## Leptogenesis from Spin-Gravity Coupling following Inflation

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The energy levels of the left- and the right-handed neutrinos are split in the background of gravitational waves generated during inflation, which, in presence of lepton-number-violating interactions, gives rise to a net lepton asymmetry at equilibrium. Lepton number violation is achieved by the same dimension five operator which gives rise to neutrino masses after electroweak symmetry breaking. A net baryon asymmetry of the same magnitude can be generated from this lepton asymmetry by electroweak sphaleron processes.

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The successful prediction of light element abundance by the big-bang nucleosynthesis [1] depends on the assumption that the net baryon number to entropy ratio  $\eta$  as determined by the recent WMAP [2] results is  $\eta = 9.2^{+0.6}_{-0.4} \times 10^{-11}$ . Sakharov's well-known condition demands that in a *CPT* conserving theory, a net lepton/baryon asymmetry can be generated if there is (a) *B/L*-violating interactions, (b) *C* and *CP* violation so that *B/L*-violating reactions in the forward and reverse channels do not cancel, and (c) departure from thermal equilibrium as the statistical distribution of particles and antiparticles is the same if the Hamiltonian commutes with *CPT*. If in the theory we have *CPT* violation, it is possible to generate lepton/baryon asymmetry at thermal equilibrium without requiring *CP* violation.

In this Letter we show that *CPT* is violated spontaneously due to the spin-connection couplings of fermions with cosmological gravitational waves. It is well known that inflation [3] generates a nearly scale invariant spectrum of gravitational waves [4]. The spin-connection couplings split the energy levels of neutrinos compared to antineutrinos, and in the presence of lepton-numberviolating interaction there is a net asymmetry generated between neutrinos and antineutrinos at thermodynamic equilibrium. Lepton number violation is generated by the dimension five operator introduced by Weinberg [5], which also generates the neutrino masses after electroweak symmetry breaking. The lepton-asymmetry gets frozen-in when the lepton-number-violating processes decouple. Baryon asymmetry can then be generated from this lepton asymmetry by the electroweak sphaleron processes [6]. Sphaleron processes conserve (B - L), so a lepton asymmetry generated in the grand unified theory (GUT) era can be converted to baryon asymmetry of the same magnitude [7].

The general covariant coupling of spin-1/2 particles to gravity is given by the Lagrangian [8]  $\mathcal{L} = \sqrt{-g}(\bar{\psi}\gamma^a D_a\psi - m\bar{\psi}\psi)$ , where  $D_a = \partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}$  is the covariant derivative, and  $\omega_{bca}$  are the spin connections  $\omega_{bca} = e_{b\lambda}(\partial_a e_c^{\lambda} + \Gamma_{\gamma\mu}^{\lambda} e_c^{\gamma} e_a^{\mu})$ . This Lagrangian is invariant under the local Lorentz transformation of the vierbein  $e_{\mu}^a(x) \rightarrow \Lambda_b^a(x) e_{\mu}^b(x)$  and the spinor fields  $\psi(x) \rightarrow \exp[i\epsilon_{ab}(x)\sigma^{ab}]\psi(x)$ . Here  $\sigma^{bc} = \frac{i}{2}[\gamma^b, \gamma^c]$  are generators of tangent space Lorentz transformation  $(a, b, c, \text{ etc.}, \text{ denote the inertial frame "flat space" indices and <math>\alpha, \beta, \gamma$ , etc., are the coordinate frame "curved space" indices such that  $e_a^{\mu}e^{\nu a} = g^{\mu\nu}, e^{a\mu}e_{\mu}^b = \eta^{ab}, \{\gamma^a, \gamma^b\} = 2\eta^{ab}$ , where  $\eta^{ab}$  represents the inertial frame Minkowski metric, and  $g_{\mu\nu}$  is the curved space metric).

The spin-connection term in the Dirac equation is a product of three Dirac matrices which after some algebra can be reduced to a vector  $A^a \gamma_a$  and an axial vector  $iB^d \gamma_5 \gamma_d$ . The vector term turns out to be anti-Hermitian and disappears when the Hermitian conjugate part is added to the Lagrangian  $\mathcal{L}$ . The surviving interaction term which describes the spin-connection coupling of fermions to gravity can be written as an axial vector:

$$\mathcal{L} = \det(e)\bar{\psi}(i\gamma^a\partial_a - m - \gamma_5\gamma_d B^d)\psi, B^d = \epsilon^{abcd}e_{b\lambda}(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a).$$
(1)

In a local inertial frame of the fermion, the effect of a gravitational field appears as the axial-vector interaction term shown in (1). We now calculate the four vector  $B^d$  for a perturbed Robertson-Walker universe.

The general form of perturbations on a flat Robertson-Walker expanding universe can be written as [9]

$$ds^{2} = a(\tau)^{2} \{ (1 + 2\phi) d\tau^{2} - \omega_{i} dx^{i} d\tau - [(1 + 2\psi)\delta_{ij} + h_{ij}] dx^{i} dx^{j} \},$$
(2)

where  $\phi$  and  $\psi$  are the scalar,  $\omega_i$  are the vector, and  $h_{ij}$  are the tensor fluctuations of the metric. Of the 10 degrees of freedom in the metric perturbations only six are independent and the remaining four can be set to zero by suitable gauge choice. For our application we need only the tensor perturbations and we choose the transverse-traceless (TT) gauge  $h_i^i = 0$ ,  $\partial^i h_{ij} = 0$  for the tensor perturbations. In the TT gauge the perturbed Robertson-Walker metric can be expressed as

$$ds^{2} = a(\tau)^{2} [(1+2\phi)d\tau^{2} - \omega_{i}dx^{i}d\tau - (1+2\psi - h_{+})dx_{1}^{2} - (1+2\psi + h_{+})dx_{2}^{2} - 2h_{\times}dx_{1}dx_{2} - (1+2\psi)dx_{3}^{2}].$$
 (3)

An orthogonal set of vierbeins  $e^a_\mu$  for this metric is given by

$$e^{a}_{\mu} = a(\tau) \begin{pmatrix} 1+\phi & -\omega_{1} & -\omega_{2} & -\omega_{3} \\ 0 & -(1+\psi)+h_{+}/2 & h_{\times} & 0 \\ 0 & 0 & -(1+\psi)-h_{+}/2 & 0 \\ 0 & 0 & 0 & -(1+\psi) \end{pmatrix}.$$
 (4)

Using the vierbeins (4) the expression for the components of the four-vector field  $B^d$  (1) is given by

$$B^{0} = \partial_{3}h_{\times}, \qquad B^{i} = (\nabla \times \vec{\omega})^{i} + \partial_{\tau}h_{\times}\delta^{i3}.$$
 (5)

The choice of vierbeins (4) which gives the metric (2) is not unique as one can make a local Lorentz transformation (LLT)  $e_{\mu}^{m\prime} = \Lambda_n^{m\prime} e_{\mu}^n$ . Under a LLT the four vector  $B^d$  transforms as  $B^{d\prime} = \Lambda_a^{d\prime} B^a$ . The dispersion relations of the left and the right helicity fermions have  $B^d$  dependent terms of the forms  $\eta_{mn} B^m B^n$  and  $\eta_{mn} p^m B^n$ , and therefore the dispersion relations do not change with a transformation of the local inertial frame.

The fermion bilinear term  $\bar{\psi}\gamma_5\gamma_a\psi$  is odd under *CPT* transformation. When one treats  $B^a$  as a background field, then the interaction term in (1) explicitly violates *CPT*. When the primordial metric fluctuations become classical, i.e., when there is no backreaction of the microphysics involving the fermions on the metric and  $B^a$  is considered as a fixed external field, then *CPT* is violated spontaneously.

The gravitational spin-connection coupling for the neutrinos at high energy is given by

$$\mathcal{L} = \det(e)[(i\bar{\nu}_L\gamma^a\partial_a\nu_L + i\bar{\nu}_R\gamma_a\partial_a\nu_R) + m\bar{\nu}_L\nu_R + m^{\dagger}\bar{\nu}_R\nu_L + B^a(\bar{\nu}_R\gamma_a\nu_R - \bar{\nu}_L\gamma_a\nu_L)],$$
(6)

where  $B^a$  are the parameters of the gravitational waves as defined in (5). If we consider only the standard model fermions, then the right-handed neutrinos carry the opposite Lepton number compared to the left-handed neutrinos,  $\nu_R = (\nu_L)^c$ , and the mass term in (6) is of the Majorana type (we have suppressed the generation index).

The dispersion relation of left and right helicity neutrinos fields are given by  $\eta^{ab}(p_a + \xi B_a)(p_b + \xi B_b) = m^2$ , where  $\xi = -1$  for  $\nu_L$  and  $\xi = 1$  for  $\nu_R$ . Keeping terms linear in the perturbations  $B^a$ , the free particle energy of the left and right helicity states is

$$E_{L,R}(p) = p + \frac{m^2}{2p} \mp \left(B_0 - \frac{\mathbf{p} \cdot \mathbf{B}}{p}\right), \tag{7}$$

with  $p = |\mathbf{p}|$ . In the standard model,  $\nu_L$  carry lepton number +1 and  $\nu_R$  are assigned lepton number (-1). In the presence of nonzero metric fluctuations, there is a split in energy levels of  $\nu_{L,R}$  given by (7). If there are GUT processes that violate lepton number freely above some decoupling temperature  $T_d$ , then the equilibrium value of lepton asymmetry generated for all  $T > T_d$  will be

$$n(\nu_L) - n(\nu_R) = \frac{g}{2\pi^2} \int d^3p \left[ \frac{1}{1 + e^{E_L/T}} - \frac{1}{1 + e^{E_R/T}} \right].$$
(8)

The spin-connection coupling with gravitational waves also splits the energy levels between the charged leftand right-handed fermions. For example,  $E(e_R^-) - E(e_L^-) = 2(B_0 - \mathbf{p} \cdot \mathbf{B}/p)$ . But this does not lead to lepton generation of lepton asymmetry, as both  $e_L^-$  and  $e_R^-$  carry the same lepton number.

In the ultrarelativistic regime  $p \gg m_{\nu}$  and assuming that  $B_0 \ll T$ , the expression (8) for lepton asymmetry reduces to

$$\Delta n_L = \frac{gT^3}{6} \left(\frac{B_0}{T}\right). \tag{9}$$

The dependence on **B** drops out after angular integration in (8), and the lepton asymmetry depends on the tensor perturbations only through  $B^0$ .

To compute the spectrum of gravitational waves  $h(\mathbf{x}, \tau)$  during inflation, we express  $h_{\times}$  in terms of the creationannihilation operator

$$h(\mathbf{x},\tau) = \frac{\sqrt{16\pi}}{aM_p} \int \frac{d^3k}{(2\pi)^{3/2}} [a_{\mathbf{k}}f_k(\tau) + a^{\dagger}_{-\mathbf{k}}f^*_k(\tau)] e^{i\mathbf{k}\cdot\mathbf{x}},$$
(10)

where **k** is the comoving wave number,  $k = |\mathbf{k}|$ , and  $M_p = 1.22 \times 10^{19}$  GeV is the Planck mass. The mode functions  $f_k(\tau)$  obey the minimally coupled Klein-Gordon equation

$$f_k'' + \left(k^2 - \frac{a''}{a}\right)f_k = 0.$$
 (11)

During the de Sitter era, the scale factor  $a(\tau) = -1/(H_I\tau)$ , where  $H_I$  is the Hubble parameter, and Eq. (11) has the solution

$$f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right),\tag{12}$$

which matches the positive frequency flat space solutions  $e^{-ik\tau}/\sqrt{2k}$  in the limit of  $k\tau \gg 1$ . The first term of (12) represents the decaying part of *h* and can be dropped. The second term of (12) represents the amplitude constant

gravitational wave, which survives to the present era. Substituting the second term of (12) in (10) and using the canonical commutation relation for  $a_k$  and  $a_k^{\dagger}$ , we get the standard expression for two point correlation of gravitational waves generated by inflation

$$\langle h(\mathbf{x},\tau)h(\mathbf{x},\tau)\rangle^{\inf} \equiv \int \frac{dk}{k} (|h_k|^2)^{\inf},$$
 (13)

with the spectrum of gravitational waves given by the scale invariant form

$$(|h_k|^2)^{\inf} = \frac{4}{\pi} \frac{H_I^2}{M_p^2}.$$
 (14)

There is a stringent constraint  $H_I/M_p < 10^{-5}$  from cosmic microwave background (CMB) data [10]. This constraint limits the parameter space of interactions that can be used for generating the requisite lepton asymmetry. In the radiation era, when these modes reenter the horizon, the amplitude redshifts by  $a^{-1}$  from the time of reentry. The reason is that in the radiation era  $a(\tau) \sim \tau$ , and the equation for  $f_k$  [Eq. (11)] gives plane wave solutions  $f_k = (1/\sqrt{2k}) \exp(-ik\tau)$ . Therefore in the radiation era the amplitudes of h redshifts as  $a^{-1}$ . The gravitational waves inside the horizon in the radiation era will be

$$h_k^{\text{rad}} = h_k^{\inf} \frac{a_k}{a(\tau)} = h_k^{\inf} \frac{T}{T_k},$$
(15)

where  $h_k^{\text{inf}}$  are the gravitational waves generated by inflation (14), and  $a_k$  and  $T_k$  are the scale factor and the temperature when the modes of wave number k entered the horizon in the radiation era. The horizon entry of mode k occurs when

$$\frac{a_k H_k}{k} = \frac{a(T)TH_k}{T_k k} = 1,$$
(16)

where  $H_k = 1.67 \sqrt{g_*} T_k^2 / M_p$  is the Hubble parameter at the time of horizon crossing of the *k* mode.  $g_*$  is the number of relativistic degrees of freedom which for the standard model is  $g_* = 106.7$ . Solving Eq. (16) for  $T_k$ , we get

$$T_{k} = \frac{1}{1.67\sqrt{g_{*}}} \frac{kM_{p}}{a(\tau)T}.$$
(17)

The amplitude of the gravitational waves of mode *k* inside the radiation horizon is, using (17) and (15), given by  $h_k^{\text{rad}} = h_k^{\inf} \frac{a(T)}{k} \frac{T^2 1.67 \sqrt{g_*}}{M_p}$ . Note that the gravitational wave spectrum inside the radiation era horizon is no longer scale invariant. The gravitational waves in position space have the correlation function

$$\langle h(\mathbf{x}, \tau) h(\mathbf{x}, \tau) \rangle^{\text{rad}} = \int \frac{dk}{k} (h_k^{\text{rad}})^2,$$
 (18)

and hence for the spin connection  $B^0$  generated by the

inflationary gravitational waves in the radiation era, we get

$$\langle B^{0}(\mathbf{x},\tau)B^{0}(\mathbf{x},\tau)\rangle = \int \frac{dk}{k} \left(\frac{k}{a}\right)^{2} (h_{k}^{\mathrm{rad}})^{2}$$
$$= \frac{4}{\pi} \left(\frac{H_{I}}{M_{p}^{2}}T^{2}1.67\sqrt{g_{*}}\right)^{2} \int_{k_{\mathrm{min}}}^{k_{\mathrm{max}}} \frac{dk}{k}.$$
 (19)

Now we find that the spectrum of spin connection is scale invariant inside the radiation horizon. This is significant in that the lepton asymmetry generated by this mechanism depends upon the infrared and ultraviolet scales only logarithmically. The scales outside the horizon are blue tilted, which means that there will be a scale dependent anisotropy in the lepton number correlation at two different spacetime points  $\langle \Delta L(r) \Delta L(r') \rangle \sim Ak^n$ , n > 0, where  $\Delta L(r) \equiv L(r) - \overline{L}$  is the anisotropic deviation from the mean value. Unlike in the case of CMB, this anisotropy in the lepton number is unlikely to be accessible to experiments. Nucleosynthesis calculations give us only an average value at the time of nucleosynthesis (when  $T \sim$ 1 MeV). The maximum value of k are for those modes which leave the de Sitter horizon at the end of inflation. If inflation is followed by the radiation domination era starting with the reheat temperature  $T_{\rm RH}$ , then the maximum value of k in the radiation era (at temperature T) is given by  $k_{\text{max}}/a(T) = H_I(\frac{T}{T_{\text{RH}}})$ . The lower limit of k is  $k_{\text{min}} =$  $e^{-N}k_{\text{max}}$ , which are the modes which left the de Sitter horizon in the beginning of inflation (N is the total efolding of the scale factor during inflation,  $N \simeq 55-70$ ). The integration over k then yields just the factor  $\ln(k_{\rm max}/k_{\rm min}) = N$ . The rms value of spin connection that determines the lepton asymmetry through Eq. (9) is  $(B_0)_{\rm rms} = \sqrt{\langle B_0^2 \rangle},$ 

$$(B_0)_{\rm rms} = \frac{2}{\sqrt{\pi}} \left( \frac{H_I}{M_p^2} T^2 1.67 \sqrt{g_*} \right) \sqrt{N}.$$
 (20)

The lepton asymmetry (9) as a function of temperature can therefore be expressed as (taking g = 3 for the three neutrino flavors)

$$\Delta n_L(T) = \frac{1}{\sqrt{\pi}} (1.67\sqrt{g_*}) \sqrt{N} \left(\frac{T^4 H_I}{M_p^2}\right).$$
(21)

The lepton number to entropy density  $(s = 0.44g_*T^3)$  is given by

$$\Delta L \equiv \frac{\Delta n_L(T)}{s(T)} \simeq 2.14 \frac{T H_I \sqrt{N}}{M_P^2 \sqrt{g_*}}.$$
 (22)

Lepton number asymmetry will be generated as long as the lepton-number-violating interactions are in thermal equilibrium. Once these reactions decouple at some decoupling temperature  $T_d$ , which we shall determine, the  $\Delta n_L(T)/s(T)$  ratio remains fixed for all  $T < T_d$ .

To calculate the decoupling temperature of the leptonnumber-violating processes we turn to a specific effective dimension five operator which gives rise to Majorana masses for the neutrinos introduced by Weinberg [5]  $\mathcal{L}_W = \frac{C_{\alpha\beta}}{2M}(\overline{l_{L\alpha}}^c \tilde{\phi}^*)(\tilde{\phi}^{\dagger} l_{L\beta}) + \text{H.c.}$ , where  $l_{L\alpha} = (\nu_{\alpha}, e_{\alpha}^-)_L^T$  is the left-handed lepton doublet ( $\alpha$  denotes the generation),  $\phi = (\phi^+, \phi^0)^T$  is the Higgs doublet, and  $\tilde{\phi} \equiv i\sigma_2\phi^* = (-\phi^{0*}, \phi^-)^T$ . *M* is some large mass scale and  $C_{\alpha\beta}$  are of order unity.

The  $\Delta L = 2$  interactions that result from the operator  $\mathcal{L}_W$  are

$$\nu_L + \phi^0 \leftrightarrow \nu_R + \phi^0, \qquad \nu_R + \phi^{0*} \leftrightarrow \nu_L + \phi^{0*}.$$
 (23)

In the absence of the gravitational waves the forward reactions would equal the backward reactions and no net lepton number would be generated. In the presence of a background gravitational waves the energy levels of the left and right helicity neutrinos are no longer degenerate and this leads to a difference in the number density of leftand right-handed neutrinos of the magnitude given by Eq. (9) at thermal equilibrium. This process continues till the interactions (23) decouple. The decoupling temperature is estimated as follows. The cross section for the interaction  $\nu_{L\alpha} + \phi^0 \leftrightarrow \nu_{R\beta} + \phi^0$  is  $\sigma = \frac{|C_{\alpha\beta}|^2}{M^2} \frac{1}{\pi}$ , and interaction rate  $\Gamma = \langle n_{\phi} \sigma \rangle$  of the  $\Delta L = 2$  interactions is  $\Gamma = \frac{0.122}{\pi} \frac{|C_{\alpha\beta}|^2 T^3}{M^2}$ . In the electroweak era, when the Higgs field in  $\mathcal{L}_W$  acquires a vacuum expectation value,  $\langle \phi \rangle = (0, v)^T$  (where v = 174 GeV), this operator gives rise to a neutrino mass matrix  $m_{\alpha\beta} = \frac{v^2 C_{\alpha\beta}}{M}$ . We can therefore substitute the couplings  $\frac{C_{\alpha\beta}}{M}$  in terms of the light left-handed Majorana neutrino mass. At the decoupling temperature the interaction rate  $\Gamma(T)$  falls below the expansion rate  $H(T) = 1.7 \sqrt{g_*} T^2 / M_p$ . The decoupling temperature is obtained from equation  $\Gamma(T_d) = H(T_d)$  and turns out to be

$$T_d = 13.68\pi \sqrt{g_*} \frac{v^4}{m_\nu^2 M_p},$$
 (24)

where  $m_{\nu}$  is the mass of the heaviest neutrino. A lower bound on the mass of the heaviest neutrino is given by atmospheric neutrino experiments [11]  $m_{\nu}^2 > \Delta_{\text{atm}} =$  $2.5 \times 10^{-3} \text{ eV}^2$ , which means that the decoupling temperature has an upper bound given by  $T_d = 1.3 \times$  $10^{13} (\Delta_{\text{atm}}/m_{\nu}^2)$  GeV. Substituting the expression (24) for *T* in (22), we finally obtain the formula for lepton number:

$$L = 92.0 \left( \frac{v^4 H_I}{m_\nu^2 M_p^3} \right) \sqrt{N}$$
  
= 7.4 × 10<sup>-11</sup>  $\frac{H_I}{4 \times 10^{14} \text{ GeV}} \frac{2.5 \times 10^{-3} \text{ eV}^2}{m_\nu^2} \frac{\sqrt{N}}{10}.$  (25)

As first pointed out in [7], electroweak spahalerons at the temperatures  $T \sim 10^3$  GeV violate B + L maximally and

conserve B - L. Therefore a lepton asymmetry generated at an earlier epoch gets converted to baryon asymmetry of the same magnitude by the electroweak sphalerons.

The input parameters needed for generating the correct magnitude of baryogenesis ( $\eta \sim 10^{-10}$ ) are the amplitude of the  $h_{\times} \sim 10^{-6}$  (or equivalently the curvature during inflation  $H_I \sim 10^{14}$  GeV or the scale of inflation is the GUT scale,  $V^{1/4} \sim 10^{16}$  GeV which is allowed by CMB [12,13]), neutrino Majorana mass in the atmospheric neutrino scale  $m_{\nu}^2 \sim 10^{-3}$  eV<sup>2</sup> [11], and duration of inflation  $H_I t = N \sim 100$  needed to solve the horizon and entropy problems in the standard inflation paradigm. All these parameters are well within experimentally acceptable limits.

To summarize, the mechanism of baryogenesis we propose arises in the standard Einstein's gravity where spontaneous *CPT* violation is caused by gravitational waves, and the  $h_{\times}$  gravitational wave modes which give nonzero spin connection are produced in *generic* inflation scenarios (in contrast to models [14] where baryon asymmetry is created through a gravitational Chern-Simons term, which can be generated if specific *CP*-violating terms are introduced in the inflaton potential which can give rise to birefringent circularly polarized gravitational waves).

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