Screening and Attraction of Dust Particles in Plasmas

C. Castaldo,[1,*] U. de Angelis,[2] and V. N. Tsytovich[3]
[1]Associazione EURATOM-ENEA, CR ENEA-Frascati, Via E. Fermi 45, 00044 Frascati (Rome), Italy
[2]Department of Physical Sciences, University of Naples, Italy and INFN Sezione di Napoli, Italy
[3]General Physics Institute, Russian Academy of Science, Moscow, Russia
(Received 16 May 2005; published 23 February 2006)

The potential around a dust particle in a plasma is found using the collisional hydrodynamic equations of dusty plasmas, taking into account ion-dust and ion-neutral collisions and considering the plasma source proportional to the dust density. The linear screening is strongly influenced by the collisions and can substantially differ from Debye screening. Attraction of negatively charged dust particles can occur due to overscreening by the ion fluxes in the presence of friction forces.

DOI: 10.1103/PhysRevLett.96.075004 PACS numbers: 52.27.Lw

The collective effects on dust-dust interactions in plasmas [1] can modify the usual Debye screening and result in long range attraction of negative dust particles [2,3], completely different from the "shadow" and "wakefield" attractions, which are not due to collective effects. In a plasma with $\tau = T_i/T_e \ll 1$ (where T_e, T_i are the electron and ion temperatures) and the plasma Debye length is essentially given by the ion Debye length λ_{Di}, as shown in [2], for the case where the plasma source is proportional to the electron density, the screening length can be of the order of the electron Debye length λ_{De} ($\gg \lambda_{Di}$) and dust-dust attraction exists. A review of collective attraction is given in [4]. Dust ordered structures were observed in thermal dusty plasma under atmospheric pressure and temperatures of 1700–2200 K [5,6]. In such conditions, the dust charging occurs not only collecting electrons and ions, but also emitting electrons. Numerical simulations also predict that attractive potentials for positive charges are formed around positively charged grains in the presence of thermionic emission [7]. Lampe *et al.* [8,9] have shown that it is important to include the effects of trapped ions to investigate the charging and shielding of dust grains in collisional plasmas. A recent paper [10] has been dedicated to the orbital theory of the shielding potential around a dust grain, including the effects of emission, but neglecting the collisions and the collective dust-dust interactions. In many astrophysical problems and in experiments where the plasma source is radioactivity or photo-ionization of the dust particles, the plasma source is proportional to the dust—not to the electron—density and the condition $\tau \approx 1$ is often valid. It is shown here that, for negatively charged dust grains, the screening length can substantially exceed the plasma Debye length and dust-dust attraction is possible, due to collisional and collective interactions, thus extending the first investigation of [2].

The potential $\phi(r)$ around dust particles of radius a_d and negative equilibrium charge $q_{\rm eq} = -eZ_d$ ($e > 0$ is the elementary charge) in a plasma is calculated using the ion fluid equations which take into account ion-dust and ion-neutral collisions [11]:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = s - n_i \bar{\nu}_{d,i} \tag{1}$$

$$n_i\left(\frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla\right)\mathbf{u}_i + \frac{1}{m_i}\nabla(n_i T_i) - \frac{e}{m_i} n_i \mathbf{E} = -(\tilde{\nu}^{\rm ch}_{d,i} + \tilde{\nu}_{d,i} + \nu_{n,i}) n_i \mathbf{u}_i, \tag{2}$$

where m_i, e, T_i are the mass, charge, and temperature (energy units) of ions, n_i and $\mathbf{u}_i$ the density and fluid velocity, and $\mathbf{E} = -\nabla\phi$ the field due to a "test" dust particle perturbing the equilibrium state of a dusty plasma with dust, electron, and ion densities indicated by n_d, n_{e0}, n_0, respectively. In the present model, the background dust grains are essentially represented as a continuous, immovable, uniform "dust medium," which is a source and sink for ions, and produces friction forces on the ions. On this medium is then superposed a single additional discrete dust grain, and we follow the ion response. It is worth noting that the effect of trapped ions has been neglected. This assumption is well justified if the mean kinetic energy of the ions is greater than the binding potential, as occurs in the linear regime considered below. The nonlinear regime has been considered in [12]. In the continuity Eq. (1) the right-hand side accounts for the presence of a source (s) and a sink of ions, $\bar{\nu}_{d,i}$ being the fluid frequency for ion absorption on dust. In the momentum Eq. (2) the right-hand side accounts for friction forces due to ion-dust charging collisions ($\tilde{\nu}^{\rm ch}_{d,i}$), Coulomb collisions ($\tilde{\nu}_{d,i}$), and to ion-neutral collisions ($\nu_{n,i}$).

Expressions for the ion-dust collision frequencies in terms of the ion equilibrium distribution function are given in [11], where the collisional plasma hydrodynamic equations have been derived from the kinetic theory which consistently takes into account the collisions with dust particles.

The dust density is assumed not perturbed and the electrons are treated in the linearized adiabatic approximation: $n_e = n_{e0}[1 + (e\phi/T_e)]$ where T_e is the electron temperature in energy units. The following parameters will be used: $z = e^2 Z_d/a_d T_e$; $P = n_d Z_d/n_0$.

The source term is assumed to be independent on the electron density and the ratio τ is assumed enough high to neglect the contribution of the trapped ions; isotropy (dependence only on r) will be assumed and normalization of densities to n_0 ($n = n_i/n_0$), distances to the ion Debye radius ($x = r/\lambda_{Di}$, $a = a_d/\lambda_{Di}$), and velocity to $\sqrt{2}v_{Ti}$, where $v_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal speed.

The equilibrium state is defined by a dust charge Z_d^0, related parameters z_0, P_0, and a source-sink balance $s = n_0 \bar{\nu}_{d,i}^0$ is assumed in the equilibrium state. The equilibrium dust charge is calculated from the modified orbit motion limited (OML) charging equation.

Introducing the screening function $\psi(r)$ as $e\phi(r) = -e^2 Z_d \psi(r)/r$ and assuming the form $n = 1 + [N(x)/x]$ for the perturbed ion density, the steady-state linear ion fluid equations have the form:

$$\frac{1}{x^2}\frac{d}{dx}(x^2 u) = -a\alpha_{\mathrm{ch}} P_0 \left[\frac{N(x)}{x}\left(1 + \frac{\tau}{z_0}\right) + \frac{\delta P}{P_0}\right] \quad (3)$$

$$\frac{d}{dx}\frac{G(x)}{x} = \alpha_{dr} P_0 u, \quad (4)$$

where

$$\alpha_{dr} = \frac{\sqrt{2}\nu_{dr}\tau}{\omega_{pi} z_0 P_0 a}, \qquad \alpha_{\mathrm{ch}} = \frac{\bar{\nu}_{d,i}^0 z_0}{\sqrt{2}\omega_{pi} a P_0 (z_0 + \tau)}, \quad (5)$$

$\nu_{dr} = \tilde{\nu}_{d,i}^{\mathrm{ch}} + \tilde{\nu}_{d,i} + \nu_{n,i}$ is the "total" (unperturbed) collision frequency, ω_{pi} the (unperturbed) ion plasma frequency, and the function $G(x) = \psi(x) - [\tau N(x)/a z_0]$ has been introduced. The steady-state dust charge perturbation is calculated from the modified OML charging equation [3]:

$$\frac{\delta Z_d}{Z_d^0} = \frac{\delta z}{z_0} = \frac{\delta P}{P_0} = -\frac{\tau + z_0}{1 + \tau + z_0}\left[\psi(x) + \frac{N(x)}{a z_0}\right]\frac{a}{x}. \quad (6)$$

Eliminating $N(x)$ the ion equations can be reduced to the single equation:

$$\frac{d^2 G}{dx^2} = k_1 G(x) - k_2 \psi(x), \quad (7)$$

where $k_1 = h_0(z_0 + \tau)$, $k_2 = h_0 z_0$, and $h_0 = \alpha_{\mathrm{ch}}\alpha_{dr} a^2 P_0^2 (z_0 + \tau)/[(1 + z_0 + \tau)\tau]$.

The Poisson equation, taking into account neutrality in the equilibrium state and dust charge perturbations, can also be written in terms of the functions $\psi(x)$ and $G(x)$:

$$\frac{d^2\psi}{dx^2} = k_3 \psi(x) - k_4 G(x), \quad (8)$$

where $k_3 = k_4(1+\tau) - P_0\tau$, $k_4 = 1 + h_1$, $h_1 = P_0(\tau + z_0)/[z_0(1 + \tau + z_0)]$.

The boundary conditions for the system of Eqs. (7) and (8) are the following: (i) the asymptotic conditions $G(x)/x$, $\psi(x)/x$, $u(x) \to 0$ for $x \to +\infty$; (ii) the exact boundary condition for the field at the grain surface, $E(a) = -eZ_d/a_d^2$, which gives the condition for the screening function:

$$\psi(a) - a\psi'(a) = 1 \quad (9)$$

(the prime denotes differentiation with respect to distance); (iii) the requirement that the ion velocity at the dust surface is given by the value u_e: this gives, from Eq. (4), the boundary condition:

$$aG'(a) - G(a) = \alpha_{dr} a^2 P_0 u_e = w. \quad (10)$$

The solution for the screening function has the general form (dimensional units):

$$\psi(r) = g_1 e^{-r/\lambda_1} + g_2 e^{-r/\lambda_2}, \quad (11)$$

where $\lambda_1 = \lambda_{Di}/s_1$, $\lambda_2 = \lambda_{Di}/s_2$, and $s_1 > s_2$ are:

$$s_{1,2} = \left[\frac{(k_1 + k_3)}{2}(1 \pm \sqrt{1 - \gamma})\right]^{1/2}, \quad (12)$$

where $\gamma = 4(k_1 k_3 - k_2 k_4)(k_1 + k_3)^{-2}$, $0 < \gamma < 1$, and

$$g_1 = \frac{e^{as_1}}{1 + as_1}\frac{k_3 - s_2^2 + k_4 w}{s_1^2 - s_2^2} \quad (13)$$

$$g_2 = \frac{e^{as_2}}{1 + as_2}\frac{s_1^2 - k_3 - k_4 w}{s_1^2 - s_2^2}. \quad (14)$$

In the linear regime, when the ion-dust potential energy of interaction is smaller than the ion thermal energy $[|e\phi(a)|/T_i \simeq e^2 Z_d/aT_i = z/\tau < 1]$, and the ion fluid velocity is smaller than the thermal velocity ($|u_e| < 1$), the present results for the potential are valid, in general, at distances significantly larger than the grain-grain separation, so that discreteness of the grains can be neglected. However, the results are also valid at distances smaller than the grain-grain separation, if the effects of the "dust medium" can be neglected, namely, considering the limits $h_0 \to 0$, $h_1 \to 0$ and assuming that the drag is only due to the collisions with neutrals.

The screening function is a combination of exponentials with screening lengths which can substantially differ from the linear Debye length, given by $\lambda_{D\mathrm{lin}} = \lambda_{Di}[1 + (1 - P_0)\tau]^{-1/2}$. Potential wells can occur if the dust grains are sources of plasma, such that the ion velocity at the dust surface exceeds the threshold value $u_{e,o} = ({s_1}^2 - k_3)/(k_4 \alpha_{dr} a^2 P_0)$.

Thermionic, photoelectric, field, and secondary emission of both electrons and ions can indeed produce plasma fluxes escaping from the grain surface. We do not discuss in detail these processes, which depend not only on the dusty plasma parameters, but also on the physical-chemical properties of the grains, as well as on the impinging fluxes of the photons and energetic particles. We have only assumed that the relevant processes are not influenced by the charge and the potential of the dust. Moreover, ambipolar flux of electrons and ions escaping from the grain has been assumed, such that the electric current emitted is zero. Thus the emission processes are represented here by the single parameter u_e, depending on the balance between the incoming ions that are absorbed on the grain, and the flux of the ions that are ejected from the grains. Generalizations are straightforward and, in all of the cases examined, do not affect the main results reported here, concerning the occurrence of long range potentials, with screening length much larger than the linear Debye length, as well the possible formation of potential wells.

In the following estimates, Maxwellian distributions of the velocity of the ions are considered and $\nu_{n,i} = \sigma_{n,i} n_n v_{Ti}$ for the ion-neutral collisions with cross section $\sigma_{n,i}$ and n_n density of neutrals. This gives, from [11] and the definitions above, $\alpha_{\mathrm{ch}} = 1/(2\sqrt{\pi})$ and:

$$\alpha_{dr} = \frac{2}{3\sqrt{\pi}}\left[\ln\Lambda + \frac{2\tau^2}{z_0^2}\left(1 + \frac{z_0}{2\tau}\right)\right] + \alpha_{dr,n}, \tag{15}$$

where $\alpha_{dr,n} = \sqrt{2}\tau/(a\lambda_{n,i} z_0 P_0)$ is the contribution of the ion-neutral collision to the drag coefficient, $\lambda_{n,i}$ the mean free path of ion-neutral collisions normalized to λ_{Di}, and $\ln\Lambda$ the Coulomb logarithm for ion-dust collisions [13]. In equilibrium condition, it is reasonable to assume that the average ion energy at the grain surface, including the potential energy, should be equal to T_i: for average ion velocity positive at the grain surface this gives (the potential energy being approximated by the OML value) $u_e \simeq (z_0/\tau)^{1/2} > u_{e,o}$; thus potential wells occur in the overall range of dusty plasma parameters.

Numerical results concerning the linear regime are presented here for thermal hydrogen plasma, $P_0 = 0.9$ and ratio of a_d to the ion-neutral mean free path 10^{-4}. Figure 1 shows the parameters $\lambda_1, \lambda_2, g_1, g_2$ and $\rho = \alpha_{dr,n}/\alpha_{dr}$ plotted versus the dust size. In Fig. 2 the potential produced by the dust test particle is compared to the Debye-Hückel potential for the two limiting cases of ion-neutral collisions negligible or dominant with respect to ion-dust collisions.

For small dust size, such that $h_0 \ll 1$, the ordering $k_1, k_2 \ll k_3, k_4$ can be used to evaluate the coefficients and screening lengths in (11): $g_1 \simeq 1$, $g_2 \simeq (k_4/k_3) \times [(k_2/k_3) - w] < 0$, $s_1 \simeq \sqrt{k_3} s_2 \simeq k_1 - (k_2 k_4/k_3)$. In this case, the potential is given by the sum of a screened negative potential with screening length smaller than, but of the same order of $\lambda_{D\mathrm{lin}}$, and a screened positive potential with screening length much larger than $\lambda_{D\mathrm{lin}}$.

For thermal plasmas and small dust size the ordering $O(|g_2|) = O(a^2)$ holds if the ion-dust collisions are dominant; $O(|g_2|) = O(a/\lambda_{n,i})$ holds if the ion-neutral collisions are dominant. Thus potentials wells occur at a distance of the order $\lambda_1 |\ln(a^2)|$ and $\lambda_1 |\ln(a/\lambda_{n,i})|$, respectively. These distances are of the same order of the correlation distances between charged grains observed in thermal plasma experiments [5,6]. Wakefield theories re-

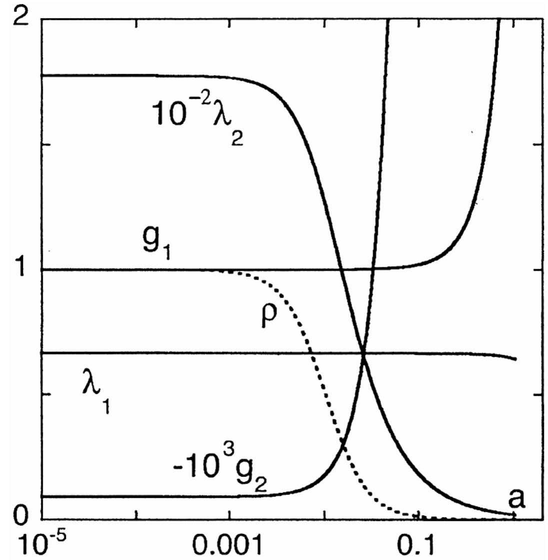

FIG. 1. Plot of the screening function parameters λ_1, λ_2, g_1, and g_2 vs a in a linear regime for hydrogen plasma, $\tau = 1$, $P_0 = 0.9$, $z_0 = 0.843$ (calculated by the charging equation), and $u_e = (z_0/\tau)^{1/2}$. The fractional contribution ρ of the term due to the collisions of the ions with the neutrals in α_{dr} is also shown (dotted line). The lengths are normalized to the linear Debye radius.

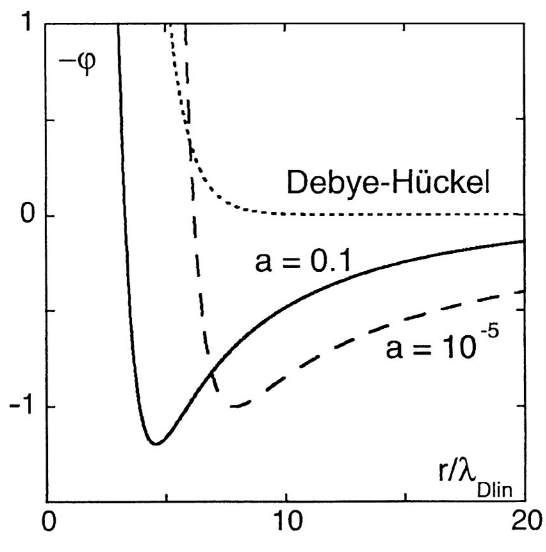

FIG. 2. The potential produced by the test dust particle for $a_d/\lambda_{D\mathrm{lin}} = 0.1$ (normalized to $10^{-3} Z_d e/\lambda_{D\mathrm{lin}}$, continuous line) and $a_d/\lambda_{D\mathrm{lin}} = 10^{-5}$ (normalized to $10^{-5} Z_d e/\lambda_{D\mathrm{lin}}$, dashed line) is compared to the Debye-Hückel potential (normalized to $10^{-3} Z_d e/\lambda_{D\mathrm{lin}}$, dotted line). Dusty plasma parameters as in Fig. 1.

quire anisotropy and cannot predict such long range potentials in isotropic dusty plasmas.

It has been shown that the shielded potential around the isolated, negatively charged grain can have an attractive well for negative charges. This effect is due to the emission of ions by the test dust grain and to the presence of friction forces: the ion density distribution is nonadiabatic and the emitted ions are much more concentrated around the grain than in the adiabatic case, thus producing overscreening and attraction of negative charges at distances larger than the linear Debye length.

It is worth noting, however, that in a real, discrete system of dust particles in plasmas the potential is formed self-consistently, and the precise evaluation of the interaction of two grains requires a numerical analysis.

There is a close analogy between the present result and the attraction found among positively charged grains in the presence of thermionic emission by numerical simulations [7]: in both cases the emission from the grain surface of a flux of charged particles, which have opposite sign to the charge on the grain, produces overscreening and attractive potentials. An accurate fit of the numerical result for the charged grain potential found in [10] is given in [14] by the same Lennard-Jones-like potential here obtained considering collisional and collective interactions. Shadow theories do not include collective effects and predict unscreened interaction potential.

The present work also suggests that attraction of dust grains can occur in dusty plasmas of astrophysical interest: nongravitationally bound systems, such as dust clouds when the ionization is produced by cosmic rays, can be formed and the attractive potential here found should be included to evaluate the virial mass.

The costs of the Letter's publishing were borne by the Euratom Communities under the contract of Association between EURATOM/ENEA. C. C. gratefully acknowledges Sergio Briguglio and Leonardo Pieroni for their helpful comments.

*Email address: castaldo@frascati.enea.it

[1] V. N. Tsytovich and U. de Angelis, Phys. Plasmas **6**, 1093 (1999).
[2] V. N. Tsytovich and G. E. Morfill, Plasma Phys. Rep. **28**, 171 (2002).
[3] G. E. Morfill, V. N. Tsytovich, and H. Thomas, Plasma Phys. Rep. **29**, 1 (2003).
[4] V. N. Tsytovich and G. E. Morfill, Plasma Phys. Controlled Fusion **46**, B527 (2004).
[5] V. E. Fortov, V. I. Moloktov, A. P. Nefedov, and O. F. Petrov, Phys. Plasmas **6**, 1759 (1999).
[6] V. E. Fortov *et al.*, Phys. Rev. E **54**, R2236 (1996).
[7] G. L. Delzanno, G. Lapenta, and M. Rosenberg, Phys. Rev. Lett. **92**, 035002 (2004).
[8] M. Lampe, V. Gavrishchaka, G. Ganguli, and G. Joyce, Phys. Rev. Lett. **86**, 5278 (2001).
[9] M. Lampe *et al.*, Phys. Plasmas **10**, 1500 (2003).
[10] G. L. Delzanno, A. Bruno, G. Sorasio, and G. Lapenta, Phys. Plasmas **12**, 062102 (2005).
[11] V. N. Tsytovich and U. de Angelis, Phys. Plasmas **11**, 496 (2004).
[12] V. N. Tsytovich, R. Kompaneets, U. de Angelis, and C. Castaldo, New Journal of Physics (to be published).
[13] U. de Angelis, C. Marmolino, and V. N. Tsytovich, Phys. Rev. Lett. **95**, 095003 (2005).
[14] G. L. Delzanno and G. Lapenta, Phys. Rev. Lett. **94**, 175005 (2005).