Leptogenesis from Gravity Waves in Models of Inflation

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We present a new mechanism for creating the observed cosmic matter-antimatter asymmetry which satisfies all three Sakharov conditions from one common thread, gravitational waves. We generate lepton number through the gravitational anomaly in the lepton number current. The source term comes from elliptically polarized gravity waves that are produced during inflation if the inflaton field contains a CP-odd component. The amount of matter asymmetry generated in our model can be of realistic size for the parameters within the range of some inflationary scenarios and grand unified theories.

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As far into the Universe as we can see, there is an excess of matter over antimatter. The recent determinations of the cosmological parameters from the cosmic microwave background by the WMAP experiment gives the baryon density of the universe as [1]

$$n_b/n_\gamma = (6.5 \pm 0.4) \times 10^{-10}. \tag{1}$$

This is a small number, but at the same time it is large enough to be a puzzle for models of particle physics. A baryon excess this large cannot be produced in the early universe within the standard model of particle physics [2]. In this Letter, we introduce a new mechanism for the creation of the matter-antimatter asymmetry, one associated with gravitational fluctuations created during cosmological inflation.

The conditions for generating a matter-antimatter asymmetry were stated by Sakharov almost 40 years ago [3]. First, baryon number should be violated. Second, CP should be violated. Third, these symmetry violations should be relevant at a time when the universe is out of thermal equilibrium. Since the 1980's, it has been realized that the standard weak interactions contain processes, mediated by *sphaelerons*, which interconvert baryons and leptons and are thermally activated at temperatures greater than 1 TeV. Thus, we can also create the baryon asymmetry by creating net lepton number at high temperature through out-of-equilibrium and CP-asymmetric processes [4,5]. Scenarios of this type are known as *leptogenesis*.

The out-of-equilibrium conditions can be created at a phase transition or through late decay of massive particles. The most attractive choice for a phase transition is that associated with electroweak symmetry breaking. However, that phase transition is probably not sufficiently strongly first order. This is known to be an obstacle to baryogenesis in the standard model. Particle decay asymmetries are loop suppressed and therefore require relatively large CP-violating phases. Such large phases are strongly constrained in supersymmetry [6] though they still could appear in the neutrino Yukawa couplings that are used in the Fukugita-Yanagida scenario for leptogenesis [5]. In any event, there is good reason to seek more effective sources of CP-violating out-of-equilibrium physics.

Our model of matter-antimatter asymmetry is assembled out of the following ingredients. First, as is well-known [7], the lepton number current, and also the total fermion number current, has a gravitational anomaly in the standard model. Explicitly,

$$\partial_\mu J^\mu_\ell = \frac{3}{16\pi^2} R\tilde{R} \tag{2}$$

where

$$J^\mu_\ell = \bar{\ell}_i \gamma^\mu \ell_i + \bar{\nu}_i \gamma^\mu \nu_i, \qquad R\tilde{R} = \tfrac{1}{2}\epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R^{\rho\sigma}_{\gamma\delta}. \tag{3}$$

The anomaly requires an imbalance of left- and right-handed leptons, so we are ignoring right-handed neutrinos. In general (2), will be correct in an effective theory valid below a scale μ. A simple guess for μ is that it is at the right-handed neutrino scale, of the order of 10^{14} GeV, but we would like to keep in mind the possibility of higher values of μ.

Next, we claim, a contribution to $R\tilde{R}$ of definite sign can be generated by gravitational fluctuations produced during inflation if the inflaton field contains a CP-odd component. This can be naturally achieved if the inflaton is a complex modulus field such as one finds in supergravity or superstring models. Such fields can have the very flat potentials required for inflation. The imaginary part ϕ of this field (which we henceforth call an ''axion'') can couple to gravity through an interaction

$$\Delta\mathcal{L} = F(\phi) R\tilde{R}, \tag{4}$$

where F is odd in ϕ, as a result of the Green-Schwarz mechanism [8]. Lue, Wang, and Kamionkowski (LWK) have studied the effects of such an interaction in generating observable parity-violation in the cosmic microwave background [9]. A simple form for $F(\phi)$ is

$$F(\phi) = \mathcal{N}\phi/(16\pi^2 M_{\rm Pl}), \tag{5}$$

where $\mathcal{N}$ is the number of stringy degrees of freedom propagating in the loops and the $M_{\rm Pl}$ in the denominator is approximately the string scale. In principle, this $M_{\rm Pl}$ can be substituted with a lower mass scale $\mathcal{F}$ with the constraint that our effective field theory is valid only for $\mu < \mathcal{F}$.

We would like to apply the interaction (4) to the dynamics of metric fluctuations during inflation. When the axion field has a slowly rolling nonzero classical value, the coupling (4) can lead to quantum fluctuations of the gravitational field that, treated to second order, generate a nonzero right-hand side for (2).

We believe that our analysis is also interesting because the Sakharov conditions are satisfied in this scenario in an unusual way. Lepton number is violated through (2). *CP* violation and out of equilibrium result from the nonzero classical value of the axion field. Before inflation, the complex modulus field varies from point to point in both modulus and phase. Inflation blows up a small region in this field to a size much greater than that of the visible universe. In this region, the modulus field is approximately constant and has a randomly chosen, fixed phase. This value then rolls slowly toward the minimum of its potential. In this process, we have out-of-equilibrium dynamics and, if the phase is nonzero, a *CP* asymmetry. We claim that no *explicit CP* violation is needed in the equations of motion. The *CP*-odd field ϕ could have zero expectation value today and need have no relation to the *CP* violation observed in particle physics.

Now we would like to quantitatively estimate the lepton number produced in inflation [10]. The general form of metric perturbations about a Friedmann-Robertson-Walker universe can be parametrized as

$$ds^2 = -(1+2\varphi)dt^2 + w_i dt dx^i + a^2(t)[((1+2\psi)\delta_{ij} + h_{ij})dx^i dx^j] \qquad (6)$$

where φ, ψ, w_i, and h_{ij} respectively parametrize the two scalar, vector, and tensor fluctuations of the metric. It is straightforward to show that the scalar and vector perturbations do not contribute to $R\tilde{R}$, and so we ignore these fluctuations in the following discussion. We can also fix a gauge so that the tensor fluctuation is parametrized by the two physical transverse traceless elements of h_{ij}. For gravity waves moving in the z direction, we write

$$ds^2 = -dt^2 + a^2(t)[(1-h_+)dx^2 + (1+h_+)dy^2 + 2h_\times dx dy + dz^2] \qquad (7)$$

where $a(t) = e^{Ht}$ during inflation and h_+, $h_\times$ are functions of t, z. To see the *CP* violation more explicitly, it is convenient to use a helicity basis

$$h_L = (h_+ - ih_\times)/\sqrt{2}, \qquad h_R = (h_+ + ih_\times)/\sqrt{2}. \qquad (8)$$

Here h_L and h_R are complex conjugate scalar fields. To be very explicit, the negative frequency part of h_L is the conjugate of the positive frequency part of h_R, and both are built from wave functions for left-handed gravitons.

The contribution of tensor perturbations to $R\tilde{R}$, up to second order in h_L and h_R, is

$$R\tilde{R} = \frac{4i}{a^3}\left[\left(\partial_z^2 h_R \partial_z \partial_t h_L + a^2 \partial_t^2 h_R \partial_t \partial_z h_L + \frac{1}{2}\partial_t a^2 \partial_t h_R \partial_t \partial_z h_L\right) - (L \leftrightarrow R)\right]. \qquad (9)$$

If h_L and h_R have the same dispersion relation, this expression vanishes. Thus, for $R\tilde{R}$ to be nonzero, we need a ‘‘cosmological birefringence’’ during inflation. Such an effect is induced by the addition of (4) to the gravitational equations [9].

Specifically, by adding (4) to the Einstein action, inserting (7), and varying with respect to the metric fluctuations, we find the equations of motion

$$\Box h_L = -2i\frac{\Theta}{a}\dot{h}_L', \qquad \Box h_R = +2i\frac{\Theta}{a}\dot{h}_R', \qquad (10)$$

where

$$\Theta = 4(F''\dot{\phi}^2 + 2HF'\dot{\phi})/M_{\rm Pl}^2, \qquad (11)$$

dots denote time derivatives, and primes denote differentiation of F with respect to ϕ. Note that (4) with a constant ϕ is a total divergence that cannot affect the equations of motion; thus, all terms in Θ involve derivatives of ϕ. We have dropped terms with third-order derivatives of h_L and h_R and terms with $\ddot{\phi}$. In fact, it is also permissible to ignore the F'' term in (11), since in slow-roll inflation, $\dot{\phi} \ll M_{\rm Pl}H$ and each derivative on F brings a dimensionful factor of order $1/M_{\rm Pl}$ [11]. These equations should be compared to those for evolution in flat space given by LWK [9]. The new term proportional to $H\dot{\phi}$ leads to a substantial enhancement in the size of Θ. With this simplification, and the approximate form (5),

$$\Theta = \sqrt{2\epsilon}\mathcal{N}(H/M_{\rm Pl})^2/2\pi^2, \qquad (12)$$

where $\epsilon = \frac{1}{2}(\dot{\phi})^2/(HM_{\rm Pl})^2$ is the slow-roll parameter of inflation [10].

Let us now focus on the evolution of h_L and, more specifically, on its positive frequency component. It is convenient to introduce conformal time

$$\eta = 1/Ha = e^{-Ht}/H. \qquad (13)$$

(Note that conformal time η runs in the opposite direction from t.) The evolution equation for h_L becomes

$$\frac{d^2}{d\eta^2}h_L - 2\frac{1}{\eta}\frac{d}{d\eta}h_L - \frac{d^2}{dz^2}h_L = -2i\Theta\frac{d^2}{d\eta dz}h_L. \qquad (14)$$

If we ignore Θ for the moment and let $h_L \sim e^{ikz}$, this becomes the equation of a spherical Bessel function:

$$\frac{d^2}{d\eta^2}h_L - 2\frac{1}{\eta}\frac{d}{d\eta}h_L + k^2 h_L = 0 \qquad (15)$$

for which the positive frequency solution is

$$h_L^+(k,\eta) = e^{+ik(\eta+z)}(1 - ik\eta). \tag{16}$$

We now look for solutions to (14) with $h_L \sim e^{ikz}$. To do this, let

$$h_L = e^{ikz}(-ik\eta)e^{k\Theta\eta}g(\eta). \tag{17}$$

Then $g(\eta)$ satisfies the equation

$$\frac{d^2}{d\eta^2}g + \left[k^2(1-\Theta^2) - \frac{2}{\eta^2} - \frac{2k\Theta}{\eta}\right]g = 0. \tag{18}$$

This is the equation of a Schrödinger particle with $\ell = 1$ in a weak Coulomb potential. For h_L, the Coulomb term is repulsive; for h_R, with the opposite sign of the Θ term, the Coulomb potential is attractive. This leads to attentuation of h_L and amplification of h_R in the early universe. This is just the cosmological birefringence described by LWK [9].

It will turn out that the generation of the matter asymmetry is dominated by modes at short distances (subhorizon modes) and at early times. This corresponds to the limit $k\eta \gg 1$. In this region, we can ignore the potential terms in (18) and take the solution to be approximately a plane wave. More explicitly,

$$g(\eta) = \exp[ik(1-\Theta^2)^{1/2}\eta(1+\alpha(\eta))], \tag{19}$$

where $\alpha(\eta) \sim \log\eta/\eta$.

We would like to apply the results of (17) to compute the expectation value of $R\tilde{R}$ in the inflationary space-time. Our expression will be dominated by the quantum part of the gravity-wave evolution. For this regime, we can calculate the expectation value by contracting h_L and h_R in $R\tilde{R}$ using an appropriate Green's function. Define

$$\begin{aligned} G(x,t;x',t') &= \langle h_L(x,t)h_R(x',t')\rangle \\ &= \int \frac{d^3k}{(2\pi)^3} e^{ik(x-x')}G_k(\eta,\eta'). \end{aligned} \tag{20}$$

For k parallel to z, the Fourier component G_k satisfies (14) with a delta-function source

$$\left[\frac{d^2}{d\eta^2} - 2\left(\frac{1}{\eta} + k\Theta\right)\frac{d}{d\eta} + k^2\right]G_k(\eta,\eta') = i\frac{(H\eta)^2}{M_{\rm Pl}^2}\delta(\eta-\eta'). \tag{21}$$

For $\Theta = 0$, the solution of this equation is

$$G_{k0}(\eta,\eta') = \begin{cases} (H^2/2k^3M_{\rm Pl}^2)h_L^+(k,\eta)h_R^-(-k,\eta') & \eta < \eta' \\ (H^2/2k^3M_{\rm Pl}^2)h_L^-(k,\eta)h_R^+(-k,\eta') & \eta' < \eta, \end{cases} \tag{22}$$

where h_L^- is the complex conjugate of (16), and h_R^+, h_R^- are the corresponding solutions of the h_R equation. For $\Theta = 0$, these solutions are the same as for h_L, but the structure of (22) will be preserved when we go to the case $\Theta \neq 0$. The leading effect of Θ is to introduce the exponential dependence from (17),

$$G_k = e^{-k\Theta\eta}G_{k0}e^{+k\Theta\eta'} \tag{23}$$

for both $\eta > \eta'$ and $\eta < \eta'$. The prefactor is modified in order Θ^2, and the wave functions acquire additional corrections that are subleading for $k\eta \gg 1$. Neither of these effects will be important for our result.

The Green's function (23) can now be used to contract h_L and h_R to evaluate the quantum expectation value of $R\tilde{R}$. The result is

$$\langle R\tilde{R}\rangle = \frac{16}{a}\int\frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3M_{\rm Pl}^2}(k\eta)^2k^4\Theta + \mathcal{O}(\Theta^3) \tag{24}$$

where we pick up only the leading behavior for $k\eta \gg 1$.

We note again that our expression for $\langle R\tilde{R}\rangle$ is nonzero because of the effect of inflation in producing a CP asymmetry out of equilibrium. The original quantum state for the inflaton might have had nonzero amplitude for a range of values of ϕ and might even have been CP invariant. However, inflation collapses the wave function onto a particular value of ϕ that is caught up in the local expansion of the universe. This value gives us a classical background that is CP asymmetric.

The above result and computations seem to be crucially depending on the form of the Green's function or the vacuum state we have used. To resolve the possible ambiguity in this regard, one may perform the above computation using a different method, the fermion level crossing, e.g., following [12]. This computation confirms the above results [13].

Inserting (24) into (2) and integrating over the time period of inflation, we find for the net lepton number density

$$n = \int_0^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3}\frac{1}{16\pi^2}\frac{8H^2k^3\eta^2\Theta}{M_{\rm Pl}^2}. \tag{25}$$

The integral over k runs over all of momentum space, up to the scale μ at which our effective Lagrangian description breaks down. The dominant effect comes not from the usual modes outside the horizon at the end of inflation (superhorizon modes), $k/H < 1$, but rather from very short distances compared to these scales. The integral over η is dominated at large values of η, early times. The integral represents a compromise between two effects of inflation, first, to blow up distances and thus carry us to smaller physical momenta and, second, to dilute the generated lepton number through expansion. It is now clear that the dominant contribution to the right-hand side comes from $k\eta \gg 1$, as we had anticipated. Performing the integrals, we find

$$n = \frac{1}{72\pi^4}\left(\frac{H}{M_{\rm Pl}}\right)^2\Theta H^3\left(\frac{\mu}{H}\right)^6. \tag{26}$$

We might interpret this result physically in the following way. The factor $(H/M_{\rm Pl})^2$ is the usual magnitude of the gravity-wave power spectrum. The factor Θ gives the magnitude of effective CP violation. The factor H^3 is the inverse horizon size at inflation; this gives the density n appropriate units. Finally, the factor $(\mu/H)^6$ gives the enhancement over one's first guess due to our use of strongly quantum, short distance fluctuations to generate $R\tilde{R}$, rather than the superhorizon modes which effectively behave classically.

To understand the significance of this estimate, we should compare it to the entropy density of the universe just after reheating, assuming that the energy of the inflationary phase has been converted to the heat of a gas of massless particles. To estimate this, assume very naively that reheating is instantaneous. Then reheating converts an energy density $\rho = 3H^2M_{\rm Pl}^2$ to radiation with $\rho = \pi^2 g_* T^4/30$ and $s = 2\pi^2 g_* T^3/45$, where g_* is the effective number of massless degrees of freedom. This gives $s = 2.3g^{1/4}(HM_{\rm Pl})^{3/2}$. With this value [14],

$$n/s = 6\times 10^{-5} g^{-1/4}(H/M_{\rm Pl})^{7/2}\Theta(\mu/H)^6. \tag{27}$$

Assuming that there have been no large increases in the entropy of the universe since the end of reheating, (27) can be compared directly to the present value of n/s inferred from (1). For this one should note that the ratio of the present baryon number to the lepton number originally generated in leptogenesis is approximately $n_B/n = 4/11$ [4]; then (1) implies $n/s = 2.4\times 10^{-10}$.

Inserting the estimate for Θ given in (12) and setting $g_* \sim 100$, we find

$$n/s \sim 1\times 10^{-6}\sqrt{\epsilon}\mathcal{N}(H/M_{\rm Pl})^{11/2}(\mu/H)^6. \tag{28}$$

In principle, $\mathcal{N}$ can be a large dimensionless number, within string theory typically $\mathcal{N} \gtrsim 100$. The ratio $(H/M_{\rm Pl})$ is limited in simple slow-roll inflation from the relation $\delta\rho/\rho \sim (H/M_{\rm Pl})/\sqrt{\epsilon} \sim 10^{-5}$. The WMAP results give a more precise version of this bound for the case of single-field inflation: $H/M_{\rm Pl} < 1\times 10^{-4}$ [15], implying that $\epsilon \sim 10^{-2}$ and hence

$$n/s \sim 1\times 10^{-5}(H/M_{\rm Pl})^{-1/2}(\mu/M_{\rm Pl})^6. \tag{29}$$

Note that in (29), we assumed that the mass scale from the kinetic term for the modulus is the Planck or string scale. If this mass scale is set at a lower mass $\mathcal{F}$, Θ can be larger, scaling as

$$\Theta \sim \sqrt{2\epsilon}\mathcal{N}(H^2/M_{\rm Pl}\mathcal{F})/2\pi^2 \sim (H/M_{\rm Pl})^2(M_{\rm Pl}/\mathcal{F}). \tag{30}$$

Assuming that $\mathcal{F} \sim \mu$ we obtain

$$\frac{n}{s} \sim 1\times 10^{-5}\left(\frac{H}{M_{\rm Pl}}\right)^{-1/2}\left(\frac{\mu}{M_{\rm Pl}}\right)^5. \tag{31}$$

Equation (31) is our final result in which $n/s \propto H^{-1/2}$, corresponding to $n \propto H^1$. As the first estimate, put $\mu \sim H \sim 2\times 10^{14}$ GeV; this yields $n/s \sim 10^{-27}$, a very small and unsatisfactory result. Noting (31), this estimate can be improved by taking higher values of μ and/or lower values of H. On the other hand, we need sphaelerons to be active after inflation. This happens for reheat temperature $T_r \gtrsim 1$ TeV, or $H \gtrsim 10^{-3}$ eV. The acceptable range for H is then $10^{-30} \lesssim H/M_{\rm Pl} < 10^{-4}$. To recover the observed value of n/s, $3\times 10^{14} < \mu \lesssim 10^{17}$ GeV. The conventional scale of supersymmetric grand unification scale is within this range.

In summary, we have presented a new mechanism for leptogenesis, which relies on the axial vector anomaly to violate fermion number, the initial state of inflation for CP violation and out-of-equilibrium dynamics. These are very minimal ingredients that might be found in a wide variety of models of physics at short distances. It is interesting to ask whether the conditions we have found can be embedded in a grand unification model in a natural way.

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