Predicting Narrow States in the Spectrum of a Nucleus beyond the Proton Drip Line

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Properties of particle-unstable nuclei lying beyond the proton drip line can be ascertained by considering the (usually known) properties of its mirror neutron-rich system. We have used a multichannel algebraic scattering theory to map the known properties of the neutron-^{14}C system to those of the proton-^{14}O one from which we deduce that the particle-unstable ^{15}F will have a spectrum of two low-lying broad resonances of positive parity and, at higher excitation, three narrow negative-parity ones. A key feature is to use coupling to Pauli-hindered states in the target.

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There is much current interest in the properties of and reactions with nuclei that lie out of the valley of stability. The masses of hundreds of such nuclei that lie between the nucleon drip lines are now known. So are some spectral properties of those that can be formed with sufficient intensity for a radioactive ion beam (RIB) to be made. Besides the inherent interest in studying the properties of weakly bound many-nucleon systems, many of these radioactive nuclei are crucial in current investigations of energy and mass production in astrophysics. But little is known about nuclear systems at, and especially beyond, the drip lines. Those particle-unstable systems are difficult to study as they may only be formed during nuclear reaction processes.

Recently, data have been obtained [1,2] from elastic scattering of radioactive ^{14}O ions from hydrogen which reveal two states in the proton-rich nucleus ^{15}F; a nucleus that lies beyond the proton drip line. Those data indicate that besides the resonant ground state, ^{15}F has a narrower first excited resonant state 1.3 MeV above the broad resonance that is the ground state. Herein we report on our analysis of those data and predict that there should be even narrower resonances in ^{15}F lying in an energy range just above the limit of the reported data.

As method of analysis we use the multichannel algebraic scattering theory (MCAS) [3]. It has the distinctive capacity to embrace in the scattering equations single-particle aspects, collective-type coupled-channel dynamics, and the Pauli principle. The Pauli principle is taken into account using the orthogonalizing-pseudo-potential (OPP) method [4]. Past studies [3–5] used that OPP scheme to deal only with Pauli-blocked and Pauli-allowed states. In this Letter we use the OPP scheme to consider also Pauli-hindered states, namely, states where the Pauli blocking is partially relaxed (Pauli hindrance). With this new feature, and with the instructive property of considering results in the limit of zero deformation [5], our analyses of the $p + {}^{14}$O system and of its mirror, the $n + {}^{14}$C system, infer new spectroscopy of the exotic nucleus, ^{15}F.

The concept of Pauli hindrance relates to levels that are neither Pauli forbidden nor Pauli allowed but are somewhere in between. This concept naturally arises in cluster-dynamics formulations based, for example, on the resonating group method (RGM). Therein, such conditions can be studied in detail, even analytically, starting from the properties of the eigenvalues of the RGM norm kernel [6]. The technique based on the introduction of the OPP method, which we have adopted and generalized to multichannel dynamics in the MCAS formulation, is particularly suited for treating such intermediate situations. For reference, Pauli-allowed states relate to zero coupling in the OPP term and complete Pauli blocking is the limit of infinite OPP couplings. In practice, blocking effects can be obtained numerically by having large (of order GeV) values to the OPP couplings, while for Pauli-hindrance couplings of the order of a few MeV are required in the strength of the corresponding OPP term. In our current formulation of the MCAS approach, we had to include this concept of Pauli hindrance in the OPP scheme to deal with breaking effects in shell closures, particularly of $0p_{1/2}$ proton orbits, which is a physical phenomenon expected in weakly bound light exotic nuclei.

Shell-closure aspects in unbound systems represent not only a fundamental question in current research in nuclear structure and reactions involving exotic nuclei, but are also of great relevance for atomic and molecular physics in general. In addition, breaking signals in the full occupancy of deep and well-packed orbits are the subject of a new proposal of studies in atomic physics [7], specifically regarding possible upper limits in the violation of the Pauli principle (VIP). We stress, in this respect, that the shell-breaking phenomena in weakly bound (or unbound) nuclei that we consider in this Letter, and the related concept of Pauli hindrance, are entirely consistent with the validity of the Pauli principle.

With the MCAS approach, an algebraic form for the multichannel scattering matrices is obtained by means of the Sturmian expansion of the full nuclear interactions. Sturmians are solutions of homogeneous Schrödinger equations for the chosen matrix of interaction potentials. The method allows determination of bound states and resonance structures that emerge from multichannel dynamics; this is obtained without the use of complex-energy techniques by studying the trajectories of the Sturmian eigenvalues in the Gauss plane. All resonances up to the limit energy considered are so obtained (spin, parity, centroid energy, and width). Importantly, use of the OPP method in the construction of the Sturmian functions ensures that the Pauli principle is not violated even when a collective-model coupled-channel prescription of the nucleon-nucleus interactions is used.

Low-excitation bound states and resonances in the spectra of nuclei in the mass region $A \sim 13$–31 include many that are expected to be due to weak coupling of a nucleon in the sd shell to the $(A - 1)$ nucleon core. Such states have been found in the spectrum of ^{15}C. The ground and first excited states are bound and have spin parities of $\frac{1}{2}^+$ and $\frac{5}{2}^+$ and they have energies lying below the $n + {}^{14}$C threshold by 1.218 and 0.478 MeV, respectively [8]. On the other hand, the observed two resonances in ^{15}F are centered about 1.47 and 2.78 MeV above the $p + {}^{14}$O threshold. They have the same spin-parity values of the two bound states in ^{15}C and so are considered as analogues. Hence we consider the $n + {}^{14}$C system and the states in ^{15}C first and then add a Coulomb field in the calculations to specify the spectrum and scattering cross section for $p + {}^{14}$O. Both mass-14 nuclei have a 0^+ ground state and then a cluster of excited states $\sim$6 MeV away. In that cluster there are a second 0^+, a 2^+, a 1^-, and a 3^- state. Of those, for simplicity in calculations, we consider only the 0_2^+ and the 2^+ states.

We use a standard collective model for the coupling potentials, with details given in Ref. [3]. It is similar to the potential prescription that we used previously [3–5] for the nucleon-^{12}C system, but now we use only central (0), spin-orbit (so), and l^2 (ll) deformed potentials,

$$V_{cc'}(r) = V_0 v_{cc'}^{(0)}(r, \beta_2) + V_{\text{so}} v_{cc'}^{(\text{so})}(r, \beta_2) + V_{\text{ll}} v_{cc'}^{(\text{ll})}(r, \beta_2). \tag{1}$$

The radial functions in Eq. (1) are derived from the Woods-Saxon function, $[1 + \exp(\frac{r-R}{a})]^{-1}$ and its derivatives. Quadrupole-type geometrical deformation of the nuclear surface was assumed by using $R(\theta) = R_0[1 + \beta_2 P_2(\theta)]$ in the Woods-Saxon function where P_2 is the Legendre polynomial of rank 2 and β_2 is the quadrupole deformation parameter [3]. In evaluations, an undeformed radius parameter $R_0 = 3.1$ fm and diffuseness $a = 0.65$ fm were used. The effects of such surface deformation in $V_{c'c}(r)$ have been taken to second order in β_2. We have chosen $\beta_2 = -0.5$, a value similar to that used before for the $n + {}^{12}$C system [3–5], because there is little information on ^{14}C, and none for ^{14}O, to suggest any different. However, the role of this deformation is primarily to specify the transition interaction for excitation or deexcitation of the 2^+ state.

For the neutron-^{14}C system, the potential strengths have been set to $V_0 = -45.0$ MeV, $V_{\text{so}} = 7.0$ MeV, and $V_{\text{ll}} = 0.42$ MeV, independent of the parity of states. The Coulomb radius used in the $p + {}^{14}$O calculations was 3.1 fm and, with respect to the $n + {}^{14}$C system, the results we show were obtained by reducing the central potential strength V_0 to -44.2 MeV. The reason for this (slight) difference is unclear. It may be a reflection of residual Coulomb interactions not described by the mean electrostatic field considered.

To consider Pauli effects, we need to interpret the structure of the target in terms of shell orbits and their occupancies. We presume that the ground states are described dominantly by two holes in an otherwise closed ^{16}O. Thus for ^{14}C (neutrons) and ^{14}O (protons), the $0s_{1/2}$, $0p_{3/2}$, and $0p_{1/2}$ relevant nucleon orbits in the ground states are considered full. For the ground-state channels in the nucleon-nucleus systems those orbits (of the relevant 8 nucleons) are Pauli blocked while all other orbits are treated as Pauli allowed. However, we presume that the excited states have considerable $2p$-$4h$ (and higher) configurations with the occupancies of the $0p_{1/2}$ orbits most affected. Thus we treat that orbit, for the relevant nucleon type and in the channels involving the excited states, as Pauli hindered. Hence the OPP scheme generates solutions of the Schrödinger equation that are orthogonal to any blocked state and are affected by the Pauli-hindered ones.

In coordinate space, the full nuclear potential is given by the standard local term of Eq. (1), plus the highly nonlocal OPP term

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r') + \lambda_c A_c(r) A_c(r')\delta_{cc'}, \tag{2}$$

where $A_c(r)$ is the normalized radial part of the single-particle bound-state wave function in channel c spanning the phase space excluded by the Pauli principle. The channel indices c designate all relevant quantum numbers. The OPP method for treating Pauli-blocked state effects holds in the limit $\lambda_c \rightarrow \infty$, but use of $\lambda_c = 1$ GeV suffices. For Pauli-allowed states, of course, $\lambda_c = 0$. But for Pauli-hindered states specific values of $1\ \text{GeV} \gg \lambda_c > 0$ are required and which, for the single orbit of relevance, we treat as adjustable parameters.

The spectra, known and calculated using MCAS, are shown in Fig. 1. The specific cases are as indicated in the diagram. Consider the experimental information on the ^{14,15}C nuclei [8], and the results we have obtained for the $n + {}^{14}$C system. The excited states of ^{14}C are clustered and well separated by $\sim$6 MeV from the ground. The spectrum of ^{15}C has two bound states of spin parities $\frac{1}{2}^+$ (ground) and $\frac{5}{2}^+$ and which are dominantly described by a single sd shell neutron on the ^{14}C ground state. Then there are three quite narrow resonances, all having negative parity, which lie

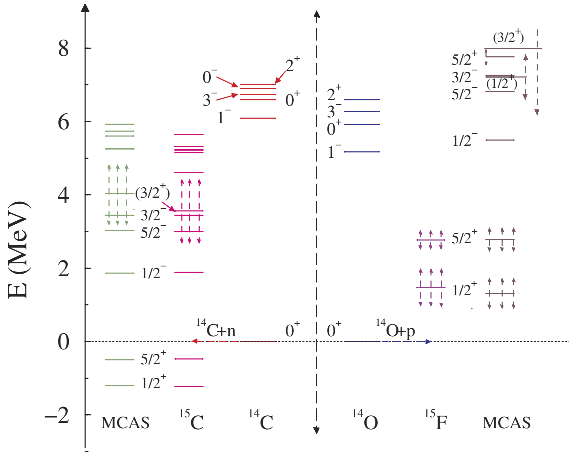

FIG. 1 (color online). Low energy spectra of ^{14,15}C and of ^{14,15}O, and from results of our MCAS calculations. The zero of the energy scale is set to that of the relevant mass-14 ground states.

within the spread of a broad $\frac{3}{2}^+$ resonant state. That broad $\frac{3}{2}^+$ was seen very clearly in the cross section from a measurement [9] of the ^{14}C(d, p) reaction. The MCAS result matches all of those features well. In the zero deformation limit ($\beta_2 \rightarrow 0$), the MCAS results reveal that the bound ($\frac{1}{2}^+$ and $\frac{5}{2}^+$) and resonant $\frac{3}{2}^+$ states are due to the coupling of a $1s_{1/2}$, of a $0d_{5/2}$, and of a $0d_{3/2}$ neutron to the ground state of ^{14}C. It is noteworthy that there are no other bound states; in particular, none having negative parity. Such would occur if in the $n + {}^{14}$C system the $0p_{1/2}$ neutron orbit were not Pauli blocked. The negative-parity states have as their progenitor a $0p_{1/2}$ neutron coupled to the 0_2^+ state (for the $\frac{1}{2}^-$ state) and to the 2^+ state (for the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states). To find these states at this excitation in ^{15}C required that the Pauli hindrance of the neutron $0p_{1/2}$ orbit in the 0_2^+ and 2^+ states of ^{14}C target be generated with $\lambda_c(0p_{1/2})$ values of 3.11 and 3.87 MeV, respectively.

Scattering cross-section results are shown in Fig. 2. In the top panel the cross sections from ^{14}O scattering from hydrogen (in inverse scattering of protons from ^{14}O) at 180° in the center of mass are given. Therein our MCAS result (solid curve) is compared with the recent data of both Goldberg *et al.* [1] (open circles) and Guo *et al.* [2] (filled squares). Clearly the MCAS fit to the measured differential cross section is good and as good as has been found with other analyses [1,10]. The Guo data were in arbitrary units and so we normalized them to the $\frac{5}{2}^+$ resonance values of Ref. [1]. Though the more recent experiment obtained results to 6 MeV, the authors indicate that such are reliable to about 5 MeV. In the bottom panel of Fig. 2 we show our prediction of the total scattering cross section of neutrons from ^{14}C for energies to 5 MeV. That cross section has four obvious resonances, three quite narrow (the negative-parity resonances) and the very broad $\frac{3}{2}^+$ one. The calculated widths of the $\frac{1}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$, and $\frac{3}{2}^+$ states are 0.002, 0.002, 0.09, and 3.4 MeV, respectively. The corresponding experimental values [8] of states are indicated by the arrows with the relevant spin-parities given alongside. One easily identifies the three narrow partners in the 180° cross section for the $p + {}^{14}$O system that is shown in the top panel, while the broad $\frac{3}{2}^+$ structure becomes more complex and overlaps with two other states. The zero of the energy scale has been placed to optimally match the $\frac{5}{2}^+$ bound state in ^{14}C to the centroid of the analogous resonance state in ^{15}F. The centroids and widths found for the ^{15}F resonances are shown in Table I.

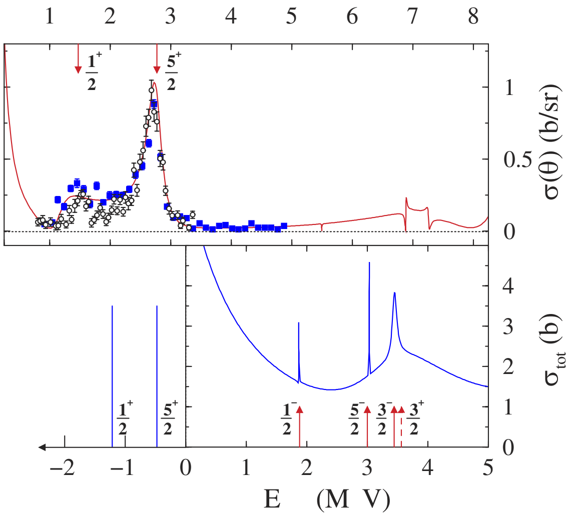

FIG. 2 (color online). The elastic cross sections from scattering of ^{14}O ions from hydrogen at 180° in the center of mass (top) and our predicted total cross section for the scattering of neutrons from ^{14}C (bottom). In both cases the known spectral values are indicated by the arrows.

Noteworthy is that the ground state of the particle-unstable ^{15}F is an *s*-wave resonance. That is so only because of the Coulomb barrier in the $p + {}^{14}$O system. Without the Coulomb barrier there would be no *s*-wave resonance, only a virtual bound state [11]. That criticality was the reason we needed a small reduction in the central interaction strength (of but 0.8 MeV) in order to describe this resonance correctly. Otherwise the interactions used were exactly those determined by our study of the $n + {}^{14}$C system. The two bound states found for ^{15}C have become resonances which, nonetheless, have single-particle-like

TABLE I. Energies (and widths) of resonances in ^{15}F, in MeV.

J^π	Theory	Experiment
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$; $\frac{5}{2}^-$; $\frac{3}{2}^-$	5.49 (0.005); 6.88 (0.01); 7.25 (0.04)	
$\frac{1}{2}^+$; $\frac{5}{2}^+$; $\frac{3}{2}^+$	7.21 (1.2); 7.75 (0.4); 7.99 (3.6)	

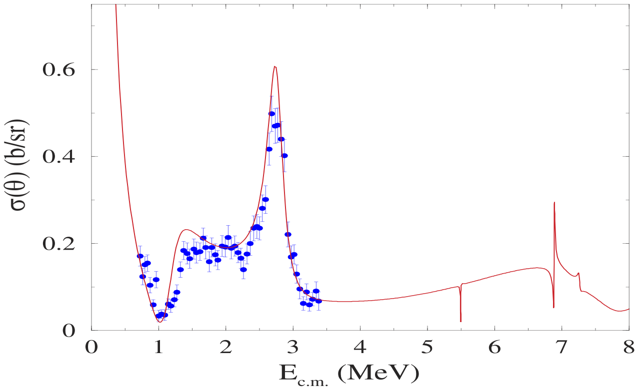

FIG. 3 (color online). The elastic cross sections from scattering of ^{14}O ions from hydrogen at 147° in the center of mass. The data were taken from Ref. [1].

nature. There are other resonance features in our calculated results lying just above the highest energy at which experimental results are known to date. These have negative parities and are analogues of the negative-parity resonances seen in ^{15}C. Thus the origin of these new, narrow negative-parity resonances in ^{15}F differ from those of the observed low-lying ones. They are compound resonances and, as with those identified in ^{15}C, are due to the Pauli hindrance of the proton $0p_{1/2}$ orbit in the 0_2^+ and 2^+ excited states of ^{14}O. Finally, we note that these new resonances persist and are relatively more noticeable in cross sections at other scattering angles. As an example, we show in Fig. 3 results from our MCAS calculation compared with data [1] taken at 147°. Again the two low-lying, broad resonances are predicted well (location, width, and magnitude) and now the higher, narrow, negative-parity resonances are clearly seen to reside on a broad resonant structure. Such is interpreted as a mixture of $\frac{1}{2}^+$, $\frac{5}{2}^+$, and $\frac{3}{2}^+$ broad states, with the last being the analogue of that observed in ^{15}C.

Since the broad resonances are single-particle-like, they are effects of the diagonal interactions in which deformation occurs only in second order. The narrow resonances, on the other hand, have widths that increase with the coupling strength. So our ansatz for β_2 is a conservative one and a smaller β_2 would make *a fortiori* our claim about them.

In conclusion, the MCAS approach has been used with isospin mirror mass-15 systems to define the spectroscopy of the particle-unstable nucleus, ^{15}F. The procedure involved first making an analysis of the neutron-rich ^{15}C system for which experimental information is known. Crucial to the description of the experimental spectrum was the concept of Pauli hindrance of single-particle orbits coupled to the collective 0_2^+ and 2^+ excitations in the mass-14 nuclei. It leads to an appropriate description of the observed three low-lying negative-parity resonances. Then, by incorporating Coulomb interactions, the same nuclear force was used to analyze the proton-^{14}O case and thus to predict the spectroscopy of ^{15}F up to 8 MeV excitation. We clearly see three narrow negative-parity resonances in the calculated cross section. This demands further experiments to test the theoretical interpretation.

The MCAS scheme may be used to estimate spectroscopy of other nuclei that are just outside of the proton drip line given that the numbers of neutron-rich isotopes within the neutron drip line usually exceed those on the proton-rich side. Thus the mirror system against which the proton-rich, unstable, system spectroscopy is to be compared will not be particle unstable and may possibly have experimentally known and detailed properties.

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