Direct Experimental Evidence of Free-Fermion Antibunching

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Fermion antibunching was observed on a beam of free noninteracting neutrons. A monochromatic beam of thermal neutrons was first split by a graphite single crystal, then fed to two detectors, displaying a reduced coincidence rate. The result is a fermionic complement to the Hanbury Brown and Twiss effect for photons.

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Over the past three decades, the research on the foundations of quantum mechanics has been enriched by many experiments on thermal neutrons, in particular, by several enlightening results about the coherence properties and the physical nature of the wave function describing the behavior of a massive particle [1]. A general property of fermions is that of being characterized by an antisymmetric wave function: the second-order correlation function of a fermion gas exhibits an anticorrelation in the intensity fluctuations, in particular, interference in the coincidence distributions of identical particles. This Letter describes a new contribution in this field: an experiment on thermal neutrons that brings to light the fermion antibunching effect in a beam of free noninteracting particles. The result is a fermionic complement to the seminal Hanbury Brown and Twiss effect for bosons (photons) [2].

The consequences of antisymmetry are well-known in condensed matter physics, where the electronic states display a strong quantum entanglement and are confined within the Fermi surface. Interesting experiments with electron beams have confirmed these effects [3–5]. In the case of almost free particles, an anticorrelation was observed in the coincidence spectrum of neutrons from compound-nuclear reaction at small relative momentum [6,7]. However, such a physical system is not a good representative sample of a statistical ensemble of noninteracting identical fermions. A monochromatic beam of thermal neutrons from a nuclear reactor represents much better a statistical ensemble of free particles. Nevertheless, the observation of thermal-neutron antibunching by means of coincidence measurements on such beams with the available instrumentation did not appear to be feasible up to now, mainly because the mean number of fermions obtainable per unit cell of phase space, to which the signal-to-noise ratio is proportional, was so low that a measurement time of several years was estimated [8].

We shall show below that, with present-day available advanced instrumentation, a very accurately designed setup, and a precise knowledge of the statistical properties of the neutron source, the experiment is feasible. In this Letter we shall describe some measurements carried out at the Institute Laue Langevin, Grenoble, France.

How can one directly bring to light an anticorrelation effect in a neutron beam? In a gas of fermions there is a certain tendency for particles of the same spin to avoid each other, a tendency arising from the exchange antisymmetry of the wave function: two fermions in the same spin state cannot occupy at the same time the same point in space, and therefore the probability amplitude for their being close together must be small. We just want to observe such an effect in a beam of thermal neutrons.

Let us start by considering the conceptual scheme of our experiment, which is schematically represented in Fig. 1, and is a massive particle analogue of the seminal optical Hanbury Brown and Twiss experiment [2], which yielded the first direct observation of the bunching effect in light beams and is a direct consequence of the symmetric wave function of a bosonic state. The semiclassical and quantum

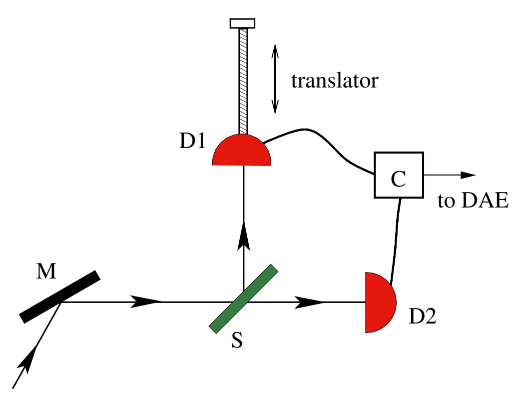

FIG. 1 (color online). Schematic drawing of the experimental setup: M, monochromator; S, beam splitter; D1 and D2, detectors; C, coincidence counter; DAE, Data Acquisition Electronics. The two detectors can be positioned at the same distance from S, and one of them can be moved across this distance. The collimators are not shown.

interpretations of this experiment are very clearly discussed in classic textbooks [9,10]: light from uncorrelated parts of the source yields interference effects that are associated with intensity correlations but not with the intensities themselves.

In the present experiment, a monochromatic beam of thermal neutrons is split by a pyrolytic graphite single crystal into a transmitted and a reflected beam. The intensity of each component is measured by a detector, and the coincidence rate of the outcomes at the two detectors is recorded as a function of their relative distance from the splitting crystal. Of course, also accidental coincidences occur in the apparatus and their rate must be subtracted from the total rate. The quantity of interest is the average value of these rates over a long enough period of observation.

The nature of the emission of thermal neutrons in the source is expected to be Poissonian, so that there is a small but finite probability of having two neutrons within the detection time interval of the apparatus. With reference to Fig. 1, for an average total rate n of neutrons impinging on the splitter crystal S and a total measuring time T_0, the predicted number of coincidences $N_{\rm bg}$ (where the subscript stands for background) measured at the two detectors can be calculated from the joint probability that a neutron is transmitted while a second neutron is reflected from S. By neglecting the fermion antibunching, one readily obtains

$$N_{\rm bg} = \tau_w n_t n_r T_0, \qquad (1)$$

where n_t and $n_r = n - n_t$ are the average rates of the transmitted and reflected beam, respectively, that have been assumed to be constant during the acquisition time T_0, and we have assumed a short enough coincidence window $\tau_w/2$, such that $n\tau_w \ll 1$ (in our experiment $n\tau_w \simeq 10^{-2}$–10^{-3}). When the distances of the two detectors from the beam splitter, $S_{\rm D1}$ and $S_{\rm D2}$, are different enough, Eq. (1) gives the expected number of *random* coincidences, since the measured coincidences are associated only with the simultaneous detection of two particles emerging from the splitter at different times.

Let us now qualitatively analyze the expected consequences of antibunching. Again referring to an ideal experiment, let $S_{\rm D1} = S_{\rm D2}$ and τ_c be the coherence time of the neutron wave packet. It should be remarked that the coherence time of the wave packet is defined by its energy distribution and it is longer when the energy width is small. Equation (1) can be modified to yield the expected number of *correlated* coincidences: observe that these are only those due to two neutrons with different spins that emerge from the beam splitter at the same time, because two neutrons with the same spin, due to the Pauli exclusion principle, cannot impinge on the beam splitter at the same time. If the incident beam is spin-unpolarized, it is equally likely that a neutron pair will either occur in one of the three triplet states or in the singlet state; i.e., the triplet states will occur 3/4 of the time. Thus, the average number of coincidences expected for $S_{\rm D1} = S_{\rm D2}$, as a consequence of fermion antisymmetry, is reduced from that of Eq. (1) by the following quantity

$$-\Delta N_{\rm fa} = -\tau_c n_t n_r T_0/2, \qquad (2)$$

where $-1/2 = -3/4 + 1/4$, $-3/4$ being due to the antibunching of the triplet states and $1/4$ to the bunching of the singlet state. Such a depression of the coincidence rate involves a two-particle state, and it is essential that both members of the pair be detected. Since the relative directions of the two particles may not be known in advance, one might think that something near a 4π detector might generally be needed. But in the present experiment the two-particle state to be tested is only that emerging (within a small solid angle) from the collimator and the monochromator, and the expression $\tau_c n_t n_r$ in Eq. (2) is simply the rate of such emerging state. This must be taken into account if one plans to perform a neutron-spin test of the Bell inequality [11].

Of course, in a real coincidence experiment, one must take τ_w much longer than τ_c, in order to account for various instrumental effects that force one to broaden the coincidence window. Actually, two particles arriving at the beam splitter at the same time may be absorbed and recorded at the two detectors within a rather long time interval τ_D, because of the finite thickness of the beam splitter and detectors and of small differences in their speeds. Moreover, the finite detection resolution $\tau_D < \tau_w$ modifies the value of $\Delta N_{\rm fa}$ in Eq. (2) by the factor τ_w/τ_D, making the use of intrinsically fast detectors highly desirable. The experimental data require therefore a careful analysis, as we shall discuss in detail in the following.

In order to detect the expected fermion antibunching effect, we have performed an optimized experiment based on the general scheme of Fig. 1. The main limitations of the experiment arise from the random fluctuations, which can mask the *difference* signal of Eq. (2). We assume that there are no accidental coincidences due to nonrandom processes. Therefore the expected random fluctuations are those due to the intrinsic statistics of the number of measured coincidences. The expected root mean square fluctuation of the total number of coincidences is $\Delta N_{\rm bg} = \sqrt{\tau_w n_t n_r T_0}$ and in order to detect a signal it is necessary that the noise to signal ratio, $\Delta N_{\rm bg}/\Delta N_{\rm fa} = 2\sqrt{\tau_w}/(\tau_c\sqrt{n_t n_r T_0})$, be smaller than unity. We see that the noise to signal ratio decreases when the coherence time of the incoming beam is long, so that the experiment should be performed by employing the most monochromatic available beam. We chose to use the primary spectrometer of the IN10 beam line [12], which produces a monochromatic beam by using an almost perfect Si(111) single crystal in the backscattering configuration. This monochromator produces a flux $n \simeq 3000\ {\rm sec}^{-1}$, at an energy $E \simeq 2.08$ meV with a (nominal) energy spread $\Delta E \simeq 0.13\ \mu$eV (0.3 μeV FWHM), and a beam size at the beam splitter of 1.5 (horizontal) $\times$ 4.0 (vertical) cm^2.

More precisely, ΔE is the total energy spread produced by the monochromator and the energy analyzer used at the IN10 instrument, the former giving a finer contribution than the latter, mainly because of the different crystal dimensions. An estimate of the energy spread produced by the monochromator only can be calculated from its geometry and yields an effective energy spread $\Delta E \lesssim 0.02\ \mu\text{eV}$ [13]. The coherence time of the incoming neutron beam is therefore $\tau_c \gtrsim \hbar/2\Delta E \simeq 16$ ns. Considering that the neutron speed v is about 630 m/s, the neutron coherence length is larger than 10 μm, a very small value. It is clear that both the beam splitter and the detectors must be as thin as possible, in order to reduce any additional spread of the signal. We therefore employed a 0.3 mm thick graphite crystal as beam splitter. At the wavelength of the present experiment the crystal has a good reflectivity so that the transmitted and diffracted beam are of the same order of magnitude. It should also be remarked that the possible velocity difference between the two particles of a pair, originating from the energy spread of the monochromator, contributes a negligible difference in the corresponding time of flight along the 40 cm path from the splitter to the detectors.

Two different detection systems were employed. The first one was based on two squashed ^{3}He 2 mm thick, 1.2 cm wide and 10 cm high detectors, whose neutron absorption length was $\simeq 1.4$ mm. The second one was based on two scintillator detectors having thickness 0.2 mm, width 1.5 cm and height 5 cm, whose neutron absorption length was $\simeq 0.25$ mm. The scintillator was a ^{6}Li 98% enriched ZnS glass, directly coupled to a 5 cm diameter fast photomultiplier. The shaping time was about 2 μs in the case of ^{3}He detectors, while it was 0.3 μs in the case of the scintillators. The coincidences were measured within an electronic time window $\tau_w/2$ of $\pm 10\ \mu$s in the case of gas detectors and $\pm 0.8\ \mu$s for the scintillators. Therefore the total time windows, that include the shaping time of the detectors, were $\pm 12\ \mu$s and $\pm 1.1\ \mu$s respectively.

Using this arrangement we have been able to perform two meaningful determinations of the actual antibunching effect in the incoming neutron beam. In order to do this, one detector was kept at a fixed position, at a distance of 40 cm from the graphite splitter along the diffracted beam, while the other detector was scanned through the transmitted beam, at approximately the same distance. For gas detectors, whose spatial resolution is of order 2 mm, we used a coarse translation step of 1 mm, while for scintillator detectors the translation step was 0.2 mm.

The data acquisition took several days; we therefore had to take into account the effect of the incoming beam fluctuations. As can be seen from Eq. (1), such an effect is nonlinear and directly related to the instantaneous value of the incoming beam rate. Since the instantaneous rate cannot be measured with adequate accuracy, one can perform a correction of the actual data by assuming that the beam fluctuations are small. In such a case, assuming that the acquisition time T_0 is much longer and the detection window τ_w much shorter than the correlation time of the noise and neglecting second-order effects, Eqs. (1) and (2) yield $N = \bar{N}(N_t/\bar{N}_t + N_r/\bar{N}_r - 1)$, where N is the actual number of coincidences, $\bar{N}$ is the number of coincidences detected in an (ideal) experiment with a constant rate on both the transmitted and incoming beam, and N_t and N_r are the actual numbers of neutrons detected on the two beams. The experimental data collected with the two detecting systems were corrected for the beam fluctuations and the results are reported in Fig. 2. In both cases a small dip is observed in the number of coincidences detected as a function of the relative distance of the two detectors from the beam splitter. We attribute this small dip to the antibunching effect due to the Fermion nature of the neutron.

It is interesting to perform a quantitative analysis, using the experimentally observed width of the dip in order to get an estimate of the coherence time of the incoming beam neutron wave packet. As a spinoff, this will yield a consistency check of our experimental results. Let us first consider the global response function of our experimental arrangement. We assume that the neutrons are diffracted within the thickness of the beam splitter according to the secondary extinction law [14] and are absorbed by the detectors, within a few mean free paths inside the absorbing medium. For simplicity, let us assume that the total response function of the detection system is (a normalized) Gaussian, $R(t) = \exp(-t^2/2\tau_D^2)/\sqrt{2\pi}\tau_D$, with a charac-

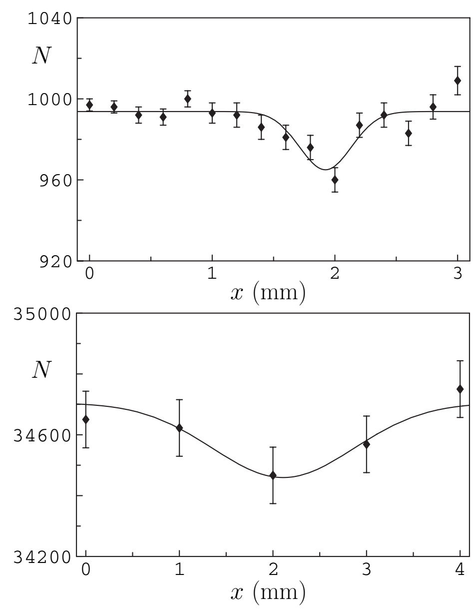

FIG. 2. Number of coincidences N as a function of the path difference $x = S_{D1} - S_{D2}$ of the detectors from the splitter. Top panel: scintillator detectors, translation step 0.2 mm; Bottom panel: gas detector, translation step 1 mm. The dip appears at $x_0 \simeq 2$ mm due to calibration, which could not be determined with a resolution better than 1.5–2.0 mm. The parameters τ_D, τ_c, N_{bg}, and x_0, by substituting $t = (x - x_0)/v$ in Eq. (3), are determined by a best fit (see text).

teristic response time $\tau_D \gg \tau_c$, where τ_c is the coherence time defined in Eq. (2). The global response time τ_D of the detection system will be obtained by fitting the experimental data: it is, however, expected to be close to the shaping time of the detectors (2 μs for ^{3}He detectors, 0.3 μs for scintillators).

The neutron pair correlation function, describing the antibunching effect, will also be taken to be Gaussian $C(t) = 1 - (1/2)\exp(-t^2/2\tau_c^2)$, the factor $1/2 = 3/4 - 1/4$ being due to the difference between the triplet and the singlet contributions [see Eq. (2)]. The total number of counts is therefore given by the convolution

$$\frac{N(t)}{N_{\rm bg}} = [R * C](t) \simeq 1 - \frac{1}{2}\frac{\tau_c}{\tau_D}\exp\left(-\frac{t^2}{2\tau_D^2}\right). \quad (3)$$

This must be compared with the *observed* number of coincidences in Fig. 2. Looking at the experimental data, we see that in both cases there is a small but appreciable dip, which is broader in the case of the experiment performed using the (thicker) gas detectors, as expected. The above formula implies that the width of the dip is τ_D, its depth being $\tau_c/2\tau_D = (\Delta N_{\rm fa}/N_{\rm bg})(\tau_w/\tau_D)$, in agreement with Eqs. (1) and (2).

An accurate fit yields $\tau_D = 1.3 \pm 0.4$ μs, $x_0 = 2.1 \pm 0.2$ mm, and $N_{\rm bg} = 34720 \pm 44$ for ^{3}He detectors, and $\tau_D = 0.33 \pm 0.07$ μs, $x_0 = 1.93 \pm 0.02$ mm, and $N_{\rm bg} = 993.7 \pm 0.6$ for the scintillators. Notice that the values obtained for $N_{\rm bg}$ agree with those calculated from Eq. (1) (acquisition times $T_0 = 600$ s for ^{3}He detectors and 300 s for the scintillators, and t_w corrected in order to include the shaping time of the detectors). Moreover,

$$\tau_c = 20 \pm 7 \text{ ns} \quad \text{for } ^3\text{He detectors}, \quad (4)$$

$$\tau_c = 19 \pm 3 \text{ ns} \quad \text{for scintillators}, \quad (5)$$

both values being fully consistent with each other and with the bound obtained by the energy spread of the beam ($\gtrsim 16$ ns). It is worth emphasizing that any additional attenuation factor (due, for example, to transversal coherence effects) multiplying the exponential in (3) can only yield larger values of τ_c: Equations (4) and (5) are therefore conservative estimates. The fitting curve is shown as a full line and is in very good agreement with the data: We obtained $\chi^2 = 19.38$ with 17 degrees of freedom, yielding $P_{17}(\chi^2 \geq 19.38) = 0.31$, for the scintillators, and $\chi^2 = 0.7083$ with 1 degree of freedom, yielding $P_1(\chi^2 \geq 0.7083) = 0.40$, for ^{3}He. A flat fit would yield $\chi^2 = 61.85$ with 20 degrees of freedom, yielding $P_{20}(\chi^2 \geq 61.85) = 4 \times 10^{-6}$, for the scintillators and $\chi^2 = 5.049$ with 4 degrees of freedom, yielding $P_4(\chi^2 \geq 5.049) = 0.28$, for ^{3}He. Note also that the value $\tau_D \simeq 0.33$ μs is in full accord with the nominal shaping time of the scintillator (0.3 μs), while the value $\tau_D \simeq 1.3$ μs is smaller than the nominal shaping time of the ^{3}He detectors (2 μs): this can be understood by remarking that ^{3}He detectors tend to absorb neutrons in the initial section. Finally, one also obtains results that are consistent with those above by performing a convolution with more realistic (non-Gaussian) shape functions describing the response function of the detectors and of the beam splitter.

It is useful to clarify in what sense this experiment performed with neutrons is complementary to its photon [2,15] and electron [3–5] counterparts. The first, obvious observation is that neutrons are fermions that are not affected by Coulomb interaction that plays, by contrast, an important role in condensed matter systems. Second, neutrons have very low phase-space densities, so that all the effects we have brought to light are due to two-particle correlations, three or more particle effects being completely negligible. This experiment, providing a firm experimental evidence of the Pauli exclusion principle, displaying its effects on free neutrons in real space, has a very basic importance, because it is directly related to the quantum mechanics of identical particles.

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