# Ion Viscous Heating in a Magnetohydrodynamically Unstable Z Pinch at Over $2 \times 10^9$ Kelvin

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Pulsed power driven metallic wire-array Z pinches are the most powerful and efficient laboratory x-ray sources. Furthermore, under certain conditions the soft x-ray energy radiated in a 5 ns pulse at stagnation can exceed the estimated kinetic energy of the radial implosion phase by a factor of 3 to 4. A theoretical model is developed here to explain this, allowing the rapid conversion of magnetic energy to a very high ion temperature plasma through the generation of fine scale, fast-growing $m = 0$ interchange MHD instabilities at stagnation. These saturate nonlinearly and provide associated ion viscous heating. Next the ion energy is transferred by equipartition to the electrons and thus to soft x-ray radiation. Recent time-resolved iron spectra at Sandia confirm an ion temperature $T_i$ of over 200 keV ($2 \times 10^9$ degrees), as predicted by theory. These are believed to be record temperatures for a magnetically confined plasma.

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There has been some difficulty in understanding how the radiated energy in a wire-array Z pinch implosion could be up to 4 times the kinetic energy [1–4], and also how the plasma pressure could be sufficient to balance the magnetic pressure at stagnation if the ion and electron temperatures were equal. In fact, theoretically the excess magnetic pressure should continue to compress the plasma leading to a radiative collapse. Some theories [5,6] have been developed to explain the additional heating, but neither of these have addressed the pressure imbalance.

Taking the pressure balance issue first, we consider the example, shot Z1141 on the Z accelerator at Sandia, which employed a 450 $\mu$g/cm nested stainless-steel wire array with an initial outer array diameter of 55 mm, inner of 27.5 mm and a 2:1 mass ratio. The measured electron temperature $T_e$ from line ratios and continuum slope is found to be 3 keV at stagnation. If no axial magnetic fields are present, pressure balance requires that the Bennett relation [7]

$$\mu_0 I^2 = 8\pi N_i e(T_i + ZT_e) \qquad (1)$$

holds at least approximately at stagnation, where $Z$ is the ion charge, $I$ is the current, and $N_i$ is the number of ions per unit length. To indicate the scale of the problem, for a current rising in 100 ns to 18 MA at stagnation on the Z accelerator at Sandia with $N_i = 3.41 \times 10^{20}$ m$^{-1}$ (70% of initial mass) and $Z = 26$ requires a value of $(T_i + ZT_e)$ of 297 keV while $ZT_e$ accounts for only 78 keV.

This could be resolved either if the ion temperature were 219 keV or if the Bennett relation were not applicable because of generation of an internal axial magnetic field and associated magnetic pressure through the development of a helical $m = 1$ unstable kink mode which was not observed on this shot. There is, indeed, some evidence of a helical structure at late times in the discharge, but it does not appear to be significant at the time of the main 5 ns FWHM soft x-ray radiation pulse where mainly long wavelength $m = 0$ modes can be and are observed. There is evidence from other wire-array experiments [8] that not all the mass and perhaps not all the current arrives on axis to the main pinch but resides in the trailing mass arising from axially nonuniform erosion and ablation of the wire cores. However, while we assume that 30% of the initial mass is left behind as trailing mass a much smaller fraction of the current would be associated with the trailing mass because it pinches in on a longer time scale compared to the main implosion, despite having the benefit of the magnetic field produced by the main pinch. However, the same pressure balance discrepancy arises in gas puff Z pinch implosions [9] in which the density and temperature profiles have actually been measured at stagnation, but which also have a hitherto unexplained high measured ion temperature of 36 keV.

Turning to the issue of energy imbalance, the only significant source of energy available to add to the kinetic energy of implosion is the magnetic energy. This will be a fixed source if the magnetically insulated pulse-forming lines are shorted out at this time, but otherwise more energy will be available. But the classical Spitzer resistive heating time for a pinch of radius $a$ of 2 mm at $T_e = 3$ keV is 8 $\mu$s, and there is little possibility of anomalous resistivity because the mean ratio of drift velocity to ion sound speed is 0.03. There is no evidence at the time of the main radiation pulse that there is, for example, a narrow, low line-density region carrying the current which would immediately precede a disruption. In this Letter a new approach to the rapid conversion of magnetic energy to radiation via short wavelength $m = 0$ interchange MHD instabilities, the subsequent ion viscous heating, then equipartition to the electrons, and finally to soft x-ray radiation is proposed.

 

Ideal MHD instabilities grow at a rate $\gamma_{\rm MHD} \approx c_A(k/a)^{1/2}$, i.e., proportional to $k^{1/2}$ where $k$ is the axial wave number, $a$ is the pinch radius, and $c_A$ is the Alfvén speed. The shortest wavelength for the compressional $m = 0$ interchange mode under the above conditions will be determined by a viscous Lundquist number $L_\mu$ defined in terms also of the sound speed $c_s$ by

$$L_\mu = \frac{2\rho({c_A}^2 + {c_s}^2)^{1/2}}{(1/3\mu_\parallel + \nu_1)k} \tag{2}$$

where $\mu_\parallel$ is the parallel ion viscosity, proportional to $T_i^{5/2}$, and $\nu_1$ is the perpendicular ion viscosity given approximately by $\mu_\parallel/(1 + 4\Omega_i^2\tau_{ii}^2)$ where $\Omega_i$ and $\tau_{ii}$ are the ion cyclotron frequency and ion-ion collision time, respectively. When $L_\mu$ is equal to 1, MHD perturbations are critically damped. The fastest growing mode is likely to be at $L_\mu$ approximately equal to 2 resulting in $(ka) \approx 10^2$. The precise value of $L_\mu$ to be used requires further theoretical work [10] and simulations, but this value is adequate to illustrate the importance of this heating mechanism, and will determine the wave number of the instability which will be important for viscous heating. It can be shown later that the ion Hall parameter $\Omega_i\tau_{ii}$ is greater than one and so ion magnetization and the full ion stress tensor [11] should be used in the analysis. This will also limit the wavelength of the instability to the ion Larmor radius, but the wavelength given by $L_\mu = 2$ is usually larger and perhaps more appropriate.

To estimate the amplitude of the MHD instability, the nonlinear saturation of the mode will occur when $(\tilde{\nu} \cdot \nabla)\tilde{\nu}$ (the convective derivative) is comparable with $\gamma_{\rm MHD}\tilde{\nu}$. Then the eddy velocity magnitude $\tilde{\nu}$ will be $c_A/\sqrt{(ka)}$. Since we will be dealing with $(ka) \approx 10^2$ it follows that the eddy velocity is a small fraction of the ion thermal speed. The fractional magnetic field fluctuation level is $1/(ka)$, which will also be small.

The viscous heating rate for the $m = 0$ mode has a component that is not reduced by ion magnetization because this mode is an interchange mode and is compressible. Furthermore, its growth rate is generally larger than the $m \geq 1$ MHD modes. Equating the ion viscous heating rate per unit volume to the equipartition rate and choosing $k$ so that $L_\mu$ equals 2 leads to

$$\rho\frac{c_A^2}{a}(c_A^2 + c_s^2)^{1/2} = \frac{3n_e e}{2\tau_{\rm eq}}(T_i - T_e) \tag{3}$$

where the equipartition time $\tau_{\rm eq}$ is given by

$$\frac{1}{\tau_{\rm eq}} = \frac{8\sqrt{2\pi m_e}e^{5/2}}{3m_i(4\pi\varepsilon_0)^2}\frac{Z^2 n_i \ell n\Lambda_{ei}}{T_e^{3/2}}. \tag{4}$$

$n_e$, $n_i$, $m_e$ and $m_i$ are the electron and ion number densities and masses, $T_e[\gg T_i(m_e/m_i)]$ is the electron temperature in electron volts, and $\Lambda_{ei}$ is the ratio of the Debye length to the Landau parameter. $\tau_{\rm eq}$ is approximately 5 ns for the example given and this together with the assembly time for the plasma to arrive at stagnation will be the controlling time scale determining the rise time of the soft x-ray pulse. The Alfvén transit time $a/c_A$ is 1 to 2 ns and [see Eq. (3)] represents the viscous heating time. Thus for stagnated $Z$ pinches where $\tau_{\rm eq}$ is significantly longer than $a/c_A$ the ion temperature will greatly exceed the electron temperature. In contrast, the ion-ion collision time is 37 ps, allowing rapid thermalization. The theoretical wavelength range for these experimental conditions is 10 to 100 $\mu$m.

In terms of global quantities and for a parabolic density profile with uniform temperature and current profiles, Eqs. (3) and (4) combine to give

$$(T_i - T_e) = 2.1 \times 10^{36}\frac{aI^3T_e^{3/2}A^{1/2}}{Z^3N_i^{5/2}\ell n\Lambda_{ei}} \tag{5}$$

where $A$ is the atomic mass number (55.8 for iron). This and Eq. (1) are consistent with $T_i$ of 219 keV and a pinch radius of 3.6 mm for the conditions described above. Further refinements to the model, including a summation over all unstable modes, predict further viscous heating and a smaller pinch radius. The viscous heating and the equipartition to the electrons give an energy conversion in excess of the kinetic energy of the implosion. In the earlier Saturn experiments [3] factors of 3 to 4 were found, but no ion temperature measurements were made; however, simulations required an artificially high $T_i$ ($\gg T_e$) to give agreement between simulated and measured x-ray radiation.

Indeed, without this artificial fix no codes have been able to model these large array diameter experiments. 2D and 3D simulations of wire-array implosions in general [9] require, as input parameters, the wavelength and initial amplitude of modes and a value of the resistivity of the "vacuum," defined as where the plasma density falls below a given value. In addition, no simulation currently includes ion viscosity (let alone the full stress tensor) or a fine enough mesh to model the short wavelength instabilities proposed here. Often an *ad hoc* procedure is used to prevent radiative collapse.

The Doppler-width measurements employed a time-resolved crystal spectrometer with an elliptically bent LiF crystal located 6.64 m from the $Z$ pinch plasma. The detector, 0.16 m from the crystal, was a micro-channel plate (MCP) with channel separation of 12 $\mu$m, backed with Kodak Tmax film. The MCP had 7 detector strips, each consisting of 0.5 $\mu$m of copper over-coated with 0.1 $\mu$m of gold. A minimum of 6 film pixels, (corresponding to 29 eV at the He-like Fe $\delta$ line of 8.45 keV), could determine a line's width with a spectral resolution $(E/\Delta E)$ of 290. The elliptic design has advantages through its focusing ability, allowing slits and filters to be employed, and the creation of an array of Johann focus points where

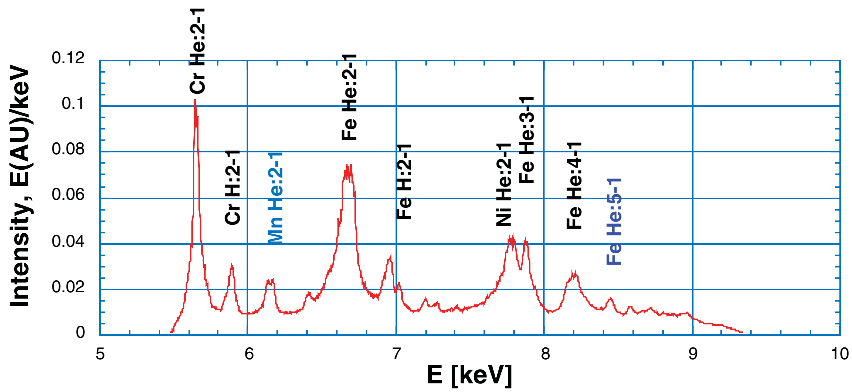


FIG. 1 (color). *K*-shell emission lines from the stainless-steel plasma of Z1141. In addition to the dominating Cr and Fe lines, Mn and Ni are apparent. Ion temperatures were obtained from emission lines with reduced opacity: Fe He-$\delta$ at 8.49 keV and Mn He-$\alpha$ at 6.18 keV.

off-axis, monoenergetic x rays are brought to a focus. (This minimizes source-broadening effects.)

The Doppler-width measurements were made from spectra captured on two shots, Z1137 and Z1141. An example of one of the spectra from shot Z1141 is shown in Fig. 1 and shows the many *K*-shell emissions from the stainless-steel plasma. In addition to the dominating Cr and Fe lines, Mn and Ni lines are apparent. The relatively weak Fe He-$\delta$ line, $1s^2$-$1s5p$ at 1.451 Å (8.49 keV) and the Mn He-$\alpha$ line, $1s^2$-$1s2p$ at 2.006 Å (6.18 keV)were chosen for examination. These lines are less susceptible to opacity effects because the Fe He-$\delta$ is a high order transition (5 to 1) and the Mn line is emitted from a low concentration element of the alloy (2% maximum).

The calculation of ion temperatures used a collisional radiative-equilibrium [12] (CRE) atomic physics model with a plasma diameter of 2.0 mm, $T_e = 3.6$ keV, and $n_i = 10^{26}$ m$^{-3}$, and included opacity broadening. For an assumed ion temperature of 300 keV, the calculated line center optical depth of the Fe He-$\alpha$ line, measured along the radius, is 15. The optical depths of the Mn He-$\alpha$ and Fe He-$\delta$ lines are, respectively, 0.3 and 0.5. By contrast, if the ion temperature is assumed to be equal to the 3.6 keV electron temperature, the optical depth of the Fe He-$\alpha$ line is 130. This variation of the optical depth as the inverse square-root of the ion temperature is a consequence of the dominance of Doppler broadening. Figure 2 compares the experimental measurement of the Fe He-$\delta$ line (from shot Z1137) captured at peak output with a CRE calculation using an ion temperature of 300 keV.

Analysis of these time-resolved lines gives strong support to the validity of the model described in this Letter. The Doppler-width temperatures from these lines ranged from above 200 keV to just above 300 keV, as shown in Fig. 3, for times before and past peak emission. For the data shown in this figure, from shot Z1137, there is good agreement between the temperatures extracted from the iron lines and the manganese lines. The second Fe ion temperature measurement (at 109 ns) was obscured by background noise, and was therefore omitted from the plot. An error of $\pm 35$ keV is assigned to the temperature measurements based on uncertainty in measuring line-widths. On examining the ion temperature for the same lines of shot Z1141, the agreement was again quite good, well within the error associated with the measurement. For this lighter massed load, the temperature evolved from just below 200 keV to the 300 keV level.

A later experiment, shot Z1386, employed a 1 mm axial aperture, which confirmed that the iron spectra were Doppler broadened rather than a shift due to an axially varying c.m. velocity as might occur in a $m = 1$ kink instability.

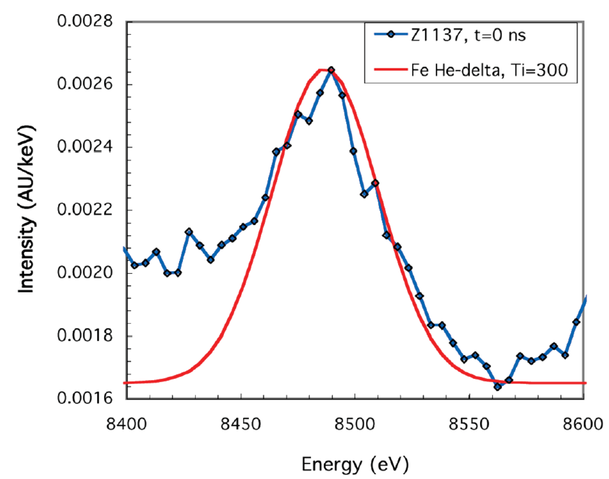


FIG. 2 (color). The measured Fe He-$\delta$ line at 8.488 keV (from shot Z1137) compared to a CRE calculation using an ion temperature of 300 keV. For this He-$\delta$ line, the low energy range of the data does not match the calculation because the intensity of the adjacent He-$\gamma$ line has raised the baseline level. The best fit to the data indicates a slightly lower temperature of 280 keV.

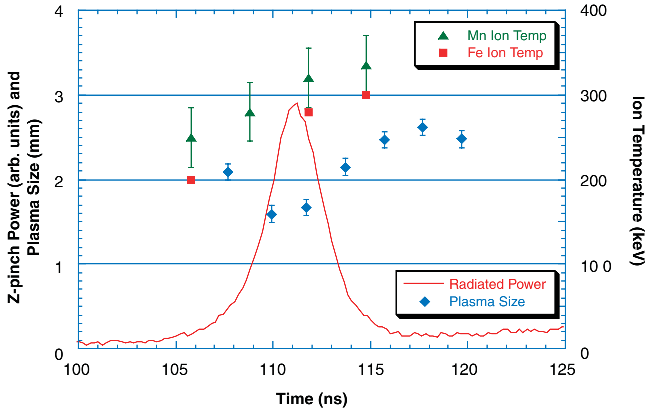


FIG. 3 (color). Measurements of ion temperature, plasma size, and radiated power as a function of time. The plasma size reached a minimum 1 to 2 ns before the x-ray output peaked, and enlarged from this time up to 2.5 mm. The ion temperature rose from 230 to 320 keV. The calculated 0D kinetic energy was reached 7 ns after the peak output. After this time 500 kJ was emitted by the plasma.

In addition to ion temperature, Fig. 3 also shows measurements of plasma size and radiated power as a function of time. The plasma size, measured with a framing pinhole camera, reached a minimum of 1.5 mm (FWHM) just 1 to 2 ns before the radiation power peaked, at which time the density and the equipartition would be a maximum. The plasma enlarged from this time to 2.5 mm 6 ns after peak output, while the ion temperature increased gradually from 240 keV to 320 keV, consistent with the viscous heating model. The slowly evolving pinch radius has a velocity of only about 15% of the ion thermal speed and justifies the earlier assumption of pressure balance.

This heating mechanism might have applications to astrophysical plasmas and also to magnetic reconnection in general. Indeed at much lower temperatures and with kink MHD modes, ion viscous heating has also been suggested for the reversed field pinch [13,14].

In conclusion, it appears that short wavelength $m = 0$ MHD instabilities at stagnation in low mass implosions provide fast viscous heating of ions to record temperatures of over 200 keV. Such temperatures have been measured, the energy coming from conversion of magnetic energy on a 5 ns time scale. The ions heat the electrons which immediately radiate the energy. Furthermore, the broadened spectral lines arising from the high ion temperature will permit a greater radiative power to occur due to decreased opacities. The proposed mechanism provides a plausible explanation of several phenomena of fundamental importance to $Z$ pinch dynamics including pressure balance at stagnation, the absence of radiative collapse, the significant excess of x-ray radiation emitted, and, as now found, record ion temperatures.

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