BEC-BCS Crossover in "Magnetized" Feshbach-Resonantly Paired Superfluids

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We map out the detuning-magnetization phase diagram for a magnetized (unequal number of atoms in two pairing hyperfine states) gas of fermionic atoms interacting via an *s*-wave Feshbach resonance (FR). The phase diagram is dominated by the coexistence of a magnetized normal gas and a singlet-paired superfluid with the latter exhibiting a BCS-Bose Einstein condensate crossover with reduced FR detuning. On the BCS side of strongly overlapping Cooper pairs, a sliver of finite-momentum paired Fulde-Ferrell-Larkin-Ovchinnikov magnetized phase intervenes between the phase-separated and normal states. In contrast, for large negative detuning a uniform, polarized superfluid, that is, a coherent mixture of singlet Bose-Einstein-condensed molecules and fully magnetized single-species Fermi sea, is a stable ground state.

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Recent experimental realizations of paired superfluidity in trapped fermionic atoms interacting via a Feshbach resonance (FR) [1,2] have opened a new chapter of many-body atomic physics. Almost exclusively, the focus has been on *equal* mixtures of two hyperfine states exhibiting pseudospin singlet superfluidity that can be tuned from the momentum-pairing BCS regime of strongly overlapping Cooper pairs (for large positive detuning) to the coordinate-space pairing Bose-Einstein condensate (BEC) regime of dilute molecules (for negative detuning) [3].

In contrast, *s*-wave pairing for *unequal* numbers of atoms in the two pairing hyperfine states has received virtually no experimental attention and only some recent theoretical activity [4–9]. Associating the two pairing hyperfine states with up ($\uparrow$) and down ($\downarrow$) pseudospin σ, the density difference $\delta n = n_\uparrow - n_\downarrow$ is isomorphic to "magnetization" $m \equiv \delta n$ and the corresponding chemical potential difference $\delta\mu = \mu_\uparrow - \mu_\downarrow$ to a purely Zeeman field $h \equiv \delta\mu/2$.

This subject dates back to the works of Fulde and Ferrell (FF) [10] and Larkin and Ovchinnikov (LO) [11], who proposed that, in the presence of a Zeeman field, an *s*-wave BCS superconductor is unstable to magnetized pairing at a finite momentum $Q \approx k_{F\uparrow} - k_{F\downarrow}$ with $k_{F\sigma}$ the Fermi wave vector of fermion σ. This FFLO state, which remains elusive in condensed matter systems where it is obscured by orbital and disorder effects, spontaneously breaks rotational and translational symmetry and emerges as a compromise between competing singlet pairing and Pauli paramagnetism.

Thus atomic fermion gases (where the above deleterious effects are absent), tuned near an *s*-wave FR, are promising ideal systems for a realization of the FFLO and related finite-magnetization paired states, which can be studied throughout the full BCS-BEC crossover.

In this Letter, we map out the detuning-magnetization phase diagram (Fig. 1) of such paired superfluids. We find that for positive detuning δ and arbitrarily small m, the system phase separates into a magnetized normal gas (N) and a singlet-paired BCS superfluid that exhibits a BCS-BEC crossover with reduced δ. The FFLO state intervenes in a sliver on the boundary between this coexistence region and the N state. For large negative detuning, a uniform magnetized superfluid (SF_M), that is, a coherent mixture of singlet Bose-Einstein-condensed molecules and fully magnetized single-species Fermi sea, is a stable ground state. Our predictions of these states and transitions between

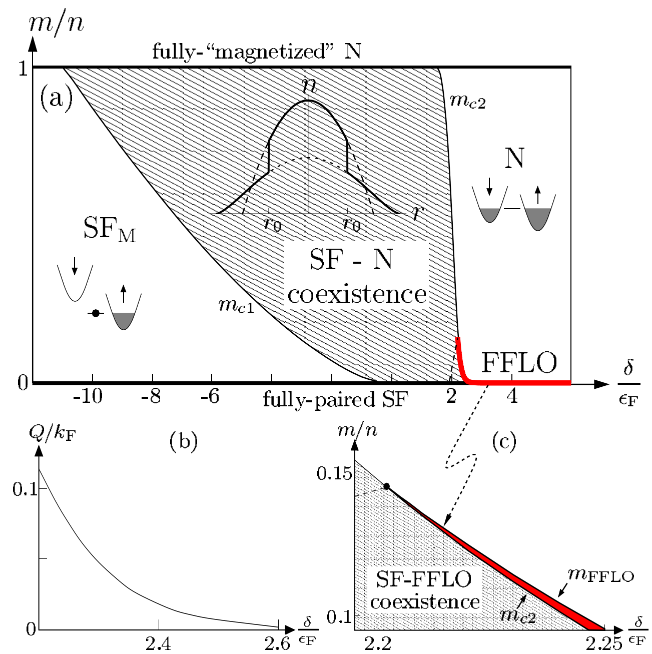

FIG. 1 (color online). Detuning, δ-population difference, $m/n = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ phase diagram (for coupling $\gamma = 0.1$) in (a) displaying "normal" (N), magnetized superfluid (SF_M), FFLO (thick red line) and SF-N coexistence states, (b) showing the FFLO wave vector $Q(\delta)$ along the FFLO-N phase boundary, and (c) zoom-in on the FFLO state, stable only for $\delta > \delta_* \simeq 2.2\epsilon_F$. To the right of the dashed lines in (a) and (c), the SF-N coexistence undergoes a transition to SF-FFLO coexistence.

them are testable via thermodynamics (qualitatively modified by gapless atomic excitations inside the SF_M and FFLO states), sound propagation (with zeroth sound velocity vanishing at the SF_M-N transition), and time-of-flight imaging (displaying density discontinuity and striking Bragg peaks associated with the finite-momentum pairing in the FFLO state).

We now sketch the analysis that led to these results. A gas of fermionic atoms, $\hat{a}_{k\sigma}$, resonantly interacting through a diatomic (closed-channel) molecule, $\hat{b}_q$, is described by a two-channel Hamiltonian [3]:

$$H = \sum_{k,\sigma}(\epsilon_k - \mu_\sigma)\hat{a}^\dagger_{k\sigma}\hat{a}_{k\sigma} + \sum_q\left(\frac{\epsilon_q}{2} + \delta_0 - 2\mu\right)\hat{b}^\dagger_q\hat{b}_q + g\sum_{k,q}(\hat{b}^\dagger_q\hat{a}_{k+q/2\downarrow}\hat{a}_{-k+q/2\uparrow} + \text{H.c.}), \quad (1)$$

where $\epsilon_k \equiv k^2/2m_a$, $\mu_{\uparrow,\downarrow} = \mu \pm h$ are the chemical potentials to impose atom number in hyperfine states $\uparrow, \downarrow$ or the total atom density $n = n_\uparrow + n_\downarrow + 2\sum_q\langle\hat{b}^\dagger_q\hat{b}_q\rangle$ (imposed by μ, with $n_\sigma = \sum_{q,\sigma}\langle\hat{a}^\dagger_{k\sigma}\hat{a}_{k\sigma}\rangle$), and density difference (magnetization) $m = n_\uparrow - n_\downarrow$ (imposed by h). Here, δ_0 is the bare FR detuning, g is the FR coupling determining the resonance width, and the system volume is unity.

For a narrow FR (small g), H can be accurately analyzed by treating $\hat{b}_q$ as a single-momentum [10–13] c-number mode $\langle\hat{b}_q\rangle \equiv b_Q\delta_{q,Q}$ with corrections small [14] in powers of $\gamma \equiv \gamma(\epsilon_F) = g^2\mathcal{N}(\epsilon_F)/\epsilon_F$, the ratio of the FR width to Fermi energy, with $\mathcal{N}(\epsilon_F) = m_a^{3/2}\sqrt{\epsilon_F}/\sqrt{2}\pi^2 \equiv c\sqrt{\epsilon_F}$ the density of states at the Fermi energy $\epsilon_F = k_F^2/2m$ set by the total atom density $n = \frac{4}{3}c\epsilon_F^{3/2}$. To lowest order in γ, standard Bogoliubov analysis [15] gives the ground-state energy ($\hbar = 1$):

$$E_G = \langle H\rangle = \left(\frac{\epsilon_Q}{2} + \delta_0 - 2\mu\right)\frac{\Delta_Q^2}{g^2} - \sum_{\mathbf{k}}(E_k - \varepsilon_k) + \sum_{\mathbf{k}}[E_{\mathbf{k}\uparrow}\Theta(-E_{\mathbf{k}\uparrow}) + E_{\mathbf{k}\downarrow}\Theta(-E_{\mathbf{k}\downarrow})], \quad (2)$$

where $E_{\mathbf{k}\sigma} = E_k \mp (h + \mathbf{k}\cdot\mathbf{Q}/2m_a)$ is the excitation spectrum for a hyperfine state σ, with "gap" $\Delta_Q \equiv gb_Q$ and $E_k \equiv (\varepsilon_k^2 + \Delta_Q^2)^{1/2}$, $\varepsilon_k \equiv \frac{k^2}{2m_a} - \mu + Q^2/8m_a$, and $\Theta(x)$ the Heaviside step function. The corresponding ground state is of the BCS form, but with pairing limited to momenta $\mathbf{k}$ satisfying $E_{\mathbf{k}\sigma} > 0$.

The phase diagram is determined by minimizing E_G over Q and Δ_Q at fixed average total density n, population difference m, and physical detuning $\delta = \delta_0 - g^2\sum_k 1/2\epsilon_k$ (determined by the 2-body scattering amplitude). The competing ground states are (i) a normal Fermi gas (N) with $\Delta_Q = 0$, (ii) a "nonmagnetic" fully paired BCS-BEC superfluid with $\Delta_Q \neq 0$, $\mathbf{Q} = 0$, and $m = 0$, (iii) a "magnetized" partially paired superfluid (SF_M) with $\Delta_Q \neq 0$, $\mathbf{Q} = 0$, and $m \neq 0$, and (iv) a magnetized, finite-momentum paired superfluid (FFLO) with $\Delta_Q \neq 0$, $\mathbf{Q} \neq 0$, and $m \neq 0$. Anticipating the existence of first-order transitions, across which m, n are discontinuous, in order to guarantee a solution everywhere it is essential to also include phase-separated states where two of above pure states coexist as a mixture in fractions $1 - x$ and x to be determined.

The computation of the ground-state energy is simplified by noting that $E_G(h) = E_G(0) - \int_0^h m(h')dh'$, where $E_G(0)$ is the well-studied fully paired $h = 0$ energy and $m(h) = -\partial E_G/\partial h$ is the atom species imbalance number. We compute E_G by first neglecting the FFLO state (i.e., $Q = 0$), which, as we shall show, is stable only for a narrow window of parameters (see Fig. 1). Then,

$$m(h) = \frac{2}{3}c\Theta(h - \Delta)[(\mu + \sqrt{h^2 - \Delta^2})^{3/2} - (\mu - \sqrt{h^2 - \Delta^2})^{3/2}\Theta(\mu - \sqrt{h^2 - \Delta^2})]. \quad (3)$$

For *positive* detuning $\delta \gg \epsilon_F\gamma^{1/2}$, appropriate in the BCS and throughout most of the crossover regimes, $\Delta \ll \mu$ and the density of states inside $E_G(0)$ can be well approximated by a constant $\mathcal{N}(\mu)$, giving

$$E_G^+ \approx \frac{1}{g^2}(\delta - 2\mu)\Delta^2 + \mathcal{N}(\mu)\left[-\frac{1}{2}\Delta^2 + \Delta^2\log\frac{\Delta}{8e^{-2}\mu}\right] - 8\mathcal{N}(\mu)\mu^2/15 - \int_0^h m(h')dh'. \quad (4)$$

For small $h \ll \mu$ the species imbalance contribution to E_G is well approximated by $\int_0^h m(h')dh' \approx \mathcal{N}(\mu)\Theta(h - \Delta)[h\sqrt{h^2 - \Delta^2} - \Delta^2\cosh^{-1}(h/\Delta)]$. For $0 < h < \Delta_{BCS}/2$, E_G^+ exhibits a single minimum at a standard ($h = 0$) BCS value $\Delta_{BCS} = 8e^{-2}\mu e^{-\gamma^{-1}(\delta-2\mu)/(\epsilon_F\mu)^{1/2}}$ and a maximum at $\Delta = 0$. For a higher Zeeman field $\Delta_{BCS}/2 < h < \Delta_{BCS}/\sqrt{2}$, the normal state at $\Delta = 0$ becomes a local minimum separated from the h-independent global minimum at Δ_{BCS} by a maximum at $\Delta_{Sarma} = \Delta_{BCS}\sqrt{2h/\Delta_{BCS} - 1}$ [16]. For $h > \Delta_{BCS}/\sqrt{2}$ the minimum at $\Delta = 0$ lowers below that of the BCS state. For a fixed μ, this predicts a first-order SF-N transition at $h_c(\mu, \delta)$, with asymptotic form in the narrow FR limit given by

$$h_c(\mu, \delta) \approx a_1\mu e^{-a_2\gamma^{-1}(\delta-2\mu)/\sqrt{\epsilon_F\mu}}, \quad (5)$$

where $a_{1,2} = 8e^{-2}/\sqrt{2}, 1$ $(120^{2/5}e^{-8/5}, 4/5)$ for $\mu \ll \delta/2$ ($\mu \gg \delta/2$). The transition is accompanied by a jump in atom density from $n^{(S)}(\mu, \delta) \approx \frac{4}{3}\mathcal{N}(\mu)\mu + 2g^{-2}\Delta_{BCS}^2$ down to $n^{(N)}(\mu, h_c) = \frac{2}{3}c\{(\mu + h_c)^{3/2} + (\mu - h_c)^{3/2}\Theta(\mu - h_c)\} \approx n^{(S)}(\mu, \delta) - \frac{4}{g^2}h_c^2[1 - \gamma(\mu)/8]$, a jump in species imbalance from 0 to $m \approx 2\mathcal{N}(\mu)h_c$, as well as other standard thermodynamic singularities.

In a more experimentally relevant ensemble of fixed total atom number $n = -\partial E_G/\partial\mu$, for $h_{c1} \equiv h_c(\mu^{(S)}(n, \delta), \delta) < h < h_{c2} \equiv h_c(\mu^{(N)}(n, h), \delta)$ neither SF nor N states can satisfy the atom number constraint while remaining a ground state; $\mu^{(S,N)}$ are SF and N chemical potentials at density n, Zeeman field h, and detuning δ, obtained by solving $n = n^{(S,N)}(\mu^{(S,N)})$ above. For a narrow FR, $\gamma \ll 1$, we find

$$h_{c1}(\delta, n) \approx \begin{cases} \frac{1}{\sqrt{2}}\Delta_F(\delta)e^{-[\delta\Delta_F(\delta)/2\sqrt{2}\gamma\epsilon_F^2]^2}, & \text{for } \delta \gg 2\epsilon_F, \\ \frac{1}{2}g[n - \frac{4}{3}c(\delta/2)^{3/2}]^{1/2}, & \text{for } \delta \ll 2\epsilon_F, \end{cases} \tag{6}$$

$$h_{c2}(\delta, n) \approx \begin{cases} \frac{1}{\sqrt{2}}\Delta_F(\delta)e^{-\delta[\Delta_F(\delta)/\epsilon_F]^2/16\gamma\epsilon_F}, & \text{for } \delta \gg 2\epsilon_F, \\ h^{(N)}(\delta/2, n), & \text{for } \delta \ll 2\epsilon_F, \end{cases} \tag{7}$$

where $\Delta_F \equiv \Delta_{\rm BCS}(\delta, \epsilon_F)$ and $h^{(N)}(\delta/2, n)$ is the solution of $n = n^{(N)}(\delta/2, h^{(N)})$. Hence for $h_{c1} < h < h_{c2}$ the gas phase separates [5] into SF and N rich regions in $1 - x(h, \delta)$ and $x(h, \delta)$ proportions, determined by the atom number constraint $xn^{(N)} + (1 - x)n^{(S)} = n$. In above $n^{(S)}(\mu_c, \delta) > n > n^{(N)}(\mu_c, \delta)$ are the SF and N state densities computed along the critical chemical potential $\mu_c(h, \delta)$ determined by Eq. (5) with limiting values $\mu_c(h_{c1,2}, \delta) = \mu^{(S,N)}(n, \delta)$. The fraction of the N state admixture is then given by $x(h, \delta) = [n^{(S)}(\mu_c, \delta) - n] \times [n^{(S)}(\mu_c, \delta) - n^{(N)}(\mu_c, \delta)]^{-1}$ ranging between 0 and 1 for $h_{c1} < h < h_{c2}$ spanning the coexistence region.

A single-valued relation between the magnetization (species imbalance) $m(h, \delta) = \frac{2}{3}c\{[\mu_c(h, \delta) + h]^{3/2} - [\mu_c(h, \delta) - h]^{3/2}\Theta(\mu_c - h)\}$ and Zeeman field h in the normal paramagnetic state allows us to reexpress above predictions in terms the species imbalance number $M = xm$, that is, the quantity (rather than h) that we anticipate to be kept fixed in atomic gas experiments. As illustrated in Fig. 1 in a phase diagram expressed in terms of $m \equiv \delta n$ the fully paired SF state is confined to the detuning axis ($m = 0$) and the boundary between the coexistence region and the N state is given by $m_{c2}(n, \delta) \equiv m(h_{c2}, \delta)$.

We now turn to the *negative* detuning (BEC) regime. Although E_G, Eq. (2) and the phase diagram that follows from it can be accurately computed numerically (Fig. 1), considerable insight can be gained by analytical analysis. This is particularly simple in the $\gamma \to 0$ limit in which $E_G^- \approx (\delta - 2\mu)|b|^2 - (4c/15)(h + \mu)^{5/2}\Theta(h + \mu)$, $m(h, \mu) = (2c/3)(h + \mu)^{3/2}\Theta(h + \mu)$, and $n = 2|b|^2 + m$.

For $h = 0$ and $\delta < 0$, this shows that the BCS superfluid ground state smoothly crosses over to a BEC of closed-channel molecules, with a finite atom excitation gap $\sqrt{\mu^2 + g^2|b|^2} \approx |\mu|$ enforcing atom vacuum and the condensate density $|b|^2 \approx n/2 + \mathcal{O}(\gamma)$. The gap equation $\partial E_G/\partial b = 2(\delta - 2\mu)b \approx 0$ then determines $\mu \approx \delta/2$, as in the crossover region, $\epsilon_F\gamma^{1/2} < \delta < 2\epsilon_F$, above. From the excitation spectrum $E_{k\sigma}$ it is clear that this ground state remains stable for $0 < h < h_m(\delta)$, with $h_m(\delta) = \sqrt{g^2n/2 + (\delta/2)^2} \approx -\delta/2$ determined by $E_{0,\uparrow}(\delta, h_m) = 0$. However, for $h > h_m$, finite species imbalance $m \approx (2c/3)(h - |\delta|/2)^{3/2}$ develops, depleting the SF condensate $|b|^2 = n/2 - (c/3)(h - |\delta|/2)^{3/2}$. The resulting magnetized SF_M is stable for $h_m < h < h_{c2}$, with $h_{c2}(\delta) = (3n/2c)^{2/3} + |\delta|/2 = 2^{2/3}\epsilon_F - \delta/2$ determined by $m(h_{c2}, \delta/2) = n$, giving a smooth extension into the BEC regime of Eq. (7), computed inside the BCS and crossover regimes.

For a narrow FR $E_G^-(\Delta, \mu, \delta, h)$ can be accurately computed analytically giving

$$E_G^- \approx \frac{1}{g^2}(\delta - 2\mu)\Delta^2 + c|\mu|^{5/2}\frac{\pi}{2}\left[\left(\frac{\Delta}{|\mu|}\right)^2 + \frac{1}{2^5}\left(\frac{\Delta}{|\mu|}\right)^4 - \frac{5}{2^{10}}\left(\frac{\Delta}{|\mu|}\right)^6 + \frac{105}{2^{16}}\left(\frac{\Delta}{|\mu|}\right)^8\right] - \int_0^h m(h')dh'. \tag{8}$$

Its minimization together with the atom number constraint fixes $\mu \approx \delta/2 + \mathcal{O}(\gamma)$ and leads to the phase diagram in Fig. 1. We find [15] that above expressions for $h_m(\delta)$ and $h_{c2}(\delta)$ receive only small $\mathcal{O}(\gamma)$ correction for $\delta < -\gamma^{1/2}\epsilon_F$. Hence in contrast to the BCS side, where the system undergoes phase separation for an arbitrary small $m \neq 0$, on the BEC side the transition at $h_m(\delta)$ is into a uniform magnetized superfluid (SF_M) that persists over a finite range of m and is a coherent superposition of a singlet molecular condensate and fully spin-polarized Fermi gas. The sequence $\text{SF} \to \text{SF}_M \to N$ of continuous transitions remains unchanged for $\delta < \delta_c \approx -10.6\epsilon_F$. However, for a finite γ and $\delta_c \approx -10.6\epsilon_F < \delta < -\gamma^{1/2}\epsilon_F$ a secondary local (N state) minimum develops at $\Delta = 0$ leading to a first-order $\text{SF}_M \to N$ transition at $h_{c1}(\delta) \approx -0.65\delta < h_{c2}(\delta)$, preempting a continuous one at $h_{c2}(\delta)$. For a fixed atom density n and $h_{c1}(\delta) < h < h_{c2}(\delta)$ the gas phase separates into coexisting SF_M and N states.

This $h_{c1}(\delta)$ boundary [equivalent to $m_{c1}(\delta) \equiv m[h_{c1}(\delta), \delta/2] \approx 0.029n|\delta/\epsilon_F|^{3/2}$] is accurately [to $\mathcal{O}(\gamma^2)$] located by the vanishing of the coefficient of Δ^4 in $E_G^-(\Delta)$, proportional to the effective molecular scattering length $a_m(h, \delta) = (\sqrt{2}\gamma^2\hbar\epsilon_F\pi^2/64\sqrt{m_a}|\mu|^{3/2})F(h/|\mu|)$. We then predict [15] that the Bogoliubov sound velocity $u(\delta, m)$ vanishes [to $\mathcal{O}(\gamma)$, followed by a small jump to 0] at the

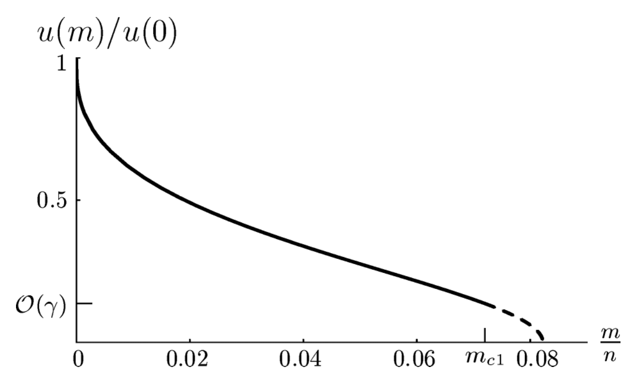

FIG. 2. Bogoliubov sound speed u in the BEC regime as a function of species imbalance m for $\delta = -2\epsilon_F$, vanishing at boundary of the SF_M with the SF-N coexistence region.

first-order SF_M-N transition and exhibits a $-m^{1/3}$ cusp singularity at the SF-SF_M boundary. The full expression is illustrated in Fig. 2 and given by (with $\hat{\delta} \equiv \delta/\epsilon_F$)

$$u \approx u_0\sqrt{1 - m/n}\sqrt{F\left(1 + \frac{2^{5/3}}{|\hat{\delta}|}\left(\frac{m}{n}\right)^{2/3}\right)}, \qquad (9)$$

with $F(x) \equiv 1 - \frac{2}{\pi x^2}[\sqrt{x-1}(x+2) + x^2 \tan^{-1}\sqrt{x-1}]$ and $u_0 = (2^{3/4}\gamma/8\sqrt{3})(v_F\sqrt{\pi}/|\hat{\delta}|^{3/4})$ the sound velocity at $m = 0$.

We now turn to the FFLO state. Because $\mathbf{Q} \neq \mathbf{0}$ pairing is driven by the mismatch of the up $\uparrow$ and $\downarrow$ Fermi surfaces, with $Q \approx k_{F\uparrow} - k_{F\downarrow}$, it is clear that the FFLO state can be stable only at large positive detuning. Computing E_G^+ for $\mathbf{Q} \neq \mathbf{0}$ to leading order in $\Delta_Q \ll \mu$ (using dimensionless quantities $\hat{\Delta}_Q \equiv \Delta_Q/\epsilon_F$, $\hat{\mu} \equiv \mu/\epsilon_F$ $\hat{h} \equiv h/\epsilon_F$, $\hat{Q} = Q\sqrt{\hat{\mu}}/k_F$, and $\varepsilon_G \equiv E_G/c\epsilon_F^{5/2}$), we find

$$\varepsilon_G \approx -\frac{8}{15}\hat{\mu}^{5/2} + \frac{\hat{Q}^2\hat{\Delta}_Q^2}{2\gamma\hat{\mu}} + \sqrt{\hat{\mu}}\left[-\hat{\Delta}_Q^2 - \hat{h}^2 + \frac{\hat{\Delta}_Q^2}{2} \times \ln\frac{4(\hat{Q}+\hat{h})(\hat{Q}-\hat{h})}{\hat{\Delta}_{BCS}^2} + \frac{h\hat{\Delta}_Q^2}{2\hat{Q}}\ln\frac{\hat{Q}+\hat{h}}{\hat{Q}-\hat{h}} + \frac{\hat{\Delta}_Q^4/8}{\hat{Q}^2-\hat{h}^2}\right]. \qquad (10)$$

At fixed μ, for a given $\hat{\delta}$ and $\hat{h}$, the ground state is determined by minimizing $\varepsilon_G(\hat{\Delta}_Q, \hat{Q})$ over $\hat{\Delta}_Q$ (the gap equation) and $\hat{Q}$ (equivalent to vanishing of the ground-state momentum). For $\delta \gtrsim 2\epsilon_F$ we find a first-order SF-FFLO (preempting SF-N) transition approximately at $h_c(\delta, \mu)$, Eq. (5). At fixed atom number, for $h > h_c$ the gas phase separates into coexisting SF and FFLO states, approximately bounded above by $h_{c2}(\delta)$, computed for $Q = 0$ above. At slightly higher field, $h_{FFLO}(\delta)$, we find that the FFLO state undergoes a continuous transition (that on general grounds we expect to be driven first-order by fluctuations) into the N state. Numerical solution of the gap, number and momentum equations yields $h_{FFLO}(\delta)$ [and thus m_{FFLO} via Eq. (3), plotted in Fig. 1] that interpolates between $0.754\Delta_F(\delta)$ for large δ (in agreement with FF [10]) and $h_{c1}(\delta, n)$ for $\delta \to \delta_*$, with the crossing point $\delta_* \approx 2\epsilon_F$. This microscopically calculated value of $\delta^* > 0$ contrasts with the conclusion of Ref. [9], the latter based on a purely *qualitative* discussion, that has little *quantitative* predictive power, e.g., in determining the precise location of phases.

In free expansion experiments, the FFLO state, most easily observed with a trap having a typical size that is large compared to Q^{-1}, should exhibit a BEC peak (observed by its projection onto the molecular condensate [1]) shifted by $\hbar\mathbf{Q}t/m_a$ (t expansion time) corresponding to the finite momentum $\hbar\mathbf{Q}(\delta)$ [Fig. 1(b)] of its condensate, and a (spontaneous) Bragg lattice of peaks in the more-likely case of multiple-$\mathbf{Q}$ pairing [11–13,17]. The anisotropy of the FFLO pairing should also be reflected in "noise" experiments [18] sensitive to *angle dependence* of pairing correlations across the Fermi surface. Our predictions of gapless atomic excitations in the SF_M and FFLO states, as well as the vanishing of the molecular scattering length a_m and of the zeroth sound velocity u at the SF_M-N phase boundary should be observable through Bragg spectroscopy and reflected in thermodynamics (e.g., heat capacity that is power law in T). We also expect standard thermodynamic anomalies across phase transitions in Fig. 1(a), and phase separation accompanied by density discontinuity and local density variation with detuning and atom imbalance across the coexistence region. Finally, because a gas trapped in a smooth potential $V(r)$ is well characterized by a local chemical potential $\mu(r) \equiv \mu - V(r)$ (the Thomas-Fermi approximation), our fixed μ analysis is directly experimentally relevant. For negative detuning and finite species imbalance we predict SF state in the cloud's core of radius $r_0(\delta)$, with density discontinuity to the outer-shell N state, with $r_0(\delta)$ determined by $\mu(r_0) = \mu_c(\delta, m)$ [see center inset of Fig. 1(a)]. We expect that this shell structure should be readily observable, particularly if different hyperfine states and closed and open channels can be imaged independently. Details of these experimental predictions will be presented in a forthcoming publication [15].

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