Consistent Theory of Turbulent Transport in Two-Dimensional Magnetohydrodynamics

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A theory of turbulent transport is presented in two-dimensional magnetohydrodynamics with background shear and magnetic fields. We provide theoretical predictions for the transport of magnetic flux, momentum, and particles and turbulent intensities, which show stronger reduction compared with the hydrodynamic case, with different dependences on shearing rate, magnetic field, and values of viscosity, Ohmic diffusion, and particle diffusivity. In particular, particle transport is more severely suppressed than momentum transport, effectively leading to a more efficient momentum transport. The role of magnetic fields in quenching transport without altering the amplitude of flow velocity and in inhibiting the generation of shear flows is elucidated. Implications of the results are discussed.

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Shear flows and magnetic fields are commonly occurring in a variety of systems and play a crucial role in determining the overall turbulent transport in these systems. One of their important effects is the formation of transport barriers as they can reduce turbulent transport dramatically via shear stabilization and via Alfvénization. The crucial effect of shear stabilization has been highlighted in laboratory plasmas in recent years (e.g., [1–3]) and is now thought to be an essential ingredient for the improved confinement, which is necessary for a future economic reactor. It is also important for controlling turbulent transport in many other systems, including Earth's atmosphere [4] and ocean [5], major planets [6], the galaxies, and the Sun [7,8]. The effect of magnetic fields on turbulent transport is no less important, as demonstrated by a series of recent works. In particular, it leads to a strong reduction in the so-called alpha effect for a highly conducting fluid, considered to be critical to the generation of mean magnetic fields (e.g., [9,10]). Furthermore, the backreaction of magnetic fields on fluids can significantly slow down the effective dissipation rate (e.g., reconnection rate) of mean magnetic field [9,11], the cross-field diffusion of passive scalar concentration [12], and momentum transport [13].

In the mean electrodynamics, turbulent transport is characterized by transport coefficients such as turbulent (eddy) viscosity, and magnetic and particle diffusivities. The values of these coefficients are often estimated to be of order $vl \sim \tau_c v^2$, based on the characteristic amplitude v, length scale l, and correlation time τ_c of a turbulent flow. The characteristics of the turbulence (v, l, and τ_c) are, however, altered by shear flows and magnetic fields in important, nontrivial manners. In the case of shear flows, a strong shearing can considerably reduce v, l, and τ_c through the distortion and breakup of turbulent eddies, for example, rendering τ_c inversely proportional to shearing rate [8]. As a result, both turbulent transport and intensity can be reduced [3]. In the case of magnetic fields, the suppression of transport comes in mainly due to the change in τ_c, as Lorentz force turns random turbulent flows into a packet of Alfvén waves via magnetic tension. Therefore, magnetic fields can lead to a considerable reduction in transport without much affecting turbulence intensity. A critical, but poorly understood, issue is how shearing acting together with magnetic fields will modify turbulence and thus transport. This problem has important implications for turbulent transport in magnetized plasmas. Note that the transport of magnetic fields and momentum was previously studied in the two-dimensional magnetohydrodynamics (2D MHD), but only in the special case of the unit magnetic Prandtl number ($P_m = \eta/\nu$) [13]. Here, ν and η are the viscosity and Ohmic diffusivity, respectively. Since P_m is hardly of order unity in most physically relevant situations, it is crucial to develop a proper theory which is valid in general. On the other hand, the transport of particles has been studied mainly to understand the role of shear flows in the formation of transport barrier in laboratory plasmas [3]. The effect of magnetic field fluctuations, however, becomes important as plasma beta (pressure) increases, and thus should be incorporated for a theory of the formation of transport barriers (e.g., [14]).

In this Letter, we investigate the transport of momentum, magnetic flux, and particles in 2D MHD. We shall focus on the strong shear limit (weak turbulence) where shearing is more efficient than viscous dissipation and particle diffusivity in the system, and obtain theoretical predictions for turbulent transport via a quasilinear analysis. Note that the results obtained using similar theoretical methods in [3] were partially confirmed by a numerical computation [15]. We examine how and to what levels the transport of particles, momentum, and magnetic flux and turbulent intensities are reduced by shearing and magnetic fields, affected by the disparity in the values of viscosity, Ohmic diffusivity, and particle diffusivity. In particular, we show that particle transport is suppressed more severely than momentum transport, thereby effectively leading to a more efficient momentum transport than mixing of chemical elements. And we elucidate the role of magnetic fields in reducing transport without much effect on the fluctuation level of turbulent flow and show via a simplified model that magnetic fields slow down the generation of shear flows by reducing the amplitude of Reynolds stress as well as the cancellation of Reynolds stress by Maxwell stress. Overall, various transport tends to be quenched more significantly, compared with the hydrodynamic case. Some of the interesting implications for astrophysical and laboratory plasmas shall be discussed.

The evolution equations for the vorticity ω and magnetic vector potential a in 2D MHD in (x, z) domain are as follows:

$$(\partial_t + \mathbf{u}\cdot\nabla)\omega = -(\mathbf{b}\cdot\nabla)\nabla^2 a + \nu\nabla^2\omega + F, \quad (1)$$

$$(\partial_t + \mathbf{u}\cdot\nabla)a = \eta\nabla^2 a, \quad (2)$$

$$(\partial_t + \mathbf{u}\cdot\nabla)n = D\nabla^2 n. \quad (3)$$

Here, $\mathbf{b} = \nabla\times a\hat{y} = (-\partial_z a, 0, \partial_x a)$, $\omega\hat{y} = \nabla\times\mathbf{u} = (\partial_x u_z - \partial_z u_x)\hat{y}$, and F is the external small-scale forcing. ν, η, and D are the viscosity, Ohmic diffusion, and diffusivity of chemical elements n. In the following, we assume that $\nu < \eta$, which is likely to be the case in the interior of the Sun and laboratory plasmas, and consider the quasilinear evolution of fluctuations a', $\mathbf{b}'$, $\mathbf{u}'$, and ω' around mean fields $\mathbf{U} = -z\Omega\hat{x}(\Omega > 0)$, $\mathbf{B} = B\hat{x}$, and $n_0(z)$. We capture the strong shearing effect nonperturbatively by using the Gabor transform for fluctuations ω', a', $\mathbf{b}'$, $\mathbf{u}'$ and F (see [3,13]), denoted by $\hat{\omega}$, $\hat{a}$, $\hat{\mathbf{b}}$, $\hat{\mathbf{u}}$, and $\hat{F}$, and obtain

$$[D_t + \nu(k^2 + p^2)]\hat{\omega} = iBk(k^2 + p^2)\hat{a} + \hat{F}, \quad (4)$$

$$[D_t + \eta(k^2 + p^2)]\hat{a} = \frac{ik}{k^2 + p^2}\hat{\omega}B = \hat{u}_z B, \quad (5)$$

$$[D_t + D(k^2 + p^2)]\hat{n} = (-\partial_z n_0)\hat{u}_z. \quad (6)$$

Here, $D_t = \partial_t + U\partial_x + k\Omega\partial_p$ is the total time derivative, and $\mathbf{k} = (k, 0, p)$. We are interested in the case of strong shear where shearing occurs much faster than any dissipative process in the system with $\xi_\nu = \nu k^2/\Omega \ll 1$, $\xi_D = Dk^2/\Omega \ll 1$, and $\xi_\eta = \eta k^2/\Omega \ll 1$. Note that $\xi_\nu < \xi_\eta$ due to the assumption $\nu < \eta$. In this limit, the coupled equations (4) and (5) can be solved for strong magnetic field such that Alfvén frequency associated with typical forcing characteristic scale $1/k$ is larger than shearing rate, i.e., $\gamma = |Bk/\Omega| \gg 1$. Note that this is likely to be the case for a wide range of $k > 1/H_0$ in the solar tachocline where strong toroidal magnetic field of order 10^4–10^5 G and radial rotational shear of $\sim 3\times 10^{-6}$ s^{-1} are thought to be present. Here, $H_0 \sim 6\times 10^9$ cm is the pressure scale height at the bottom of convection zone. A similar analysis performed in [13] then gives us the solution:

$$\hat{a}(\mathbf{k}, \mathbf{x}, t) = \frac{i}{k\sqrt{k^2 + p^2}}\int d^2x_1 d^2k_1 \int_0^t dt_1 g(t{:}t_1) \times \frac{|k_1|}{\sqrt{k_1^2 + p_1^2}}\psi_1 \sin\varphi e^{\chi}\hat{F}(\mathbf{k}_1, \mathbf{x}_1, t_1). \quad (7)$$

Here, $\psi_1 = [1 - \frac{k_1^4}{2\gamma^2(k_1^2+p_1^2)^2}]^{-1}$, $\varphi = \gamma\Omega(t - t_1) - \frac{1}{4\gamma}\times [\tan^{-1}(\frac{p}{k}) - \tan^{-1}(\frac{p_1}{k_1}) + \frac{pk}{(k^2+p^2)} - \frac{p_1 k_1}{(k_1^2+p_1^2)}]$, $\chi = -\frac{\eta}{2}\times [k^2 t + \frac{p^3}{3\Omega k} - (k_1^2 t_1 + \frac{p_1^3}{3\Omega k_1})] + \frac{1}{4\gamma^2}[\frac{k^4}{(k^2+p^2)^2} - \frac{k_1^4}{(k_1^2+p_1^2)^2}]$, and $g(t{:}t_1) = \delta(k - k_1)\delta(p - p_1 - k\Omega(t - t_1))\delta(z - z_1)\times \delta(x - x_1 - U(t - t_1))$. From Eqs. (4)–(7), various correlation functions for η_T, D_T, and ν_T ($\langle a' u_z'\rangle = \eta_T B$, $\langle n' u_z'\rangle = -D_T\partial_z n_0$, and $\langle u_x' u_z'\rangle = \nu_T\Omega$) and turbulence intensities can be obtained for a given statistics of the forcing, which is, for simplicity, taken to be homogeneous and stationary with a short correlation time τ_f. The results obtained to leading order in small parameters $1/\gamma$, ξ_ν, ξ_D, and ξ_η are as follows:

$$\eta_T \sim \frac{\tau_f}{4B^2}\int\frac{d^2k}{(2\pi)^2}\frac{\hat{\phi}(\mathbf{k})}{k^4}\frac{2\Gamma(\frac{1}{3})}{3^{2/3}}\xi_\eta^{2/3} \sim \nu\xi_\eta^{2/3}\frac{v^2}{B^2}, \quad (8)$$

$$\nu_T \sim \frac{\tau_f}{4B^2}\int\frac{d^2k}{(2\pi)^2}\frac{\hat{\phi}(\mathbf{k})}{k^4} \sim \nu\frac{v^2}{B^2}, \quad (9)$$

$$D_T \sim \frac{\tau_f}{4B^2}\int\frac{d^2k}{(2\pi)^2}\frac{\hat{\phi}(\mathbf{k})}{k^2}\frac{2\Gamma(\frac{1}{3})}{3^{2/3}}\xi_D\xi_\eta^{-1/3} \sim \frac{D}{\eta}\eta_T, \quad (10)$$

$$\langle n'^2\rangle \sim \frac{\tau_f\pi(\partial_z n_0)^2}{4\Omega B^2}\int\frac{d^2k}{(2\pi)^2}\frac{\hat{\phi}(\mathbf{k})}{k^4} \sim (\partial_z n_0)^2\xi_\nu\frac{v^2}{B^2}, \quad (11)$$

$$\langle u_z'^2\rangle \sim \frac{\tau_f\pi}{4\Omega}\int\frac{d^2k}{(2\pi)^2}\frac{\hat{\phi}(\mathbf{k})}{k^2} \sim \xi_\eta^{1/3}\langle u'^2\rangle \sim \xi_\nu v^2. \quad (12)$$

Here, $\Gamma(x)$ is a Gamma function; $\hat{\phi}(\mathbf{k})$ is the power spectrum of the forcing ($\langle F^2\rangle = \int d^2k\hat{\phi}(\mathbf{k})/(2\pi)^2$), which is assumed to be dominated by modes with $p/k \ll 1$ for simplicity. In light of the Goldreich-Sridhar theory [16], initial wave packets driven by the forcing thus have small aspect ratio $p/k = k_\perp/k_\parallel \ll 1$, justifying the assumption of weak turbulence. A large $k_\perp$ is, however, developed mainly due to shearing effect in our weak turbulence theory, which is valid when the shearing rate is larger than the eddy turnover time. To appreciate the effects of shear and magnetic fields, the last terms in (8)–(12) are the estimates given in terms of the amplitude of the flow velocity v in the absence of magnetic fields and shear flow ($\Omega = 0$ and $B = 0$), which can easily be shown to be $v^2 \sim \tau_f\langle F^2\rangle/\nu k^4$. Here, $1/k$ is the characteristic scale of the forcing. Equations (8)–(12) clearly show how turbulent transport and intensities are reduced for strong magnetic field (large B) and for strong shear (small ξ_η, ξ_ν, and ξ_D). First, while all transport coefficients (η_T, D_T, and ν_T) as well as density fluctuation ($\langle n'^2\rangle$) are quenched via magnetic fields, the amplitude of turbulent flow $\langle u'^2\rangle$ or $\langle u_z'^2\rangle$ in Eq. (12) is suppressed only by shearing. This is because of the subtle effect of magnetic fields in increasing the memory time of a turbulent flow without affecting its amplitude. This suggests an interesting possibility of a significant reduction in transport by magnetic fields with less change in turbulence intensity. To quantify this, we obtain the cross phase—the flux normalized by turbulence intensity—as follows:

$$\cos\delta = \frac{\langle n' u_z'\rangle}{\sqrt{\langle n'^2\rangle\langle u_z'^2\rangle}} \sim \xi_D^{2/3}\gamma^{-1}\left(\frac{D}{\eta}\right)^{1/3}. \quad (13)$$

For a fixed value of D/η and $\gamma = Bk/\Omega$, Eq. (13) becomes small as shearing increases (i.e., for small ξ_D), with the scaling $\cos\delta \propto \Omega^{-2/3}$. In our model, $\cos\delta$ is further reduced for strong magnetic fields ($\gamma \gg 1$) as well as small particle diffusivity ($D < \eta$). Note that shear stabilization tends to reduce both turbulent transport and intensity with only modest reduction in the cross phase [3] since the distortion and breakup of eddies by shearing tend to suppress turbulence intensity through enhanced dissipation in addition to transport. Second, unlike other quantities, the eddy viscosity ν_T in Eq. (9) depends only on B. This small positive ν_T is a result of the cancellation of Reynolds stress by Maxwell stress, while Reynolds stress alone would have resulted in $\nu_T < 0$. The effect of magnetic field on ν_T will later be elaborated more in a simpler model. Third, the scaling of η_T can be made more transparent by expressing it in terms of total flow velocity amplitude $\langle u'^2\rangle \sim v^2\xi_\nu\xi_\eta^{-1/3}$, as $\eta_T \sim \eta\langle u'^2\rangle/B^2$. This is the Zeldovich theorem as a result of conservation of square of magnetic potential. Fourth, compared with the diffusion of magnetic field, particle diffusion is slower by a factor of D/η, as can be seen from Eq. (10). When $\eta = D$, Eq. (10) recovers the expected result $\eta_T = D_T$, since, in this case, particles evolve exactly the same as magnetic fields. Similar results were also found in the absence of shear flows [12]. Finally, the comparison between Eqs. (9) and (10) reveals that

$$\frac{D_T}{\nu_T} \sim \xi_D^{2/3}(D/\eta)^{1/3} \ll 1,$$

for $\xi_D^2 \ll \eta/D$, which is true for strong shear ($\xi_D \ll 1$). That is, shear flows acting together with magnetic fields inhibit the mixing of particles more severely than momentum transport, effectively leading to a more efficient momentum transport than particle transport. Importantly, this is in contrast to the three-dimensional hydrodynamics (HD), where the transport of momentum and particles is reduced to a similar level with the scaling $\nu_T \sim D_T \sim \Omega^{-2}$ [8]. Note that while the result of a more efficient momentum transport than particle transport is still valid for $D/\eta = 1$ (i.e., $D_T/\nu_T \sim \xi_\eta^{2/3} \ll 1$), when $D/\eta < 1$ (e.g., relevant to the solar interior), this effect is further boosted by a factor of D/η as $D_T/\nu_T \sim \xi_D^{2/3}(D/\eta)^{1/3} = \xi_\eta^{2/3}(D/\eta) < \xi_\eta^{2/3}$ for fixed η. One of the interesting implications of this result concerns the angular momentum transport versus mixing of light elements (lithium) in stellar interior. Mixing of light elements taking place in stellar interior can be inferred from their surface depletion, and observational evidences have suggested that the mixing of these elements occur on longer time scale than the redistribution of angular momentum (see, e.g., [17]). Unfortunately, for the lack of a fundamental theory derived from the first principle, this observational constraint has often been crudely parametrized in models. In contrast, our result offers a robust mechanism in which angular momentum transport can take place more efficiently than mixing of these elements in a consistent way.

We recall that the results obtained above are valid in the case of strong magnetic fields. To complement these results, we now consider a simplified model which permits us to examine various properties of turbulence as the strength of magnetic fields continuously changes. To this end, we assume that magnetic fluctuations are stationary as the advection of magnetic fields by a turbulent flow is balanced by efficient Ohmic dissipation [$D_t\hat{a} = 0$ in Eq. (5)]. By using $B\hat{u}_z - \eta(k^2 + p^2)\hat{a} = 0$ in Eq. (4), we can obtain the following exact solution:

$$\hat{\omega}(\mathbf{k}, \mathbf{x}, t) = \int d^2k_1 d^2dx_1 \int_0^t dt_1 g(t{:}t_1) e^{\psi} \hat{F}(\mathbf{k}_1, \mathbf{x}_1, t_1).$$

Here, $\alpha = B^2/\eta\Omega$; $\psi = -\alpha[\tan^{-1}(\frac{p}{k}) - \tan^{-1}(\frac{p_1}{k_1})] - \nu[k^2t + \frac{p^3}{3\Omega k} - (k_1^2 t_1 + \frac{p_1^3}{3\Omega k_1})]$; $g(t{:}t_1) = \delta(k - k_1)\delta(p - p_1 - \Omega k(t - t_1))\delta(x - x_1 - U(t - t_1))\delta(z - z_1)$. Then, by using the same forcing correlation function as before, we can obtain the following results to leading order in $\xi_\nu = \nu k^2/\Omega \ll 1$ and $\xi_D = Dk^2/\Omega \ll 1$:

$$\nu_T = \frac{-\tau_f}{4\Omega^2} \int \frac{d^2k}{(2\pi)^2} \frac{\hat{\phi}(\mathbf{k})}{k^2} \left[I_1(\alpha) - \left(\frac{B}{\eta k}\right)^2 I_2(\alpha) \right], \quad (14)$$

$$D_T = \frac{\tau_f}{2\Omega^2} \int \frac{d^2k}{(2\pi)^2} \frac{\hat{\phi}(\mathbf{k})}{k^2} I_3(\alpha), \quad (15)$$

$$\langle n'^2\rangle = \frac{\tau_f(\partial_z n_0)^2}{\Omega^3} \int \frac{d^2k}{(2\pi)^2} \frac{\hat{\phi}(\mathbf{k})}{k^2} \frac{\Gamma(1/3)}{3} I_4(\alpha), \quad (16)$$

$$\langle u_z'^2\rangle = \frac{\tau_f}{4\Omega} \int \frac{d^2k}{(2\pi)^2} \frac{\hat{\phi}(\mathbf{k})}{k^2} I_5(\alpha). \quad (17)$$

Here, $I_1 = f_2/(1 + \alpha^2)$, $I_2 = f_2[5/16(\alpha^2 + 1) + 3/16(\alpha^2 + 9)] + f_1/2(\alpha^2 + 4)$, $I_3 = (1 - e^{-\pi\alpha/2})^2/\alpha^2$, $I_4 = \{(3/2\xi_\nu)^{1/3} - 2[3/(\xi_\nu + \xi_D)]^{1/3} e^{-\pi\alpha/2} + (3/2\xi_D)^{1/3} \times e^{-\pi\alpha}\}/\alpha^2$, $I_5 = f_1/\alpha + \alpha f_2/(\alpha^2 + 1)$, $f_1 = 1 - e^{-\pi\alpha}$, and $f_2 = 1 + e^{-\pi\alpha}$. The I_i's in Eqs. (14)–(17) are the functions of $\alpha = B^2/\eta\Omega$, containing the effects of magnetic field. Figure 1(a) explicitly shows that I_i's monotonically decrease as α increases, manifesting the suppression of turbulent transport and intensities by magnetic fields. First, for small $\alpha \ll 1$, I_3 in Eq. (15) becomes $\pi^2/4$, yielding $D_T \propto \Omega^{-2}$. That is, particle transport is quenched by shearing for small α. As magnetic field (α) becomes large, I_3 becomes monotonically very small; for a sufficiently large α, D_T is suppressed mainly by magnetic fields, with the scaling $D_T \propto B^{-4}$. Second, the eddy viscosity ν_T in Eq. (14) contains the two parts, Reynolds stress ($\langle u_x' u_z'\rangle$) $\propto I_1$ and Maxwell stress ($\langle b_x' b_z'\rangle$) $\propto I_2$. As I_1 is positive, Reynolds stress gives rise to negative ν_T (inverse cascade), leading to the generation of shear flows. The amplitude of Reynolds stress I_1 decreases as the strength of magnetic field increases due to the Lorentz force, making ν_T less negative, and thus slowing down the generation of shear flows. In addition, Maxwell stress

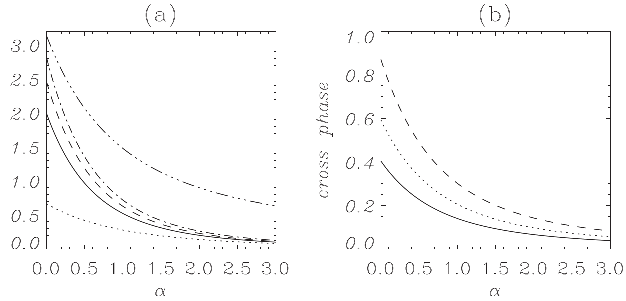

FIG. 1. (a) I_1 (solid line), I_2 (dotted line), I_3 (dashed line), $I_4/10$ (dash-dotted line), and I_5 (dash-dot-dot-dotted line) plotted as a function of $\alpha = B^2/\eta\Omega$ for $\xi_\nu = \xi_D = 10^{-3}$; (b) Cross-phase $\cos\delta$ plotted as a function of α for $\xi_D = 10^{-5}$ (solid line), 10^{-4} (dotted line), and 10^{-3} (dashed line).

with $I_2 > 0$ cancels part of Reynolds stress, further inhibiting the generation of shear flows with smaller negative ν_T. However, ν_T remains negative in this model since the contribution from Maxwell stress cannot exceed Reynolds stress for $B/\eta k \lesssim 1$, as required for the approximation of stationary magnetic fields to be valid. This is in contrast to the previous case where magnetic fluctuations are strong enough to render $\nu_T > 0$. Third, Eqs. (15)–(17) reveal how the cross-phase $\cos\delta$ for particle transport varies as α increases. To see this clearly, we plot $\cos\delta$ in Fig. 1(b) as a function of $\alpha = B^2/\eta\Omega$ for the three different values of $\xi_\nu = \xi_D = 10^{-5}$ (solid line), 10^{-4} (dotted line), and 10^{-3} (dashed line) with $\nu = D$. As can easily be seen, $\cos\delta$ becomes very small as magnetic field (α) increases. This is similar to what happens in the previous case with nonstationary magnetic fluctuations, although in that case, only the asymptotic behavior was found in the limit of strong magnetic fields. Note also that for a given value of α, $\cos\delta$ decreases for stronger shear (i.e., smaller ξ_D).

To summarize, we have presented a theory of turbulent transport of particle, momentum and magnetic flux in 2D MHD. The resulting transport coefficients and turbulence intensities, in general, are more severely reduced compared with HD case, with different dependences on shearing rate, magnetic field, and the values of viscosity, Ohmic diffusion, and particle diffusivity, some of them being quenched more strongly than others. In particular, particle transport was shown to be suppressed more severely than momentum transport, thereby leading to a more efficient transport of momentum than particles, due to the combined effects of shear and magnetic fields, together with $D/\eta < 1$. $D/\eta < 1$ also leads to smaller values of D_T/η_T and cross phase. The effect of magnetic fields was studied in detail through a simplified model of stationary magnetic fields, which explicitly showed how magnetic fields inhibit the particle transport and slow down the generation of shear flows (e.g., zonal flows). Particular attention was paid to the possibility of reducing transport of particles, without much quenching fluctuation levels due to magnetic fields. These results highlight the important role that magnetic fields as well as shear flow play in transport, which has often been overlooked. For instance, they can offer a robust mechanism to explain a more efficient angular momentum transport compared with mixing of light elements in stellar interiors. The theoretical predictions should be incorporated in constructing a consistent model of the evolution of rotation, magnetic fields, and chemical species in astrophysical plasmas, where traditionally transport coefficients are heavily parametrized and then fine-tuned to obtain agreements with observations. Of particular interest would be the application to the solar tachocline where the interplay between strong toroidal magnetic fields and rotational shear is crucial to the dynamics in that region. The results should also serve as valuable guides to understand the interaction between shear flows (e.g., zonal flows) and magnetic fields and their effects on transport reduction in the formation of transport barriers in other physical systems, including laboratory plasmas. The works addressing these issues will be published in future papers.

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