Fault-Tolerant Quantum Communication Based on Solid-State Photon Emitters

L. Childress,[1] J. M. Taylor,[1] A. S. Sørensen,[1,2,3] and M. D. Lukin[1,2]

[1]*Department of Physics, Harvard University, Cambridge, Massachusetts, 02138, USA*
[2]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts, 02138, USA*
[3]*The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark*

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We describe a novel protocol for a quantum repeater that enables long-distance quantum communication through realistic, lossy photonic channels. Contrary to previous proposals, our protocol incorporates active purification of arbitrary errors at each step of the protocol using only two qubits at each repeater station. Because of these minimal physical requirements, the present protocol can be realized in simple physical systems such as solid-state single photon emitters. As an example, we show how nitrogen-vacancy color centers in diamond can be used to implement the protocol, using the nuclear and electronic spin to form the two qubits.

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Quantum communication holds promise for transmitting secure messages via quantum cryptography, and for distributing quantum information [1]. However, attenuation in optical fibers fundamentally limits the range of direct quantum communication techniques [2]. If a photon is injected into an optical fiber, the probability to retrieve the photon after a distance L decreases exponentially, making transmission very impractical for long distances. In principle, photon losses can be overcome by introducing intermediate quantum nodes and utilizing a so-called quantum repeater protocol [3]. A repeater creates entanglement over long distances by building a backbone of entangled pairs between closely spaced nodes. Performing an entanglement swap at each intermediate node [4] leaves the outer two nodes entangled, and this long-distance entanglement can be used to teleport quantum information [5,6] or transmit secret messages via quantum key distribution [7]. Even though quantum operations are subject to errors, by incorporating entanglement purification [8,9] at each step, one can create long-distance high-fidelity entangled pairs in a time that scales polynomially with distance [3].

In practice, current long-distance quantum communication schemes remain challenging to implement in the laboratory. For example, while approaches based on photon storage in atomic ensembles [10] are being explored, and these implementations are capable of correcting errors caused by photon losses, they offer no protection against more general errors such as those due to dynamical phase fluctuations. Other approaches attempt to create an interface between light and single quantum bits (qubits) [11,12]. In order for these schemes to be fully fault tolerant, existing theories [13] require that each node must contain a small quantum computer whose size increases logarithmically with the communication distance; the construction of such quantum computers represents a considerable challenge. In this Letter, we present a protocol for a fully fault-tolerant quantum repeater in which each node is formed by a two-qubit quantum computer. We thereby avoid the increase in the number of qubits required by previous protocols, which substantially simplifies the experimental realization of quantum repeaters.

In addition to presenting an algorithm for resource-efficient entanglement propagation and purification, we also present a physical system in which it can be implemented. The general protocol is relevant to a variety of systems, including trapped atoms in a cavity [14] or trapped ions [11]. The reduced physical requirements facilitate development of a scheme that is, to our knowledge, the first realistic proposal for the construction of a quantum repeater in a solid-state environment. In particular, we describe how the repeater nodes may be constructed from single photon emitters in solid-state systems by using the nuclear spin degree of freedom to store quantum information while the electron spin is used for communication with neighboring nodes. This may be accomplished, e.g., in nitrogen-vacancy (NV) centers in diamond [15,16], single quantum dots [17–19], etc. We start with the details of our protocol for a fault-tolerant quantum repeater. Then, a specific implementation using NV centers in diamond is developed to demonstrate the scheme.

A protocol for a simple repeater is presented in Fig. 1(a). The total communication channel is divided into small segments of length L_0 by a set of quantum nodes, each containing a two-qubit quantum computer. Initially, two qubits in neighboring nodes are prepared in an entangled state $|\Psi_-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$, where $|0\rangle$ and $|1\rangle$ are the two states of the qubits [solid line between upper qubits in the first and second nodes in Fig. 1(a)]. As detailed below, we envision that such entangled states can be prepared probabilistically between distant nodes with state-selective light scattering. Because the optical fibers connecting the two nodes are lossy, this step is necessarily probabilistic, and has to be repeated until successful. Simultaneously, an entangled state is prepared between two nearby nodes [solid line between upper qubits in the third and fourth nodes in Fig. 1(a)]. Once all entanglement steps succeed,

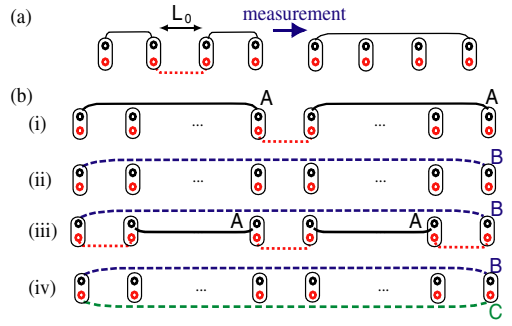

FIG. 1 (color online). Protocol for fault-tolerant quantum communication. Each node is represented by an oval containing two qubits (circles). Entangled states are represented by a solid or a dashed line between the entangled qubits. (a) Entanglement connection. (b) Nested entanglement purification. Dots indicate an arbitrary number of nodes.

the entangled pairs are mapped into storage states (i.e., nuclear spins), and the remaining qubits in the central nodes are entangled in a similar fashion [dotted line between lower qubits in Fig. 1(a)]. Finally, an entanglement swap [4] is performed at each node, which teleports the entanglement between the nodes so that eventually the outer two qubits are entangled [Fig. 1(a), right]. Since entanglement is generated only over a fixed distance L_0, this procedure avoids the exponentially small probability for a photon to travel the full length of the channel and thus allows the construction of long-distance entanglement from short range entanglement—provided that there are no errors.

A single error in the chain will destroy the final state, making the fidelity decay exponentially with distance. To extend entanglement to long distances in the presence of errors, active purification is required at each level of the repeater scheme. According to a standard purification protocol [9], we prepare two entangled pairs between two nodes. Local two-qubit operations at each node are followed by measurement of one qubit at each node. Conditioned on a successful outcome of the measurement, this procedure yields an entangled pair of higher fidelity between the remaining entangled qubits in the two nodes. In Fig. 1(b) we present a protocol for incorporating entanglement purification into a two-qubit repeater scheme. For clarity, we distinguish three types of entangled pairs A, B, and C, labeled according to their purity. A pairs are fully purified high-fidelity pairs ready to be used in the next step of the protocol, B pairs are intermediate pairs, which are being purified to A pairs, and C pairs are the lowest quality pairs, which are used to purify the B pairs.

The argument proceeds inductively: We assume that we have a method to create and purify A pairs over distances up to $L_n = nL_0$ using only two qubits per node and show that we can use these to generate and purify A pairs over a distance $L_{2n+1} = (2n + 1)L_0$. It is fairly straightforward to construct a B pair over distance L_{2n+1} by creating two purified A pairs over the distance L_n, and connecting them via an entangled pair between the central nodes [see Fig. 1(b)(i) and 1(b)(ii)]. Previous schemes [3,13] have constructed a C pair in the same manner—at the cost of requiring an extra qubit in the outermost nodes. Instead, we employ the unused nearest-neighbor nodes, creating two A pairs and three short range pairs, as shown in Fig. 1(b)(iii). Performing an entanglement swap at the central and nearest-neighbor nodes creates a C pair over the distance L_{2n+1}, which can be used to purify the B pair; see Fig. 1(b)(iv). We then perform the purification protocol [9], which, if successful, results in a B pair with higher fidelity. The generation of C pairs and purification may then be repeated. After generating C pairs for m consecutive successful purification steps (a technique sometimes referred to as "entanglement pumping"), the stored pair becomes a purified A pair over the full distance L_{2n+1}. We note that this procedure is most efficient when $n \sim 2^k$ for integer k.

The fidelity obtained at the end of this nested purification procedure, $F(m, L/L_0, F_0, p, \eta)$, depends on the number of purification steps m, the total number of nodes L/L_0, the initial fidelity F_0 between adjacent nodes, and the reliability of measurements $\eta \leq 1$ and local two-qubit operations $p \leq 1$ required for entanglement purification and connection. There is little theoretical insight gained from the mathematical form of F (see Refs. [13,20]), but it is easily evaluated numerically. In Fig. 2 we show the result of a numerical investigation of the protocol in the presence of errors. In this analysis we assume that the qubits do not decohere significantly over the communication time. In Fig. 2(a) we show the fidelity as a function of distance. The curves show the fidelity obtained by using three purification steps $m = 3$. As seen in the figure, the fidelity saturates and shows only a very limited decrease in fidelity with distance, demonstrating the ability of the protocol to correct error and the applicability of the protocol for long-distance quantum communication. As shown in Fig. 2(b), the required time scales polynomially with distance.

To determine the tolerance to the initial fidelity F_0 and to errors in gates $1 - p$ and measurements $1 - \eta$, it is useful to consider the limiting case of many purification steps and large distances. As the number of purification steps increases $m \rightarrow \infty$, the fidelity at a given distance L grows, eventually saturating at a fixed point

$$F \rightarrow F_{FP}(L, F_0, p, \eta). \tag{1}$$

Additional purification steps yield no further benefit at the fixed point, because the increase in fidelity they offer is canceled by the likelihood of errors in the purification procedure [13]. Moreover, as L increases, the fidelity may approach an asymptotic value

$$F_{FP} \rightarrow F_\infty(F_0, p, \eta), \tag{2}$$

which is independent of distance [20]. For comparison

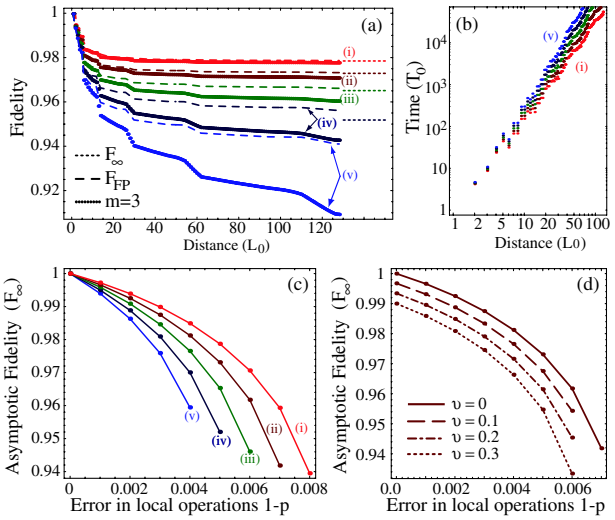

FIG. 2 (color online). (a) Fidelity scaling with distance. Points show results using 3 purification steps at each nesting level; dashed lines show the fixed point F_{FP} at each distance; dotted lines indicate the asymptotic fidelity F_∞. For (a) and (b), measurements and local two-qubit operations $\eta = p$ contain 0.5% errors. For (a), (b), and (c), the initial fidelity F_0 is (i) 100%, (ii) 99%, (iii) 98%, (iv) 97%, (v) 96% with phase errors only. (b) Time scaling with distance for $m = 3$, given in units of $T_0 \gg L_0/c$, the time required to generate entanglement between nearest neighbors, and L_0, the distance between nearest neighbors. (c) Long-distance asymptote dependence on operation and measurement errors. (d) Long-distance asymptote dependence on error type ($F_0 = 0.99$, $\upsilon = 0, 0.1, 0.2, 0.3$).

these quantities are also shown in Fig. 2(a). The asymptotic fidelity F_∞ in Fig. 2(c) shows that our scheme will operate in the presence of $1 - p \lesssim 1\%$ errors in local operations and percent-level phase errors in initial entanglement fidelity. For the specific physical system presented below, the most likely error in entanglement generation between neighboring nodes results in an incoherent admixture of the state $|\Psi_+\rangle = (01\rangle + |10\rangle)$, which we refer to as a phase error. Above we have assumed that only this type of error matters, but other types are, in principle, possible. In Fig. 2(d) we account for arbitrary errors in the initial entanglement by allowing incoherent admixtures of the other two Bell states $|\Phi_\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, each with weight $\upsilon(1 - F_0)$ [9] [the weight of phase errors is thus $(1 - 2\upsilon)(1 - F_0)$]. Although the protocol we use is most effective for purifying phase errors, Fig. 2(d) indicates that it also tolerates arbitrary errors.

For the implementations discussed below, the overall time scale is set by the classical communication time between nodes, and the fidelity is limited by the photon emission probability P_{em} from each node. As an example, using a high photon collection efficiency, a photon loss rate of $\sim$0.2 dB/km, spacing $L_0 \sim 20$ km, an initial fidelity F_0 set by an emission probability $P_{\text{em}} \sim 8\%$ [see Eq. (3) below], local errors $\eta = p = 0.5\%$, and just one purification step at each nesting level, our scheme could potentially produce entangled pairs with fidelity $F \sim 0.8$ sufficient to violate Bell's inequalities over 1000 km in a few seconds. Moreover, the bit rate could likely be significantly improved by employing optimal control theory to tailor the details of the repeater protocol to the parameters of a desired implementation.

The above analysis demonstrates that two qubits per repeater node are sufficient for fully fault-tolerant long-distance quantum communication. To illustrate the possibilities opened up by reduced physical requirements, we now turn to a specific example for implementation of the protocol in a solid-state system: the NV center in diamond. The qubits required for entanglement connection and purification are realized in the electronic triplet ground state and the nuclear spin of a nearby ^{13}C impurity. Each spin can be manipulated by magnetic resonance techniques, and strong hyperfine interactions couple the two qubits, allowing experimental demonstration of two-qubit gates [21]. A single NV center with a nearby ^{13}C can therefore effectively constitute a two-qubit quantum computer.

The remaining requirement for a quantum repeater is an entanglement generation scheme. In atoms and ions, entanglement between spatially separated systems can be generated probabilistically by, e.g., Raman scattering [22] or polarization-dependent fluorescence [11] followed by photon interference. In the spirit of reducing the physical requirements on the system, we show that probabilistic entanglement generation can succeed even in the absence of polarization selection rules, allowed Raman transitions, or even radiatively broadened optical transitions. Our scheme requires only state-selective light scattering [see Fig. 3(a), inset], and is thus applicable to a variety of solid-state emitters including the NV center [15,16,23]. Furthermore, this simple level structure facilitates entanglement of one qubit (the electron spin) while leaving the other qubit (the nuclear spin) undisturbed. In particular, by choosing a scattering transition between $M_s = 0$ electron spin states, we can eliminate sensitivity to the nuclear spin state during entanglement generation [see Fig. 3(b)]. The desire for a simple requirement for level structure combined with the

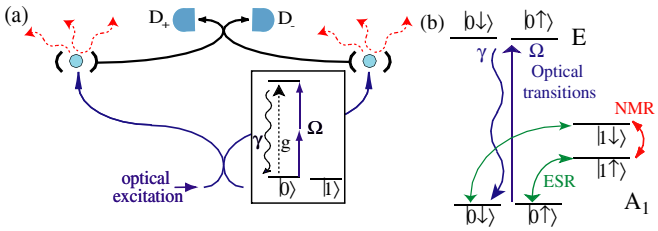

FIG. 3 (color online). (a) Interferometric arrangement for entanglement generation. Inset shows relevant level scheme. (b) Implementation with NV centers. Electronic spin ($|0\rangle, |1\rangle$) and ^{13}C nuclear spin ($|\uparrow\rangle, |\downarrow\rangle$) states are coupled by optical, microwave, and rf transitions. The $M_s = -1$ electronic state can be shifted out of resonance by a small magnetic field.

necessity of preserving the nuclear spin state motivates discussion of a new entanglement generation scheme based on state-selective elastic light scattering and photon interference.

Our entanglement generation scheme proceeds as follows: We consider two NV centers separated by a distance L_0, such that each node scatters light only if its electron spin is in state $|0\rangle$. Two adjacent nodes thus form state-selective mirrors in an interferometer [see Fig. 3(a)]. Our scheme relies on balancing this interferometer so that when both nodes are in the scattering state $|0\rangle$, the outgoing photons will always exit one detector arm D_+. A detection event in the other arm D_- can then project the spins onto an entangled state.

We now address this scheme quantitatively, and determine the entanglement fidelity it can produce. The scheme starts with each node in the state $(|0\rangle + |1\rangle)/\sqrt{2}$; state $|0\rangle$ is then coupled to an excited level that decays radiatively at a rate γ. In the weak excitation limit, we can adiabatically eliminate the excited state, resulting in coherent scattered light. The combined state of node i and the scattered light field is given by $|\psi\rangle_i \approx (|1\rangle + T_i|0\rangle)/\sqrt{2}$ with $T_i = e^{-\sqrt{P_{\rm em}}(\sqrt{1-\epsilon}\hat{b}_i^\dagger + \sqrt{\epsilon}\hat{a}_i^\dagger) - P_{\rm em}/2}$, where $P_{\rm em}$ is the total emission probability, ϵ is the total collection, propagation, and detection efficiency, and $\hat{a}_i$, $\hat{b}_i$ are the annihilation operators for the field reaching the beam splitter and other (loss) fields, respectively. In the limit $P_{\rm em} \to 0$, a detection event in D_- (mode $\hat{d}_- \propto \hat{a}_1 - \hat{a}_2$) projects the system onto a maximally entangled state $\hat{d}_-(T_1|01\rangle + T_2|10\rangle)/2 \propto |\Psi_-\rangle$. For finite $P_{\rm em}$, there is a chance $\sim P_{\rm em}$ that an undetected photon was emitted into the environment. After a detection in D_-, the nodes cannot be in the $|00\rangle$ or $|11\rangle$ state; the additional photon emission thus introduces some admixture of the state $|\Psi_+\rangle$ (a phase error). We find that the scheme succeeds with probability $P = (1/2)(1 - e^{-P_{\rm em}\epsilon/2}) \approx \epsilon P_{\rm em}/4$, producing the state $|\Psi_-\rangle$ with fidelity

$$F_0 = \frac{1}{2}(1 + e^{-P_{\rm em}(1-\epsilon)}) \approx 1 - \frac{P_{\rm em}(1-\epsilon)}{2} \qquad (3)$$

in time $T_0 \approx (t_0 + t_c)/P$.

Finally, we mention some technical aspects of the proposed implementation. First, interferometer stabilization poses a challenge, but has been achieved over $\sim$10 km distances [24]. Alternatively the interferometric setup may be replaced by a photon coincidence detection, which is less susceptible to path length fluctuations [25,26]. Another important source of error is the homogeneous broadening typically found in solid-state emitters. The effect of this broadening can, however, be reduced by sending the light through a narrow frequency filter or a using a cavity [20]. For NV centers coupled to cavities with Purcell factors $\sim$10 [17], we find that the dominant source of error is electron spin decoherence. Using an emission probability $P_{\rm em} \sim 5\%$, a collection efficiency $\epsilon \sim 0.2$, and $t_c \sim 70$ μs over $L_0 \sim 20$ km, we find $F_0 \sim 97\%$ for electron spin coherence times in the range of a few milliseconds. According to our numerical calculations, this fidelity is sufficient for long-distance quantum communication.

In conclusion, we have shown that a fully fault-tolerant quantum repeater can be constructed using only two qubits per node. This opens up the possibility to build repeaters in simple systems with only 2 degrees of freedom, such as coupled nuclear and electronic spins. We have exemplified this with a particular implementation in NV centers, but the concept can be applied to a variety of physical systems [20].

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