Harmonic Lasing in a Free-Electron-Laser Amplifier

B. W. J. McNeil,[1,*] G. R. M. Robb,[1] M. W. Poole,[2] and N. R. Thompson[2]
[1]*SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom*
[2]*ASTeC, Daresbury Laboratory, Warrington WA4 4AD, United Kingdom*
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A method is demonstrated that allows a planar wiggler high-gain Free-Electron-Laser (FEL) amplifier to lase so that the interaction with an odd harmonic of the radiation field dominates that of the fundamental. This harmonic lasing of the FEL is achieved by disrupting the electron interaction with the usually dominant fundamental while allowing that of a harmonic interaction to evolve unhindered. The disruption is achieved by a series of relative phase changes between the electrons and the ponderomotive potentials of both the fundamental and harmonic fields. Such phase changes are relatively easy to implement and some current FEL designs would require little or no structural modification to test the scheme.

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The Free-Electron Laser (FEL) is a source of high power, coherent electromagnetic radiation that currently successfully covers electromagnetic wavelengths from mm waves to $\sim$30 nm. High-gain FEL amplifiers offer the prospect of extending this range to shorter wavelengths in the x-ray regions of the spectrum. Many proposals and several funded projects exist to build such shorter wavelength sources worldwide [1]. Realization of such sources will provide great opportunities by opening up more detailed investigations of many new areas of science.

Planar wiggler FELs allow resonant interactions with radiation fields of wavelength $\lambda_h = \lambda_w(1 + a_w^2)/2h\gamma^2$, where $h = 1$ is the fundamental with odd harmonics $h = 3, 5, 7, \ldots$, the wiggler period is λ_w, the mean electron beam energy in units of $m_e c^2$ is γ, and the rms wiggler parameter is a_w. The main limiting factor in directly accessing an harmonic interaction in a high-gain FEL is the dominance of the interaction at the fundamental radiation wavelength, λ_1. In this Letter a method is proposed that will suppress the interaction at the fundamental while allowing the harmonic interaction to evolve unhindered to saturation. This offers the prospect of yet shorter wavelength FEL operation.

The method, tentatively suggested in [2], uses a series of relative phase changes between the electrons and ponderomotive potentials of the resonant fields that describe the FEL interaction. The phase of the electrons with respect to the ponderomotive potential of the fundamental resonant wavelength is defined as θ_j where $j = 1, \ldots, N$ and N is the number of electrons. The phase of the electrons with respect to the ponderomotive potential of the nth harmonic field is then $\theta_{nj} = n\theta_j + \phi_n$, where ϕ_n is the relative phase between the ponderomotive potential of the fundamental and nth harmonic. If the phase of the electrons with respect to the fundamental ponderomotive potential is changed at a pointlike region of the interaction by a relative phase $\Delta\theta_j = 2\pi/k$ then the corresponding phase change for the harmonics will be $\Delta\theta_{nj} = 2\pi n/k$. Hence, if $k = n$ the electrons will be rephased within the ponderomotive potential of the nth harmonic by 2π whereas that for the fundamental will be $2\pi/n$. While the 2π electron rephasing of the nth harmonic should not adversely effect its subsequent FEL interaction, the $2\pi/n$ electron rephasing of the fundamental can be expected to disrupt its exponential growth.

The bunching of electrons at harmonics of the fundamental in a high-gain FEL was investigated in the 1D limit in [3]. A similar notation for the FEL equations is used here:

$$\frac{d\theta_j}{d\bar{z}} = p_j \tag{1}$$

$$\frac{dp_j}{d\bar{z}} = -\sum_{h,\mathrm{odd}} F_h(\xi_w)(A_h e^{ih\theta_j} + \mathrm{c.c.}) \tag{2}$$

$$\frac{dA_h}{d\bar{z}} = F_h(\xi_w)\langle e^{-ih\theta}\rangle, \tag{3}$$

where $j = 1, \ldots, N$ are the total number of electrons, $h = 1, 3, 5, \ldots$ are the odd harmonic components of the field, $\xi_w = a_w^2/2(1 + a_w^2)$ and $F_h(\xi_w)$ are the usual difference of Bessel function factors associated with planar wiggler FELs. Other symbols have their usual meaning [3].

In the 1D scaling and analysis that follow, both the wiggler period, λ_w, and the initial electron average beam energy, γ, are assumed constant. Tuning of the fundamental resonant radiation wavelength is therefore achieved by variation in the wiggler magnetic field alone.

The FEL interaction is investigated for *single* radiation wavelength operation using both the normal mode of operation (where the wavelength is the fundamental) and in the harmonic lasing mode, as described above, where the wavelength is an odd harmonic. The wiggler must therefore have two different settings. In the first mode the rms wiggler parameter $a_w = a_1$ and is set so that the fundamental resonant wavelength is $\lambda_f = \lambda_1$ giving harmonic resonant wavelengths $\lambda_h = \lambda_1/h$, $h = 3, 5, 7, \ldots$. In the second mode the wiggler parameter is reset to $a_w = a_n$ so that the new resonant fundamental wavelength is equal to the nth harmonic wavelength of the first mode setting, i.e., $\lambda_f = \lambda_n$. For the fixed beam energy and wiggler period assumed here, it is simple to show from the FEL resonance relation that a_1 and a_n must, therefore, satisfy the relation:

$$\frac{1 + a_1^2}{1 + a_n^2} = n. \tag{4}$$

Hence, there are no real solutions a_n for $a_1 < a_c = \sqrt{n-1}$, and it is not possible to retune the wiggler to a fundamental wavelength $\lambda_f = \lambda_n$.

The FEL scaling parameter, $\rho = \gamma^{-1}(a_w \omega_p / c k_w)^{2/3}$ [3,4], of Eqs. (1)–(3) is that for the wiggler parameter $a_w = a_1$, i.e., when the wiggler is tuned so that the fundamental wavelength $\lambda_f = \lambda_1$. When the wiggler is tuned so that the fundamental wavelength $\lambda_f = \lambda_n$ then using identical scaling as Eqs. (1)–(3) and neglecting all harmonics $h > n$, the FEL equations may be rewritten in the form:

$$\frac{d\theta_j}{d\bar{z}} = \frac{p_j}{n} \tag{5}$$

$$\frac{dp_j}{d\bar{z}} = -\frac{a_n}{a_1} F_1(\xi_n)(A_n e^{in\theta_j} + \text{c.c.}) \tag{6}$$

$$\frac{dA_n}{d\bar{z}} = \frac{a_n}{a_1} F_1(\xi_n)\langle e^{-in\theta}\rangle. \tag{7}$$

The appearance of the factor a_n/a_1 describes the reduced FEL coupling as $a_n < a_1$. Equations (1)–(3), truncated at $h = n$, and Eqs. (5)–(7) form the working set of equations for the remainder of this Letter.

Linear analysis using the methods of [4] allows the 1D gain lengths of the FEL interaction to be calculated for the two wiggler settings as described above. As was shown in [3], the harmonic evolution of Eqs. (1)–(3) have two separate regimes of evolution before the fundamental saturates. For small values of the bunching at the fundamental, both fundamental and harmonics are uncoupled and evolve exponentially with gain lengths determined only by the independent parameters. However, as the exponential growth of the bunching at the fundamental wavelength λ_1 progresses, the harmonics become strongly driven by the interaction at the fundamental. In the cold-beam limit this results in a dramatic reduction in the gain length of the harmonic to $1/h$th of that of the fundamental.

Here, we assume, as will be shown in subsequent sections, that the scheme of disrupting the exponential growth of the fundamental works as described above and therefore that there is no growth of the fundamental. The gain lengths of a single wavelength $\lambda = \lambda_3$ are compared for the two cases: (1) 3rd harmonic interaction at wavelength λ_3 with gain length l_{3h} (fundamental is disrupted) and (2) wiggler retuned to fundamental $\lambda_f = \lambda_3$ with gain length l_{3f}. To aid in comparison, these gain lengths are scaled with respect to the undisrupted fundamental gain length l_{1f} of (case 1). Using the results of [3,5] it can be shown that:

$$\frac{l_{3h}}{l_{1f}} = \left(\frac{F_1^2(\xi_1)}{3F_3^2(\xi_1)}\right)^{1/3} \tag{8}$$

$$\frac{l_{3f}}{l_{1f}} = \left(\frac{a_1 F_1(\xi_1)}{a_3 F_1(\xi_3)}\right)^{2/3}. \tag{9}$$

These expressions are plotted in Fig. 1 as a function of the wiggler parameter a_1. The value of the retuned wiggler parameter (case 2) is obtained from (4) with $n = 3$. It is seen from the plot for l_{3f}/l_{1f} that the gain length $l_{3f} \to \infty$ as $a_1 \to \sqrt{2}$. This is the limit $a_3 \to 0$ in Eq. (4) where FEL coupling ceases. Furthermore, note that the gain length for the disrupted fundamental scheme of (case 1), $l_{3h} < l_{3f}$. This is generally true for all odd harmonics n. Thus, the important result is obtained that for a fixed period wiggler and in the 1D cold-beam limit, when tuning an FEL interaction to a shorter wavelength by a factor $1/n$, the FEL gain length is *always* shorter by using the disrupted fundamental scheme of lasing than by a simple retuning of the wiggler magnetic field [where that is possible under the restrictions of Eq. (4)].

The system of FEL Eqs. (1)–(3) were also solved numerically, with wiggler parameter $a_1 = 4$, to demonstrate the harmonic lasing scheme as described above. The results are shown in Fig. 2. A numerical solution of Eqs. (5)–(7) is also shown on the same scale to demonstrate the solution when the wiggler parameter is retuned to $a_3 = 2.16$ so that λ_3 is the fundamental. The resonant, cold-beam limit is assumed with an electron distribution $p_j = 0 \forall\ j$, so that the spread $\sigma_p = 0$. It is seen that for $\bar{z} > 4$ the exponential instability of the fundamental scaled power is

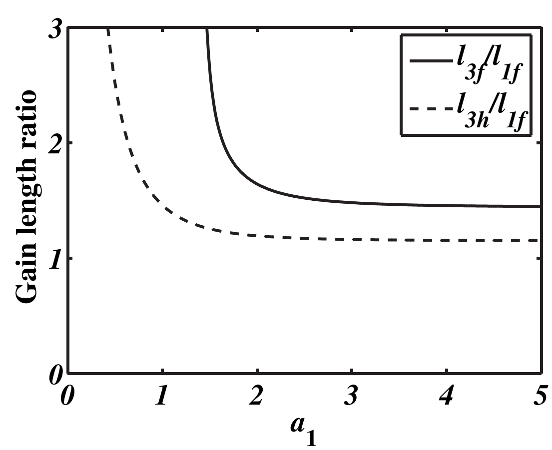

FIG. 1. Comparison of gain lengths as a function of a_1 for a fixed wavelength using the wiggler tuned for the harmonic lasing scheme of (case 1): l_{3h} (dashed line) and tuned for lasing at the fundamental (case 2): l_{3f} (solid line).

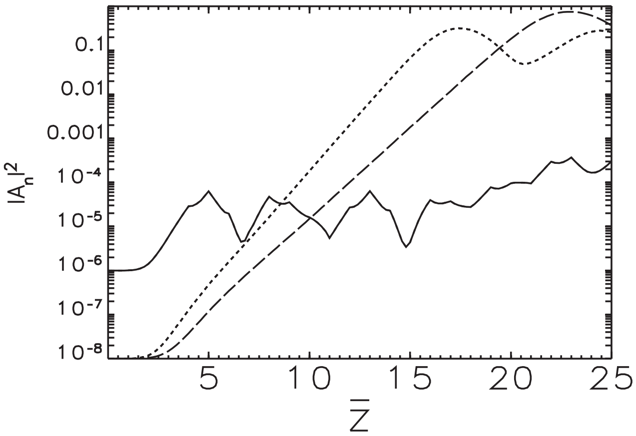

FIG. 2. Scaled powers of fundamental $|A_1|^2$ (solid line) and third harmonic $|A_3|^2$ (dotted line) for wiggler parameter $a_1 = 4$ demonstrating the effects of relative phase changes of $\Delta\theta = 2\pi/3$ at $\bar{z} = 4, 5, 6, \ldots, 24$. For the wiggler parameter retuned to $a_3 = 2.16$, A_3 is the fundamental and a separate simulation shows how $|A_3|^2$ (dashed line) evolves.

disrupted by the series of $\Delta\theta = 2\pi/3$ relative phase changes so that $|A_1|^2 \ll 1$ throughout the interaction. However, the simultaneous evolution of the third harmonic is unaffected, remaining resonant and evolving exponentially to saturation. Note that the powers $|A_1|_0^2 = 10^{-6}$ and $|A_3|_0^2 = 10^{-8}$ where the 0 subscript indicates initial values at $\bar{z} = 0$. The field A_1 may therefore act as a seed field to the electrons and transfer beneficial longitudinal coherence properties to the shorter wavelength interaction for A_3.

When the wiggler is retuned to $a_3 = 2.16$, so that λ_3 is the fundamental, it seen that the gain length is longer. The relative gain lengths agree with the linear theory of above. Despite the lower growth rate, however, the saturation power is larger. It can be seen by comparing Eqs. (1) and (5) that, for the same energy spread, the phase velocity spread is a factor n times larger for the case of harmonic lasing. This reduces the electron energy spread and so the radiation power at saturation for the case of harmonic lasing.

The effect of an n-fold increase in the phase velocity spread for harmonic lasing also increases the homogeneous energy spread requirements of the electron beam at the beginning of the interaction. For normal FEL interaction at the fundamental this may be expressed as $\sigma_\gamma/\gamma < \rho$ or equivalently in the scaling used here $\sigma_p < 1$. For harmonic lasing this requirement is increased to $\sigma_p < 1/n$. The effect of this on harmonic lasing is clearly seen by comparing the results of Figs. 3 and 4, where the results of Fig. 2 are extended to include the effects of initial Gaussian energy spreads of $\sigma_p = 0, 0.1, 0.2, \ldots, 0.5$.

One can conclude that harmonic lasing at λ_3 is more sensitive to the effects of electron beam energy spread than fundamental lasing at λ_3 (if the requirement $a_1 > a_c$ makes this possible). The benefit of tuning the wiggler so that the harmonic becomes the fundamental is therefore that the spread in p is reduced by $1/n$ [see Eq. (5)] which may well improve the growth rate above that of the harmonic lasing scheme. Nevertheless, from Eq. (4), when $a_1 < a_c$ it is not possible to retune the wiggler so that the harmonic becomes the fundamental. In the above $a_1 = 4$, well above the critical value $a_c = \sqrt{2}$.

In addition to describing the uncoupled linear evolution of the fundamental and harmonic interactions, the work of [3] showed that evolution of the bunching at the fundamental also drives the nth harmonic field at a growth rate of n times that of the fundamental. This nonlinear coupling is seen in Fig. 5, where the fundamental and harmonic fields are plotted for a harmonic lasing scheme, here with $a_1 = 1 < a_c$. A Gaussian energy spread parameter of $\sigma_p = 0.1$

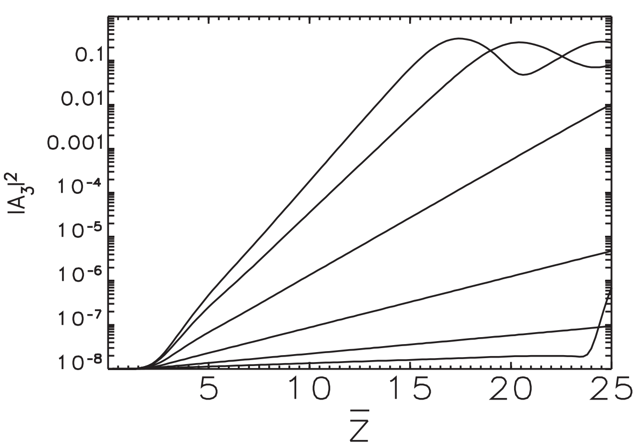

FIG. 3. The effect of energy spread on harmonic lasing. The third harmonic $|A_3|^2$ of Fig. 2 (dotted line) is plotted for scaled Gaussian energy spreads in p of $\sigma_p = 0.0$, 0.1, 0.2, 0.3, 0.4, and 0.5. The growth rate decreases monotonically with σ_p. The fundamental (solid line of Fig. 2) is not shown.

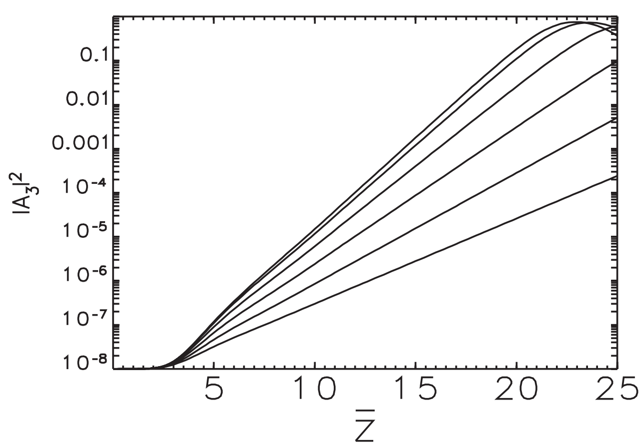

FIG. 4. The effect of energy spread for a retuned wiggler parameter of $a_3 = 2.16$ making A_3 the fundamental. The fundamental $|A_3|^2$ of Fig. 2 (dashed line) is plotted for scaled Gaussian spread in p of $\sigma_p = 0.0$, 0.1, 0.2, 0.3, 0.4, and 0.5. The growth rate decreases monotonically with σ_p.

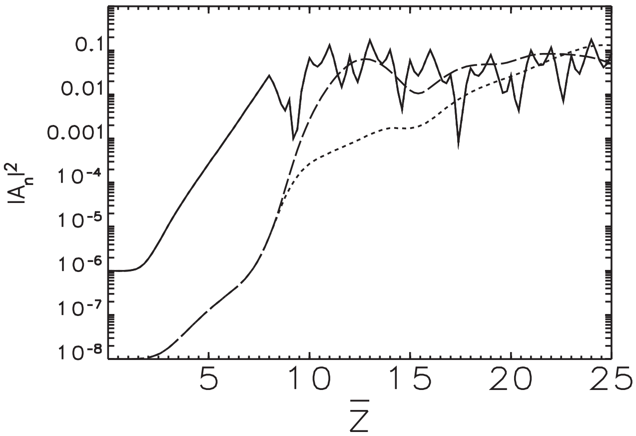

FIG. 5. The scaled powers of $|A_1|^2$ (solid line) and $|A_3|^2$ (dashed line) for the case of phase changes of $4\pi/3$ at $\bar{z} = 8, 9, \ldots, 24$—i.e., after nonlinear coupling of fundamental to the harmonic becomes significant around $\bar{z} \approx 7$. For phase changes of $2\pi/3$, the growth of the harmonic $|A_3|^2$ (dotted line) demonstrates less beneficial nonlinear coupling to the fundamental.

is used. The plot shows the evolution of the fundamental and third harmonic with phase changes of $4\pi/3$ at $\bar{z} = 8, 9, 10, \ldots$. The third harmonic is also shown for phase changes of $2\pi/3$ at $\bar{z} = 8, 9, 10, \ldots$. (The fundamental is not shown for this case.) According to linear theory, there should be no difference between the harmonic lasing for the two cases of $4\pi/3$ and $2\pi/3$ phase changes and this is indeed the case until $\bar{z} \approx 8.5$. Thereafter, for $4\pi/3$ phase changes, the harmonic is seen to attain a saturation power of approximately 2 orders of magnitude greater than that for $2\pi/3$ phase changes and also, not shown, approximately 1 order of magnitude greater than that if no phase changes are applied and the fundamental evolves to saturation in the usual way. The difference in behavior between the two cases is due to the electron coupling with the fundamental which continues to nonlinearly drive the bunching at the harmonics. For the case of $4\pi/3$ phase changes the fundamental continues to bunch the electrons in a way similar to that of [2] when phase changes of π are used with the FEL interaction to enhance the bunching at the fundamental. For $2\pi/3$ phase changes, the fundamental interaction does not bunch the electrons as well, however, greatly reducing the nonlinear driving of the harmonic.

A scheme for "taming" the fundamental high-gain FEL instability in a planar wiggler has been proposed in a way that allows an odd harmonic to remain resonant and evolve exponentially to saturation. The series of relative phase changes between electrons and the ponderomotive wells should not be difficult to implement. In particular, for FELs requiring long interaction lengths and employing many wiggler sections, typical of current vacuum ultraviolet to x-ray FEL designs, phase-changing mechanisms between wiggler sections already exist. An experiment to test the validity of the harmonic lasing scheme in such FELs would therefore incur little or no cost. As with other schemes that attempt to exploit harmonics in the reach toward shorter wavelengths, electron beam quality is a limiting factor. However, the energy spread requirement of the harmonic lasing scheme, namely $\sigma_p < 1/n$, does compare favorably with that for a High-Gain Harmonic Generation (HGHG) scheme [6] for which the energy spread requirement is $\sigma_p < 1/nD$, where $D = \rho\gamma \times d\theta/d\gamma > 1$ is the scaled strength of the dispersive section between modulator and radiator [7]. Harmonic lasing does, therefore, offer potentially improved harmonic emission and may certainly enable single wiggler FELs to outperform their original design specifications at shorter wavelengths. Of course, one can also envisage hybrid systems which incorporate the harmonic lasing interaction with multiwiggler schemes [8] including HGHG. No attempt has been made here to optimize the size of phase changes to attain higher saturation powers. Trial simulations suggest that this is possible.

Other phase-changing schemes may also offer other opportunities in controlling the electron-radiation interaction. For example, phase changes post saturation may act as a form of tapering to enable further energy extraction from the electrons. In short, the general method is a potentially useful tool and offers greater control of the FEL interaction.

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*Electronic address: b.w.j.mcneil@strath.ac.uk

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