Full Counting Statistics for a Single-Electron Transistor: Nonequilibrium Effects at Intermediate Conductance

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We evaluate the current distribution for a single-electron transistor with intermediate strength tunnel conductance. Using the Schwinger-Keldysh approach and the drone (Majorana) fermion representation, we account for the renormalization of system parameters. Nonequilibrium effects induce a lifetime broadening of the charge-state levels, which suppress large current fluctuations.

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The "full counting statistics" (FCS) of charge transport has proven to be a powerful tool in the description of current fluctuations [1]. The concept had been explored by Levitov and Lesovik [2], who expressed the FCS of an arbitrary mesoscopic structure with noninteracting electrons in terms of its S matrix. Much less is known about the FCS of interacting mesoscopic systems, a problem which has been addressed only recently [3–7].

As a fundamental example of interacting mesoscopic systems, we consider a single-electron transistor (SET). It consists of a metallic island coupled to source and drain (left and right) electrodes via low-capacitance tunnel junctions, with resistances R_L and R_R, as well as to a gate electrode. The strength of the Coulomb interaction is characterized by the charging energy $E_C = e^2/2C_\Sigma$, which depends on the total capacitance between the island and the electrodes, $C_\Sigma = C_L + C_R + C_G$. A measure for the tunneling strength is the dimensionless parameter $\alpha_0 = (R_L + R_R)/2\pi e^2 R_L R_R$ (we put $\hbar = k_B = 1$).

In Refs. [3,4], the FCS of a similar system—a quantum dot—has been studied, fully accounting for strong electron correlations, however, only for a particular setup and parameters, corresponding to the Toulouse point. A renormalization group (RG) approach had been developed for the regime $\alpha_0 \gg 1$ [5]. In the opposite limit $\alpha_0 \to 0$, the FCS has been analyzed to lowest order in tunneling in Ref. [6] and next-to-lowest order (cotunneling) in Ref. [7]. The intermediate conductance regime $\alpha_0 \lesssim 1$ has not been covered before. It is particularly interesting since it provides the unique opportunity to realize the nonequilibrium multichannel Kondo physics [8]. The aim of this Letter is to derive the FCS for a SET in this regime.

Let us further specify the situation to be considered. At low transport voltages and temperatures, $eV, T \ll E_C$, due to a Coulomb blockade, tunneling is suppressed in a SET, everywhere except near specific values of the gate voltage, e.g., near $Q_G \equiv C_G V_G = e/2$. In the neighborhood of this conductance peak, the Coulomb barrier is $\Delta_0 = E_C(1 - 2Q_G/e)$. For $\alpha_0 \ll 1$, electrons tunnel via the island sequentially only when $\mu_R < \Delta_0 < \mu_L$, where $\mu_{L/R} = \kappa_{L/R} eV$ is the voltage drop between the L/R electrode and the island, and $\kappa_{L/R} = \pm C_{R/L}(C_L + C_R)^{-1}$. With increasing α_0, higher order effects such as cotunneling and quantum fluctuations of the charge gain importance [9]. They lead to a renormalization of Δ_0 and α_0. The perturbative RG analysis [8] (for $eV = 0$) predicts a renormalization factor $z_0 = 1/\{1 + 2\alpha_0 \ln(E_C/\Lambda)\}$ to depend logarithmically on the cutoff energy $\Lambda = \max\{\Delta_0, T\}$.

The model.—We concentrate on the tunneling regime with inverse RC time $1/R_T C_\Sigma = 4\pi\alpha_0 E_C$ smaller than E_C, which ensures that the charge-state levels are well resolved. In the vicinity of the conductance peak, precisely for $|\Delta_0|/E_C \ll 1$, it is sufficient to restrict attention to two charge states with charges differing by e. The Hamiltonian can then be mapped onto the "multichannel anisotropic Kondo model" [8]. Introducing a spin-1/2 operator $\hat{\sigma}$ acting on the charge states, we write

$$\hat{H} = \sum_{r=L,R,I} \sum_{kn} \varepsilon_{rk} \hat{a}^\dagger_{rkn} \hat{a}_{rkn} + \frac{1}{2}\Delta_0 \hat{\sigma}_z + \sum_{r=L,R} \sum_{kk'n} (T_r \hat{a}^\dagger_{Ikn} \hat{a}_{rk'n} \hat{\sigma}_+ + \text{H.c.}). \quad (1)$$

Here $\hat{a}^\dagger_{rkn}$ creates an electron with wave vector k and channel index (including spin) n in the left or right electrode or island ($r = L, R, I$). Tunneling matrix elements T_r are assumed to be independent of k and n. The junction conductances are $1/R_r = 2\pi e^2 N_{\rm ch} |T_r|^2 \rho_I \rho_r$, with $N_{\rm ch}$ being the number of channels and ρ_r the electron density of states. We assume that energy and spin relaxation times are fast, and electrons obey Fermi distribution.

A convenient tool to treat the spin-1/2 operators in Eq. (1) is the "drone" (Majorana) fermion representation [10] $\hat{\sigma}_+ = \hat{c}^\dagger \hat{\phi}$, $\hat{\sigma}_z = 2\hat{c}^\dagger \hat{c} - 1$, where $\hat{\phi} = \hat{d}^\dagger + \hat{d}$ is a Majorana fermion and $\hat{c}$ and $\hat{d}$ are Dirac fermions. This formulation enables one to apply Wick's theorem and the fermionic Schwinger-Keldysh approach [11,12].

Cumulant generating functional.—The central object of our approach is the generating functional of connected Green's functions (GFs)

$$W[\varphi] \equiv -i \ln \int D[a^*_{rkn}, a_{rkn}, c^*, c, d^*, d] e^{i \int_C dt \mathcal{L}(t)}. \quad (2)$$

Here $\mathcal{L}$ is the Lagrangian corresponding to $\hat{H}$ (1), and fermion operators switched to Grassmann variables for the functional integral (see Ref. [13] for details). The closed time path C (Keldysh contour) runs from $t = -\infty$ to ∞, back to $-\infty$, and connects to the imaginary time path to end at $t = -\infty - i/T$. We introduce auxiliary source fields, the phase of the tunneling matrix element $T_r \to T_r e^{i\kappa_r \varphi(t)}$, distinguishing between forward and backward time paths $\varphi_+(t)$ and $\varphi_-(t)$. The "center-of-mass" variable $\varphi_c(t) \equiv \{\varphi_+(t) + \varphi_-(t)\}/2 = eVt$ is then fixed by the transport voltage.

From the cumulant generating functional (CGF), one finds the number of transmitted electrons q during the measurement time t_0, $\mathcal{W}(\lambda) = \sum_{n=1}^{\infty} \langle\!\langle \delta q^n \rangle\!\rangle (i\lambda)^n/n!$. Following Ref. [12], it is derived from Eq. (2) by fixing during the measurement the "counting field" (relative variable) $\varphi_\Delta(t) \equiv \varphi_+(t) - \varphi_-(t)$ at a constant value λ:

$$\mathcal{W}(\lambda) = iW[\varphi]|_{\varphi_c(t)=eVt, \varphi_\Delta(t)=\lambda\theta(t_0/2+t)\theta(t_0/2-t)}. \quad (3)$$

The distribution of q (or, equivalently, of the current $I \equiv eq/t_0$) is given by the inverse Fourier transformation

$$P = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\lambda e^{\mathcal{W}(\lambda) - iq\lambda} \approx e^{\mathcal{W}(\lambda^*) - i(t_0 I/e)\lambda^*}. \quad (4)$$

The integral can be evaluated in saddle point approximation, with λ^* following from the relation $I = -ie\partial_\lambda \mathcal{W}(\lambda^*)/t_0$ [6]. This approach is valid for long measurement times since $\mathcal{W}$ is proportional to t_0 (see below).

We proceed following Ref. [13], where a conserving approximation for the second cumulant had been developed. Tracing out the electron degrees of freedom leads to an effective action for the c and d fields, $S^\lambda \equiv S_{\text{ch}} + S_t^\lambda$, composed of a charging and a tunneling term:

$$S_{\text{ch}} = \int_C dt \{c(t)^*(i\partial_t - \Delta_0)c(t) + id(t)^* \partial_t d(t)\}, \quad (5)$$

$$S_t^\lambda = -\int_C dt dt' c^*(t)\phi(t)\alpha^\lambda(t,t')\phi(t')c(t') + O(T_r^4). \quad (6)$$

Here $\alpha^\lambda = \alpha_L^\lambda + \alpha_R^\lambda$ is a particle-hole GF describing tunneling of an electron from one electrode to the island. It depends on the counting field. The connection to the ordinary GF is established by a rotation by $\lambda_r = \kappa_r \lambda$ in the Keldysh space as follows:

$$\tilde{\alpha}_r^\lambda(\omega) = U_{\lambda_r}^\dagger \tilde{\alpha}_r(\omega) U_{\lambda_r}, \tilde{\alpha}_r(\omega) = \begin{pmatrix} 0 & \alpha_r^A(\omega) \\ \alpha_r^R(\omega) & \alpha_r^K(\omega) \end{pmatrix}, \quad (7)$$

where $U_{\lambda_r} = \exp(-i\lambda_r \boldsymbol{\tau}_1/2)$ and $[\boldsymbol{\tau}_1]_{ij} = 1 - \delta_{ij}$. The retarded and advanced components are given by

$$\alpha_r^R(\omega) = \alpha_r^A(\omega)^* = -i\pi\alpha_0^r \frac{(\omega - \mu_r)E_C^2}{(\omega - \mu_r)^2 + E_C^2},$$

where $\alpha_0^r = 1/2\pi e^2 R_r$, and the Keldysh component by $\alpha_r^K(\omega) = 2\alpha_r^R(\omega)\coth(\omega - \mu_r)/2T$. We introduced a Lorentzian cutoff to regularize the ultraviolet divergence and ignored the term $O(T_r^4)$ in the action (6), since it is small in the limit $N_{\text{ch}} \gg 1$.

The free retarded GF of the Dirac fermion $\hat{c}$, $g_c^R(\omega) = 1/(\omega + i\eta - \Delta_0)$, describes the dynamics of charge excitations (η is a positive infinitesimal). The corresponding self-energy $\Sigma_c^\lambda = \Sigma_L^\lambda + \Sigma_R^\lambda$ accounts for quantum fluctuations of the island charge caused by tunneling. Integrating out d fields, we obtain the components of the self-energy in first order in α_0:

$$\Sigma_r^K(\omega) = 2\alpha_r^R(\omega), \quad \Sigma_r^R(\omega) = \int \frac{d\omega'}{2\pi} \frac{i\alpha_r^K(\omega')}{\omega + i\eta - \omega'}. \quad (8)$$

(For simplicity, we present here only the result for $\lambda = 0$.) For a symmetric SET ($R_L = R_R$, $C_L = C_R$), at $T = 0$ and $|\omega| \ll eV$, one finds $\Sigma_c^R(\omega) \approx \alpha_0 \ln(2E_C/eV)\omega - i\Gamma/2$, where $\Gamma = \Gamma_{IL} + \Gamma_{LI} + \Gamma_{IR} + \Gamma_{RI}$ is the sum of the rates $\Gamma_{rI/Ir} = \pm(1/e^2 R_r)(\Delta_0 - \mu_r)/(e^{\pm(\Delta_0 - \mu_r)/T} - 1)$ describing tunneling into (out of) the island through the junction r, evaluated by Fermi's golden rule.

We can proceed in a systematic diagrammatic expansion in α_0 [13]. To lowest order, one obtains for the CGF: $\mathcal{W}^{[1]}(\lambda) = -\int_C dt dt' g_c(t,t')\Sigma_c^\lambda(t',t)$. We project the time from contour C to the real axis and observe that, for long enough measurement times, we can approximate $\delta_{t_0}(\omega) \equiv \int_{-t_0/2}^{t_0/2} dt e^{-i\omega t}/2\pi$ by a δ function, $\delta_{t_0}(\omega) \to \delta(\omega)$, and $(\delta_{t_0}(\omega))^2 \to t_0\delta(\omega)/2\pi$. The latter ensures that any closed diagram, and, consequently, $\mathcal{W}$, is proportional to t_0. After Fourier transformation, we obtain

$$\begin{aligned}\mathcal{W}^{[1]}(\lambda) &\approx -t_0 \int d\omega \,\text{Tr}\{\tilde{g}_c(\omega)\boldsymbol{\tau}_1 \tilde{\Sigma}_c^\lambda(\omega)\boldsymbol{\tau}_1\}/2\pi \\ &= t_0 \sum_{r=L,R} \{P_- \Gamma_{rI}(e^{i\lambda_r} - 1) + P_+ \Gamma_{Ir}(e^{-i\lambda_r} - 1)\}.\end{aligned}$$

Here we used the expression for the Keldysh component of a c-field GF, $g_c^K(\omega) = 2i\,\text{Im} g_c^R(\omega)(P_- - P_+)$, which contains equilibrium occupation probabilities of the charge states Q_G and $Q_G - e$: $P_\pm = 1/(e^{\pm\Delta_0/T} + 1)$.

At this point, we note that the naive first order expansion $\mathcal{W}^{[1]}$ is insufficient. First, it contains the equilibrium occupation probabilities rather than the stationary ones. Second, due to charge conservation, the CGF should depend only on the difference of the counting fields $\lambda_L - \lambda_R = \lambda$ [14], which is also violated. These problems are resolved if we sum up an infinite subclass of diagrams. Specifically, we sum up the geometric series in $(\tilde{g}_c \boldsymbol{\tau}_1 \tilde{\Sigma}_c^\lambda \boldsymbol{\tau}_1)$, which contains the leading logarithms, i.e., powers of $\alpha_0 \ln(2E_C/eV)$,

$$\begin{aligned}\mathcal{W}(\lambda) &= t_0 \int \frac{d\omega}{2\pi} \text{Tr} \ln[\tilde{g}_c(\omega)^{-1} - \boldsymbol{\tau}_1 \tilde{\Sigma}_c^\lambda(\omega)\boldsymbol{\tau}_1] \\ &= t_0 \int \frac{d\omega}{2\pi} \ln[1 + T^F(\omega) f_L(\omega) h_R(\omega)(e^{i\lambda} - 1) \\ &\quad + T^F(\omega) f_R(\omega) h_L(\omega)(e^{-i\lambda} - 1)]. \quad (9)\end{aligned}$$

Here $f_r(\omega) = 1/(e^{(\omega-\mu_r)/T} + 1)$, $h_r(\omega) = 1 - f_r(\omega)$, and

$$T^F(\omega) = -\alpha_L^K(\omega)\alpha_R^K(\omega)/|\omega - \Delta_0 - \Sigma_c^R(\omega)|^2. \qquad (10)$$

Note that we subtracted a constant from the CGF in order to satisfy the normalization condition $\mathcal{W}(0) = 0$.

Equation (9) is the main result of this Letter. It is similar to the Levitov-Lesovik formula [2], but the effective transmission probability (10) accounts for quantum fluctuations of the charge. Using the condition Eq. (3), Eq. (9) can also be obtained from an approximate W given in Eq. (25) of Ref. [13]. Thus, the first and second cumulants $\langle I\rangle = e\langle\langle\delta q\rangle\rangle/t_0$ and $S_{II} = 2e^2\langle\langle\delta q^2\rangle\rangle/t_0$ reproduce the average current [15] and zero-frequency noise [13].

Although we used only the first order expansion for the self-energy, Eq. (9) is exact to second order in α_0. One can check that the diagrams ignored in Eq. (9) within second order expansion, i.e., the diagrams with intersecting interaction lines, are proportional to $\Delta_0/E_C \ll 1$. Higher order terms of Eq. (9) generate the renormalization factor z_0 consistent with the RG result [8]. They are crucial close to the threshold of Coulomb blockade. In particular, they remove the logarithmic singularities adherent to finite order perturbation theory.

Limiting cases.—In the limit $\alpha_0 \to 0$, one can show, following the derivation in Ref. [6], Sec. IV, that Eq. (9) reproduces the result of the "orthodox" theory:

$$\mathcal{W}^{(1)}(\lambda) = t_0\Gamma[\sqrt{D(\lambda)} - 1]/2, \qquad (11)$$

where $D(\lambda) = 1 + 4\Gamma_{LI}\Gamma_{IR}(e^{i\lambda} - 1)/\Gamma^2 + (L \leftrightarrow R, \lambda \to -\lambda)$. The second order expansion in α_0 reads

$$\mathcal{W}^{(2)}(\lambda) = \partial_{\Delta_0}\{\mathrm{Re}\Sigma_c^R(\Delta_0)\mathcal{W}^{(1)}(\lambda)\} + \mathcal{W}^{\mathrm{cot}}(\lambda). \qquad (12)$$

The first term of this equation provides the renormalization of the system parameters up to first order in α_0. Namely, the parameters are renormalized as $\alpha_0 \to \alpha_0\{1 + \partial_{\Delta_0}\mathrm{Re}\Sigma_c^R(\Delta_0)\}$ and $\Delta_0 \to \Delta_0 + \mathrm{Re}\Sigma_c^R(\Delta_0)$. This agrees with the corresponding results obtained earlier for the average current [16]. It is also consistent with the recent results of Braggio *et al.* [7]. We also checked that Eq. (12) can be reproduced by the systematic real-time diagrammatic expansion similar to that of Ref. [15].

The second term of Eq. (12) is the CGF of a bidirectional Poissonian process

$$\mathcal{W}^{\mathrm{cot}}(\lambda) = t_0\{\gamma^+(e^{i\lambda} - 1) + \gamma^-(e^{-i\lambda} - 1)\}, \qquad (13)$$

governed by the cotunneling rates

$$\gamma^\pm = \int d\omega\, \mathrm{Re}\frac{2\pi e^{\pm(\omega-\mu_R)/T}}{(\omega + i\eta - \Delta_0)^2}\prod_{r=L,R}\frac{\alpha_0^r(\omega - \mu_r)}{e^{\pm(\omega-\mu_r)/T} - 1}.$$

This term is relevant in the Coulomb blockade regime and is consistent with the FCS of quasiparticle tunneling [17].

At the conductance peak $\Delta_0 = 0$, for a symmetric SET, and $T = 0$, the orthodox theory yields $\mathcal{W}^{(1)} \approx 2\bar{q}(e^{i\lambda/2} - 1)$, where $e\bar{q}/t_0 = V/2(R_L + R_R)$. The factor $e^{i\lambda/2}$ leads to a sub-Poissonian value of the Fano factor $S_{II}/2e\langle I\rangle \approx 1/2$, indicating that tunneling processes are correlated. The origin of this correlation can be understood from the explicit form of the distribution $P(q) = \sum_{q_L,q_R=0}^{\infty} P_{\mathrm{P}}(q_L) \times P_{\mathrm{P}}(q_R)\delta_{q,(q_L+q_R)/2}$, obtained by inverse Fourier transformation of Eq. (4) without saddle point approximation. The numbers of electrons transmitted through either junction, q_L and q_R, follow the same Poissonian distribution $P_{\mathrm{P}}(q) = \bar{q}^q e^{-\bar{q}}/q!$. The Kronecker delta implies that q_L and q_R are correlated.

With Δ_0 approaching the threshold value $\Delta_0/eV = 0.5$, the tunneling onto the island becomes the bottleneck and the CGF acquires the Poissonian form $\mathcal{W}^{(1)} \approx t_0\Gamma_{LI}(e^{i\lambda} - 1)$. It remains Poissonian in the cotunneling regime, $|\Delta_0/eV| > 0.5$, $\mathcal{W}^{\mathrm{cot}} \approx t_0\gamma^+(e^{i\lambda} - 1)$.

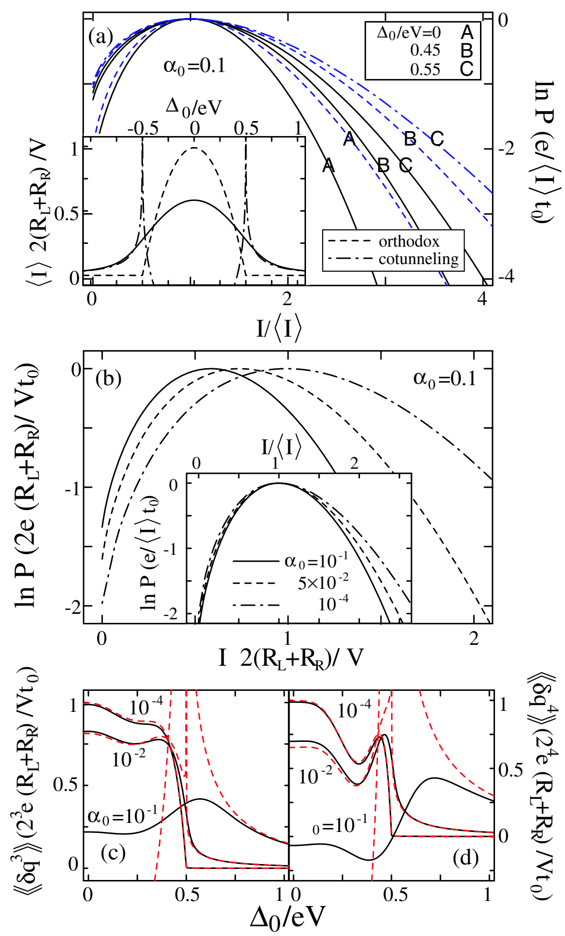

FIG. 1 (color online). Current distribution P of a symmetric SET [$T = 0$ and $eV/E_C = 0.2$]. (a) Solid lines are plots of P for various values of Δ_0 [Eq. (9)]. The dashed lines represent the orthodox theory [Eq. (11)], and the dotted-dashed line represents the cotunneling expansion [Eq. (13)]. The inset shows the average current for the same parameters. (b) Plot of P at $\Delta_0 = 0$ for various values of the conductance versus the current normalized to $V/2(R_L + R_R)$; inset: the same distribution normalized to the average current $\langle I\rangle$. (c) The skewness and (d) kurtosis for various conductances derived from Eq. (9) (solid lines) and the sum of Eqs. (11) and (12) (dashed lines).

Let us compare our result (9) to the orthodox (11) and the cotunneling (13) theories. The latter theories fail around the threshold $\Delta_0/eV = 0.5$. For example, in the average current, shown in the inset in Fig. 1(a), we observe a mismatch between the predictions of the orthodox (dotted line) and the cotunneling (dotted-dashed line) theories since $\Gamma_{LI} \to 0$ while $\gamma^+ \to \infty$. In contrast, our result (solid line), derived from Eq. (9) behaves regularly [solid lines in Fig. 1(a)]. It widens with increasing Δ_0. The orthodox and the cotunneling theories show the same trend but overestimate the width.

Renormalization and lifetime broadening effects.—For large conductance, quantum fluctuations of the charge are pronounced. However, as long as $z_0\Gamma \ll \Lambda$, where $\Lambda = \max(|z_0\Delta_0|, 2\pi T, |eV|/2)$, the orthodox CGF $\mathcal{W}^{(1)}$ with renormalized parameters $z_0\alpha_0$ and $z_0\Delta_0$ remains a good approximation. This scenario may fail in the regime $\Lambda \ll T_K = E_C e^{-1/2\alpha_0}/2\pi$, where the approximation of leading logarithms might be insufficient.

The renormalization effect is illustrated in Fig. 1(b), where the current distribution at $\Delta_0 = 0$ is plotted. Since z_0 decreases with increasing α_0, the mean value of the current, i.e., a peak position, shifts to lower values. The renormalization effect can be absorbed when we plot $\ln P$ with the horizontal axis normalized by $\langle I\rangle$ rather than $V/2(R_L + R_R)$. However, even after plotting the distribution as a function of the normalized current [inset in Fig. 1(b)], the three curves do not collapse to a single one. The remaining differences can be attributed to the non-Markovian lifetime broadening effect as described by $\mathrm{Im}\Sigma_c^R$. We observe that the current distribution shrinks with increasing α_0. This agrees with the previously noted suppression of the Fano factor [13]. FCS provides further information, showing in detail how the probability for currents exceeding the average value is suppressed.

The effect of lifetime broadening is also visible in the moments. At moderately large voltages, $eV \gg T_{\mathrm{K}}$, and at $T = 0$, the real part of the self-energy Σ_c^R is negligible and $\Sigma_c^R(\omega) \approx -i\pi\alpha_0 eV$. The CGF at $\Delta_0 = 0$ then is

$$\mathcal{W}(\lambda) \approx 2\bar{q}\{(e^{i\lambda/2} - 1) - 2\alpha_0(e^{i\lambda} - 1) + \pi^2\alpha_0^2(e^{i3\lambda/2} - e^{i\lambda/2})/2 + O(\alpha_0^3)\}, \quad (14)$$

and the ratio of higher order cumulants to the first one becomes $\langle\langle\delta q^n\rangle\rangle/\langle\langle\delta q\rangle\rangle = 2^{1-n}\{1 - 4\alpha_0(2^{n-1} - 1) + O(\alpha_0^2)\}$. As α_0 increases, higher order cumulants are suppressed as compared to the Poissonian result 2^{1-n}.

Solid lines in Figs. 1(c) and 1(d) show the skewness $\langle\langle\delta q^3\rangle\rangle$ and the kurtosis $\langle\langle\delta q^4\rangle\rangle$ as a function of Δ_0. A peak around the threshold develops with increasing conductance. Clear deviation from the second order perturbation theory, $\mathcal{W}^{(1)} + \mathcal{W}^{(2)}$ the sum of Eqs. (11) and (12) (dashed lines), appears for large α_0 around the threshold. We expect that the characteristic peak can be observed with present-day experimental techniques [18].

In conclusion, we have derived the full counting statistics for a single-electron transistor with intermediate strength conductance where quantum fluctuations of the charge play a dominant role. They are taken into account by the summation of a certain subclass of diagrams, which corresponds to the leading logarithmic approximation in the sense that the result is consistent with the RG analysis. In first order in α_0, our results reproduce the orthodox theory, while in second order they account for non-Markovian cotunneling effects, consistent with the recent analysis of Braggio *et al.* We have shown that, in nonequilibrium situations, quantum fluctuations of the charge induce lifetime broadening for the charge states of the central island. Consequences which can be detected in experiments include a suppression of the probability of currents larger than the average value.

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[1] *Quantum Noise in Mesoscopic Physics*, edited by Yu. V. Nazarov, NATO Science Series II, Vol. 97 (Kluwer, Dordrecht, 2003).
[2] L. S. Levitov and G. B. Lesovik, JETP Lett. **58**, 230 (1993); cond-mat/9401004.
[3] M. Kindermann and B. Trauzettel, Phys. Rev. Lett. **94**, 166803 (2005).
[4] A. Komnik and A. O. Gogolin, Phys. Rev. Lett. **94**, 216601 (2005).
[5] D. A. Bagrets and Yu. V. Nazarov, Phys. Rev. Lett. **94**, 056801 (2005).
[6] D. A. Bagrets and Yu. V. Nazarov, Phys. Rev. B **67**, 085316 (2003).
[7] A. Braggio, J. König, and R. Fazio, Phys. Rev. Lett. **96**, 026805 (2006).
[8] K. A. Matveev, Sov. Phys. JETP **72**, 892 (1991).
[9] D. S. Golubev *et al.*, Phys. Rev. B **56**, 15 782 (1997), and references therein.
[10] H. J. Spencer and S. Doniach, Phys. Rev. Lett. **18**, 994 (1967).
[11] K.-C. Chou *et al.*, Phys. Rep. **118**, 1 (1985); A. Kamenev, in *Nanophysics: Coherence and Transport*, edited by H. Bouchiat *et al.*, NATO ASI (Elsevier, Amsterdam, 2005).
[12] A. Kamenev, in *Strongly Correlated Fermions and Bosons in Low-Dimensional Disordered Systems*, edited by I. V. Lerner *et al.*, NATO Science Ser. II, Vol. 72 (Kluwer, Dordrecht, 2002).
[13] Y. Utsumi *et al.*, Phys. Rev. B **67**, 035317 (2003); Y. Utsumi, cond-mat/0503653.
[14] W. Belzig, in *CFN Lecture Notes on Functional Nanostructures Vol. 1*, edited by K. Busch *et al.*, Lecture Notes in Physics Vol. 658 (Springer-Verlag, Berlin, 2004).
[15] H. Schoeller and G. Schön, Phys. Rev. B **50**, 18 436 (1994).
[16] J. König, H. Schoeller, and G. Schön, Phys. Rev. Lett. **78**, 4482 (1997); Phys. Rev. B **58**, 7882 (1998).
[17] L. S. Levitov and M. Reznikov, Phys. Rev. B **70**, 115305 (2004).
[18] B. Reulet *et al.*, Phys. Rev. Lett. **91**, 196601 (2003).