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Cosmology with Massive Neutrinos Coupled to Dark Energy

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Cosmological consequences of a coupling between massive neutrinos and dark energy are investigated. In such models, the neutrino mass is a function of a scalar field, which plays the role of dark energy. The evolution of the background and cosmological perturbations are discussed. We find that mass-varying neutrinos can leave a significant imprint on the anisotropies in the cosmic microwave background and even lead to a reduction of power on large angular scales.

The discovery of the accelerated expansion of the Universe is a major challenge for particle physics (see Ref. [1] for the latest results). According to general relativity, the dynamics of the Universe is dominated by a new (dark) energy form with negative pressure. A well-motivated candidate for dark energy is a light scalar field [2,3]. From the particle physics point of view, however, such light scalar fields are problematic: first of all, why is the mass so small and how can this low mass be stabilized against radiative corrections [4]? Secondly: is this field coupled to any other matter form? And if not, why not [5]? Indeed, early papers on quintessence discussed this possibility in detail with models in which dark matter is coupled to dark energy [2,6,7]. The discovery of "dark energy" clearly requires new physics for its explanation.

In this work we explore the cosmological consequences of an idea recently put forward in Ref. [8]. According to this idea, dark energy and neutrinos are coupled such that the mass of the neutrinos is a function of the scalar field which drives the late time accelerated expansion of the Universe. In general, the field will evolve with time and, hence, the mass of the neutrinos is not constant (mass-varying neutrinos). One of the motivations for such considerations is the question of whether there is a relation between the neutrino mass scale and the dark energy scale, that has a similar order of magnitude compared to the detected neutrino mass splittings. In such models the origin of the neutrino mass and dark energy are interlinked. Astrophysical and cosmological implications of such models have recently been studied in Ref. [9]. Here we study for the first time the transition of coupled neutrinos from the relativistic to the nonrelativistic regime as well as the dynamics of the dark energy field. We also consider how the coupling affects the cosmic microwave background radiation (CMB) and large scale structures.

The dark energy sector is described by a scalar field with potential energy $V(\phi)$. This potential has to be seen as an effective, classical one, since the coupling between the scalar field and the neutrinos can lead to significant quantum corrections [4], a problem also present in models with dark matter/dark energy interaction [7]. To be specific, in this Letter we will choose a standard quintessential potential, namely, the exponential potential $V(\phi) = V_0 \exp(-\sqrt{3}\lambda\phi/\sqrt{2})$ (in the following we set $8\pi G \equiv 1$) [2]. With this choice, our theory differs from the one proposed in Ref. [8]. There, the choice of potential was such that the mass of the scalar field is much larger than the Hubble parameter H from times before big bang nucleosynthesis until today. In contrast, with our choice of potential, the mass of the field will be at most of order H. For our purposes the neutrinos can be either Dirac or Majorana particles: the details will not affect our considerations. The only necessary ingredient is that, according to Ref. [8], the neutrino mass is a function of the scalar field, i.e., $m_\nu = m_\nu(\phi)$. Here we consider three species of neutrinos with the same mass and choose a field dependence of the form $m_\nu = M_0 \exp(\beta\phi)$, with $\beta = \mathcal{O}(1)$. The form of the coupling chosen is well motivated (see, e.g., Refs. [2,6]) and has been considered in the past in models with dark matter or dark energy interaction [6,7]. We point out, however, that results for other potentials and couplings are similar to the ones presented here [10].

In the cosmological context, neutrinos cannot be described as a fluid. Instead, we must solve the distribution function $f(x^i, p^i, \tau)$ in phase space (where τ is the conformal time). We are interested in times when neutrinos are collisionless, and so the distribution function f does not depend explicitly on time. Solving the Boltzmann equation, we can then calculate the energy density stored in neutrinos (f_0 is the background neutrino distribution function):

$$\rho_\nu = \frac{1}{a^4} \int q^2 dq d\Omega \epsilon f_0(q), \qquad (1)$$

with $\epsilon^2 = q^2 + m_\nu(\phi)^2 a^2$; a is the scale factor and $q^i = ap^i$ is the comoving momentum. The pressure is

$$p_\nu = \frac{1}{3a^4} \int q^2 dq d\Omega f_0(q) \frac{q^2}{\epsilon}. \qquad (2)$$

From these equations one can easily derive that

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu}{\partial \phi} \dot{\phi}(\rho_\nu - 3p_\nu) \qquad (3)$$

(the dot representing the derivative with respect to τ). This equation is akin to the equation for matter coupled to a scalar field in scalar-tensor theories. The equation of motion for the scalar field reads

$$\ddot{\phi} + 2H\dot{\phi} + a^2 \frac{\partial V}{\partial \phi} = -a^2 \frac{\partial \ln m_\nu}{\partial \phi}(\rho_\nu - 3p_\nu), \qquad (4)$$

which can be obtained from the energy conservation equation of the *combined* fluid of neutrinos and dark energy

$$\dot{\rho}_\nu + \dot{\rho}_\phi + 3H(\rho_\nu + \rho_\phi + p_\nu + p_\phi) = 0 \qquad (5)$$

and Eq. (3). From this equation and our choice of $m_\nu(\phi)$, the dynamic of the field is specified by the effective potential

$$V_{\text{eff}} = V(\phi) + (\tilde{\rho}_\nu - 3\tilde{p}_\nu)e^{\beta\phi}, \qquad (6)$$

where $\tilde{\rho}_\nu = \rho_\nu e^{-\beta\phi}$ and $\tilde{p}_\nu = p_\nu e^{-\beta\phi}$ are independent of ϕ. With $V = V_0 e^{-\sqrt{3}\lambda\phi/\sqrt{2}}$, the field value at the minimum of the effective potential is given by $\phi_{\text{min}} = \lambda^{-1}\ln(\sqrt{3}\lambda V_0/ \sqrt{2}\beta(\rho_\nu - 3p_\nu))$. Restricting to the case $\lambda > 0$, the effective minimum only exists for $\beta > 0$.

With our choice of potential and coupling, the setup is similar to that studied in Ref. [11]. For such a system, a number of critical points have been identified of which only two of these are compatible with an accelerating Universe. However, in our work there is the major difference that the dark energy field couples to neutrinos rather than cold dark matter (CDM). Furthermore, neutrinos have never dominated the dynamics of the Universe in the past and their equation of state is not constant. When the neutrinos are relativistic, the coupling terms containing the trace of the neutrino energy momentum tensor are small, but nonzero. In particular, $(\rho_\nu - 3p_\nu)/\rho_\nu \ll 1$. Given that $\dot{\phi}$ is at most of order H in quintessence models, and β is of order one, it is obvious that the second term on the left hand side in Eq. (3) dominates over the coupling term. We have confirmed this by numerically solving the Boltzmann equation and Eq. (4). The source terms are only significant when the neutrinos become nonrelativistic.

Typically, we find that the system passes through a series of four stages. Firstly, when the neutrinos are ultrarelativistic, the field is frozen and the neutrino mass is constant. Then, as the neutrinos start to become nonrelativistic, part of the energy of the neutrinos is transferred to kinetic energy for the scalar field. Subsequently, at a temperature close to the neutrino mass, the neutrinos become nonrelativistic and begin to scale similarly to dark energy but differently compared to dark matter. Here the kinetic energy dominates the dynamics of the scalar field. During this

time the neutrino mass starts to evolve significantly. Finally, typically at a redshift of order unity, the energy density of dark energy takes over and starts to dominate, while the other energy densities decay away.

In Figs. 1 and 2 we plot the evolution of the density parameter $\Omega_i = \rho_i/\rho_{\text{cr}}$ (ρ_{cr} is the critical density) of the matter species for different choices of β and λ. Clearly, the interaction between the scalar field and the neutrinos significantly modifies the evolution of the neutrino density. The differences in the evolution of Ω_{CDM} and Ω_ν for different choices of β and λ are obvious. The neutrinos do not scale like a^{-3} for some time, since the pressure does not vanish completely at the beginning of the matter dominated epoch and also because of the coupling to the scalar field. This has important consequences for the early time integrated Sachs-Wolfe (ISW) effect, as we will discuss shortly.

For the model studied here, the neutrinos are heavier in the past and become lighter during the cosmological evolution. This results in a larger neutrino density in the past, for the times when the neutrinos are nonrelativistic. Since the mass of the neutrinos is considerably larger in some models, this affects the free-streaming length and hence the matter power spectrum (see Fig. 3). In Fig. 2 (lower panel) we have also plotted the evolution of the neutrino mass for different choices for β and λ. In models with positive β the effective potential possesses a minimum, which explains the late time increase of the masses for the cases with $\beta > 0$: the field rolls from large field values toward the mini-
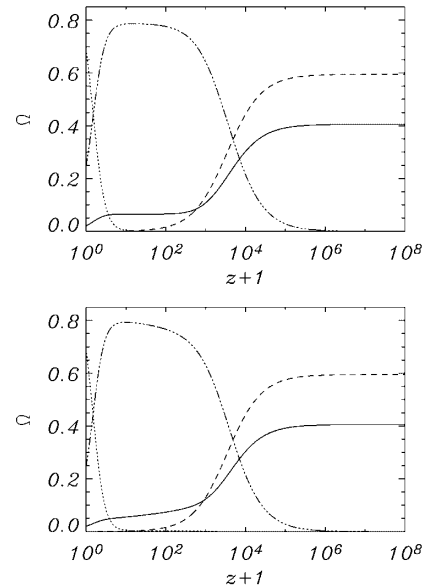

FIG. 1. Background evolution: In the upper panel, we plot the evolution of the density parameters for a model with $\beta = 0$, $\lambda = 1$. In the lower panel the corresponding plot with $\beta = 1$ is shown. (Solid line: neutrinos, dot-dashed line: CDM, dotted line: scalar field, and dashed line: radiation) In all cases, the mass of the neutrinos is $m_\nu = 0.314$ eV today. We are considering a flat Universe with $\Omega_b h^2 = 0.022$, $\Omega_c h^2 = 0.12$, $\Omega_\nu h^2 = 0.01$, and $h = 0.7$.
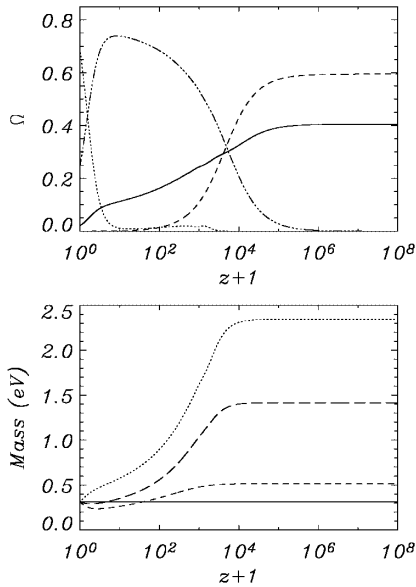
FIG. 2.   The upper plot is the same as Fig. 1, but choosing $\beta = -0.79$ and $\lambda = 1$. The cosmological parameters are chosen as in Fig. 1. The lower plot shows the evolution of the neutrino mass in the different models (solid line: $\beta = 0$, $\lambda = 1$; short dashed line: $\beta = 1$, $\lambda = 1$; dotted line: $\beta = -0.79$, $\lambda = 1$; long dashed line: $\beta = 1$, $\lambda = 0.5$.)

FIG. 3.   Upper panel: the CMB anisotropy spectrum (unnormalized). Solid line: $\beta = 0$, $\lambda = 1$; short dashed line: $\beta = 1$, $\lambda = 1$; dotted line: $\beta = -0.79$, $\lambda = 1$; long dashed line: $\beta = 1$, $\lambda = 0.5$. The lower panel shows the matter power spectrum. From the top curve to the bottom curve: ($\beta = 0$, $\lambda = 1$), ($\beta = 1$, $\lambda = 0.5$), ($\beta = -0.79$, $\lambda = 1$). The matter power spectrum for ($\beta = 1$, $\lambda = 1$) is indistinguishable from the ($\beta = 0$, $\lambda = 1$) curve.

mum, overshoots, comes to a halt, and is currently rolling back towards the minimum. In this case, starting from about redshift unity, the neutrino energy density will decay more slowly than in the cases where the minimum does not exist. This particular behavior plays a role in the late time ISW effect (see below).

In order to study the evolution of cosmological perturbations, we have extended the Boltzmann treatment of Ref. [12] to include the coupling between neutrinos and dark energy [10] (see, e.g., Ref. [13] for detailed discussions on the physics of CMB anisotropies). To calculate the power spectra, we modified the CAMB code [14] accordingly. In Fig. 3 we plot the anisotropy spectrum for different choices of β and λ. We observe a number of differences with respect to the uncoupled case. First, we see an increase in power on scales larger than a degree (multipole number $l < 100$). In some cases an interesting reduction of power can be observed on larger scales (multipole number $l < 10$). Furthermore, for some choices of parameters, the positions as well as the relative heights of the peaks are also affected. We will now discuss these effects in more detail.

As discussed above, the background evolution is modified in the presence of mass-varying neutrinos. In particular, the density of neutrinos is larger at early times in models with $\beta \neq 0$. Around the period of matter-radiation equality, the coupling of the neutrinos to the scalar field causes the neutrino density to decay faster than the energy density of CDM (even if the pressure of the neutrinos is negligible). This can be seen from Eq. (3) and using the fact that during this period $\beta\dot{\phi} < 0$ for the models under
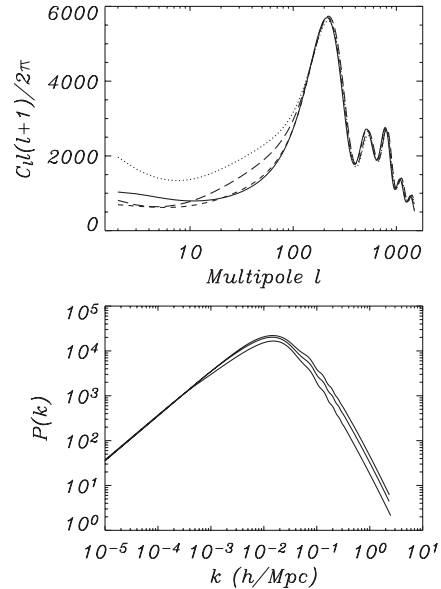
consideration. As a result, the regime between the radiation and matter dominated era is prolonged, which can be seen in Figs. 1 and 2. This implies that the evolution of the metric perturbations $\delta g_{00} = -2a^2\Psi$ and $\delta g_{ij} = -2a^2\Phi\delta_{ij}$ in this period is significantly modified, as shown in Fig. 4. The ISW is an integral of $\dot{\Psi} + \dot{\Phi}$ over conformal time and wave number k and therefore depends on the parameters β and λ. The changes in the evolution of Ψ and Φ in the redshift range $z \approx 50$–1000 imply an excess of power in the CMB spectra, which can be seen in Fig. 3 in the multipole range $10 < l < 100$ for models with $\beta \neq 0$.

The anisotropies on very large scales ($l \lesssim 20$) are dominated by the late time ISW, i.e., by the evolution of $\Psi + \Phi$ in the redshift range between $z = 0$ and $z \approx 1$, which is governed by the evolution of the background and the perturbations. In particular, ρ_ϕ and ρ_ν, as well as the equation of state of dark energy, affect the late time behavior of cosmological perturbations. As mentioned above, the evolution of the scalar field is influenced by the presence of a coupling to the neutrinos and hence the equation of state of dark energy depends upon β. Likewise, the clustering properties of dark energy depends on the coupling to neutrinos (see Ref. [15] for a discussion on the clustering of dark energy and its impact on the CMB). The neutrinos will generally tend to fall into the potential wells of dark matter, although at a rate slightly dependent on the coupling to the scalar field. The scalar field itself will cluster together with the neutrinos and thereby affect the gravitational potential. For some choices of β and λ we find a suppression of power relative to the case with $\beta = 0$.
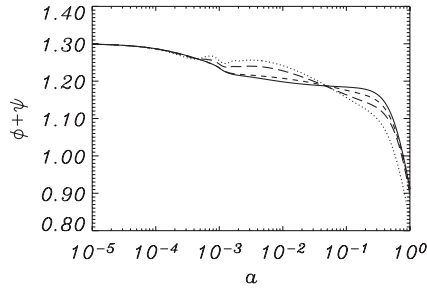
FIG. 4. Evolution of the sum of the metric perturbations $\Phi +
\Psi$. Solid line: $\beta = 0$, $\lambda = 1$; short dashed line: $\beta = 1$, $\lambda = 1$;
dotted line: $\beta = -0.79$, $\lambda = 1$; long dashed line: $\beta = 1$, $\lambda =
0.5$. The scale is $k = 10^{-3}$ Mpc^{-1}.

In particular, the anisotropy spectrum for the model with
$\beta = 1$ and $\lambda = 1$ only differs from the uncoupled case on
large angular scales, leaving the acoustic peaks almost
unmodified, and should therefore result in an improved
fit with the latest WMAP data [16]. However, the reduction
of power on large angular scales is not generic and other
choices for β and λ lead to an enhancement of power in the
region $l = 2$–100, as can be seen in Fig. 3.

Finally, the shifts and slight rescaling in the peaks is
caused by the different densities stored in massive neutri-
nos, baryons and CDM at the time of decoupling when
changing the parameters β and λ. The physics is very
similar to the cases studied in Ref. [17]. The predicted
matter power spectra look very similar to standard models
with CDM + hot dark matter. The damping observed in
the spectra can be simulated by an averaged neutrino mass
in the models considered here. However, there are new
signatures in the CMB power spectra which cannot be
obtained with an averaged neutrino mass and are due to
the coupling between dark energy and neutrinos.

We would like to point out that the decay of neutrinos
into ϕ quanta does not play a role for the parameters
chosen here. Potentially, this can have an important effect
in cosmology (see Ref. [18]). The Lagrangian for the
neutrinos is

$$\mathcal{L}_\nu = m_\nu(\phi)\nu\bar\nu \approx M_0\bar\nu\nu + \frac{\beta M_0}{M_{Pl}}(\phi - \phi_{\min})\bar\nu\nu + \cdots,$$
(7)

where we used $m_\nu(\phi) = M_0\exp(\beta\phi)$, expanded around
the minimum $\phi_{\min}$ and have neglected higher order terms.
This Lagrangian has the same form as the one used in
Ref. [18] if we identify the coupling to be $g = \beta M_0/M_{Pl}$.
For $\beta = \mathcal{O}(1)$ and $M_0 \approx \mathcal{O}(eV)$ the coupling $g \ll 1$ and
indeed much smaller than the value ($g = 10^{-5}$) used in
Ref. [18].

In conclusion, cosmologies with neutrino-dark energy
coupling have a rich phenomenology. It is clear from the
results presented in this Letter that models with mass-
varying neutrinos cannot be mimicked with an averaged
constant neutrino mass. We have found that some models

with a coupling of the order of the inverse of the Planck
mass present a reduction of power in the temperature CMB
anisotropies spectrum at low multipoles but a standard
cosmology peak structure in line with current CMB data.
Our work implies that CMB anisotropies as well as large
scale structures will be able to constrain parameters of the
theory tightly. In the future we will investigate other po-
tentials and couplings, elaborate on the degeneracies be-
tween the parameters and will use current data to constrain
such models [10].

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