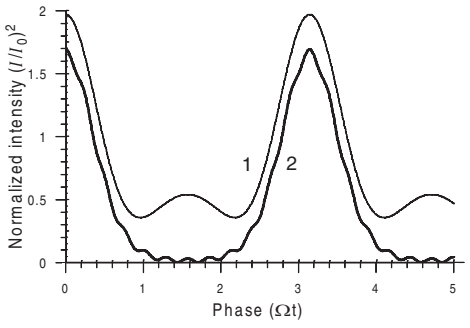

FIG. 1. Intensity of the transparent mode averaged over $2\pi/\omega_0$ for the case $\gamma_{21}/\Omega \ll 1$ [according to (20)] for $P = 3.83$ (curve 1), and for the case $\gamma_{21}/\Omega = 10$ [according to (13)] for $P = 16$ (curve 2).

nant absorption. We would like to note that if the condition $\gamma_{21}/\Omega \ll 1$, used above, is not fulfilled, one should take into account absorption at the frequencies $\omega_0 + n\Omega$, $n = \pm 1, \pm 2, \ldots, N$ (this will be discussed in more detail elsewhere). According to the general solution (13), in this case the burstlike regime becomes more pronounced and the burst duration reduces with an increase of the linewidth (Fig. 1).

Doppler modulation of the radiation frequency relative to the atomic transition frequency in the case of the vibration is equivalent to the modulation of the atomic transition frequency relative to the frequency of the incident radiation. Such modulation can be provided by an external microwave field via Zeeman or Stark effects. In the case of an adiabatic harmonic modulation the density matrix Eq. (3) takes on the following form:

$$\frac{\partial \rho_{21}}{\partial t} + i\{\omega_{21} + \Delta\cos(\Omega t)\}\rho_{21} - in_{12}\sum_L \alpha_L e^{-i(\omega_L t - k_L z)} = -\gamma_{21}\rho_{21}. \quad (21)$$

Its solution and all the effects are the same as discussed above. The only difference is the modulation index $P = \Delta/\Omega$ instead of $P = kR$ in (6)–(20).

The described model of transparency either via vibration or modulation of atomic transition frequency can be realized in solid films, gas, or plasma. It can be interesting for the gamma-ray range. For instance, as was experimentally shown in [4], quasimonochromatic gamma radiation $\hbar\omega = 14.4$ keV resonant to the nuclear transition of ^{57}Fe (total linewidth $\sim$5 MHz) after propagating through a film vibrating with frequency $\Omega/2\pi = 23.79$ MHz and amplitude up to $R \sim 0.05$ nm can generate up to 9 spectral components. If one takes a sample with linewidth $\sim$200 MHz, vibration amplitude $R \sim 0.25$ nm, and the same value of the vibration frequency, then the γ ray bursts similar to curve 2 in Fig. 1 with duration $\sim$1 ps and maximal intensity $\sim$1.75 with respect to the input intensity can be obtained. Realization of such experiments with microwave drive instead of vibration could extend possible regimes of transparency.

Modulation of the atomic transition frequency can be experimentally studied at the Balmer line of hydrogen. Irradiation of a hydrogen plasma cell by a magnetron microwave field, $\lambda \sim 3$ cm (10 GHz) and intensity up to 10 kW/cm^2, provides the modulation index of H_α line up to $P \sim 3$ via the linear Stark effect. This corresponds to generation of eight spectral components with their total width up to 80 GHz and intensity time dependence similar to curve 1 in Fig. 1.

Another experimental demonstration of transparency via modulation of atomic transition could be the control of resonant field absorption by nonresonant radiation. For example, radiation of a dye laser detuned to 10 GHz from the H_α line provides cancellation of resonant absorption for the phase matched radiation of the same intensity in the presence of 10 GHz microwave field with intensity $\sim$5 kW/cm^2.

We showed that the vibration of the two-level medium with arbitrary amplitude can produce partial or full transparency as well as amplification at the resonant atomic transition depending on spectral structure of the input radiation. There exists the optimal spectral structure of the incident field providing the full resonant transparency, which depends on a number and intensity of the frequency components as well as vibration amplitude. If an input spectral structure is not optimal, amplification and absorption of different input components, including the resonant one, as well as appearance of additional spectral components takes place. The possibility of control of the resonant absorption is shown by the example of a bichromatic input. Appearance of the spectral components is shown by the examples of nonresonant and resonant monochromatic input. In the latter case, appearance of amplitude-frequency modulated wave and possibility of a burstlike regime are shown. The most clear interpretation of the effects is Doppler modification of the spectrum of *any* radiation interacting with vibrating atoms, absorption, and destructive interference of different (parametrically coupled via vibration) spectral components of radiation at the frequency of atomic resonance in the atomic reference frame. The same effect is shown to appear in the case of adiabatic modulation of the atomic transition frequency by an external microwave field via Stark or Zeeman mechanisms.

The authors thank Yury Rostotsev and Leonid Yatsenko for fruitful discussions. This research was supported in part by RFBR (Grant No. 04-02-17042), ISTC (Grant No. A-1095), AFOSR, and NSF.

[1] J. J. Longdell *et al.*, Phys. Rev. Lett. **95**, 063601 (2005); Y. Wu and X. Yang, Phys. Rev. A **71**, 053806 (2005); L. Deng *et al.*, Appl. Phys. Lett. **81**, 1168 (2002); A. S. Zibrov *et al.*, Phys. Rev. Lett. **88**, 103601 (2002).

[2] A. G. Litvak and M. D. Tokman, Phys. Rev. Lett. **88**, 095003 (2002); G. Shvets and J. S. Wurtele, Phys. Rev. Lett. **89**, 115003 (2002).

[3] S. L. Ruby and D. I. Bolef, Phys. Rev. Lett. **5**, 5 (1960); I. Tittonen *et al.*, Phys. Rev. Lett. **69**, 2815 (1992); F. G. Vagizov *et al.*, Hyperfine Interact. **116**, 91 (1998).

[4] Yu. V. Shvyd'ko and G. V. Smirnov, J. Phys. Condens. Matter **4**, 2663 (1992).

[5] R. A. Akhmedzhanov *et al.*, JETP Lett. **41**, 384 (1985); I. N. Polushkin *et al.*, Sov. Phys. JETP **62**, 953 (1985); R. A. Akhmedzhanov *et al.*, Sov. Phys. JETP **63**, 30 (1986).

[6] A. V. Sokolov *et al.*, Phys. Rev. Lett. **87**, 033402 (2001).

[7] B. J. Feldman and M. S. Feld, Phys. Rev. A **1**, 1375 (1970).

[8] Ying Wu and X. Yang, Phys. Rev. A **70**, 053818 (2004).

[9] E. Kuznetsova, R. Kolesov, and O. Kocharovskaya, Phys. Rev. A **68**, 043825 (2003).