Nonexistence of Intrinsic Spin Currents

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We describe the electron spin dynamics in the presence of Rashba spin-orbit interaction and disorder using the spin-density matrix method. We show that in the Born approximation in the scattering amplitude the spin current is zero for an arbitrary ratio of the spin-orbit splitting and the scattering rate. Various types of the disorder potential are studied. We argue that the bulk spin current always depends explicitly on scattering by impurities. In this sense universal intrinsic spin current does not exist.

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Spin-orbit coupling brings about a number of interesting effects, one of which is generation of a spin flux in the plane perpendicular to the charge current direction. This phenomenon occurs in the paramagnetic system and is very well known for quite a long time, see Ref. [1], where the Yafet-Elliott spin-orbit mechanism was considered. It is a consequence of the fact that in the presence of spin-orbit coupling the scattering by impurities has an asymmetric character (the Mott effect) [2]. Spins with up orientation are scattered preferably to the right, and those with down orientation to the left. This phenomenon exists only beyond the Born approximation in the scattering amplitude and leads to an accumulation of the spin density near the sample surface [1]. Mutual transformation of the current and spin fluxes leads also to the renormalization of the electrical conductivity of the system, see Ref. [3], where the case of 3D holes described by the Luttinger Hamiltonian was considered.

It has been recently claimed [4,5] that an analogous phenomenon can exist even without scattering by impurities, i.e., in the ballistic regime; the corresponding contribution being called intrinsic. In particular, in the case of the 2D electron system described by the Rashba Hamiltonian, the universal value $e/8\pi\hbar$ for the spin current was derived [4]. Later, several papers appeared where the effect of scattering by impurities was taken into account [6–10] with a range of totally different results. We solve this problem [11] using the well-known method of a spin-density matrix [3]. We show by exact calculations, keeping all components of the spin-density matrix, that in the case of the Rashba Hamiltonian the intrinsic spin current does not exist. In the Born approximation (when the scattering amplitude has additional symmetry properties, see below), the spin current is found to be zero for an arbitrary value of $\Delta\tau$, where Δ is the spin splitting of the electron spectrum and τ is the transport scattering time. It should be noted that calculations for other Hamiltonians give a nonzero result for the spin current already in the Born approximation [12]. However, even in the ballistic limit $\Delta\tau \gg 1$, the value of the spin current depends explicitly on the disorder properties; the result, being independent on the spin-orbit coupling constant, is different for a different correlation radius of the disorder potential. In this sense universal intrinsic spin current does not exist.

Because of the recent observations of the spin accumulation near the sample boundary caused by the current through the sample [13,14], and the relation of this phenomenon to the spin Hall effect, I would like to mention also that the correct criterion for the identification of the extrinsic spin current compared to the "intrinsic" one (see above and [12]) is not only the value of the $\Delta\tau$ parameter, as it is frequently used, but also the strength of the scattering (Born or beyond). Therefore, certain care should be taken when trying to check by exact numerical diagonalization the robustness of the intrinsic value with respect to the disorder, since strong scattering inevitably generates an *extrinsic* contribution to the spin current.

The Hamiltonian of the problem is

$$\hat{\mathcal{H}}(\mathbf{p}) = \frac{p^2}{2m} + \frac{\alpha}{2}\vec{\sigma}\cdot\vec{\Omega}(\mathbf{p}), \qquad \epsilon_M(p) = \frac{p^2}{2m} + M\alpha p, \tag{1}$$

where $\vec{\Omega}(\mathbf{p}) = [\mathbf{n}\cdot\mathbf{p}]$, $\mathbf{n}$ is the unit vector normal to the 2D plane (z axis), $\epsilon_M(p)$ are the eigenvalues, and $M = \pm 1/2$ are the helicity values. The eigenfunctions are

$$\chi_{\pm 1/2}(\mathbf{p}) = \frac{1}{\sqrt{2}}[\pm e^{-i(\phi-\pi/2)/2}u_{1/2} + e^{i(\phi-\pi/2)/2}u_{-1/2}],$$

where ϕ is the angle of $\mathbf{p}$, and u_μ the eigenfunction of the $\hat{\sigma}_z$ operator.

Spin current, kinetic equation.—We will calculate the q_{yz} component of the spin current. This quantity is zero in the thermodynamic limit [15] and defined as

$$q_{yz} = \mathrm{Tr}\int\frac{d^2p}{(2\pi)^2}\hat{f}(\mathbf{p})\frac{1}{2}(\hat{S}_z\hat{V}_y + \hat{V}_y\hat{S}_z) = -\frac{1}{2}\int\frac{d^2p}{(2\pi)^2}\frac{p_y}{m}[f_{+-}(\mathbf{p}) + f_{-+}(\mathbf{p})]. \tag{2}$$

Here $\hat{f}(\mathbf{p})$ is the spin-density matrix [16], $\hat{V}_y$ the y component of the velocity operator and $\hat{S}_z = (1/2)\hat{\sigma}_z$ the spin operator. The last expression in Eq. (2) is given in the

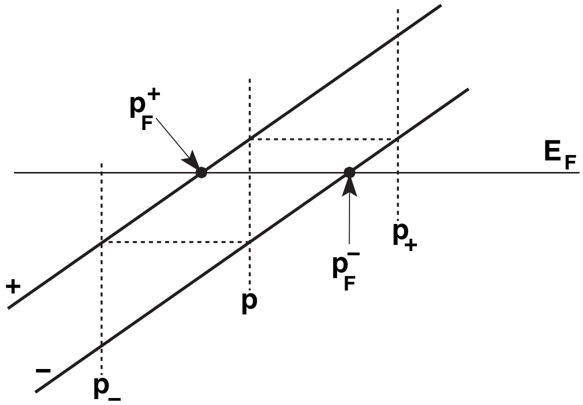

FIG. 1. Schematics of the $\pm$ energy bands. Momenta p, $p_\pm$, $p_F^\pm$ are shown, see the text.

helicity basis. The general expression for the quantum kinetic equation in the case of spin-orbit interaction, when the Hamiltonian and the Wigner distribution function are matrices over the spin indexes, was derived in Ref. [3]. When there is a magnetic field or some inhomogeneity in the problem, the field term and the gradient term must be symmetrized since the velocity operator is also a matrix. In our case, when we deal only with the electric field which is constant in space, this equation is simple and reads

$$\frac{\partial \hat{f}(\mathbf{p})}{\partial t} + e\mathbf{E}\frac{\partial \hat{f}(\mathbf{p})}{\partial \mathbf{p}} + \frac{i}{\hbar}[\hat{\mathcal{H}}(\mathbf{p}), \hat{f}] = St\{\hat{f}(\mathbf{p})\}. \quad (3)$$

It differs from the usual classical kinetic equation by the last term on the left-hand side which is the commutator of the Hamiltonian with the spin-density matrix. This commutator is due to the spin-orbit splitting of the electron spectrum and leads to the precession of the spin around the effective momentum-dependent magnetic field. The expression for the collision term is given below. Now we write Eq. (3) in the helicity basis where the Hamiltonian is diagonal. While doing that, we should take into account the fact that eigenfunctions $\chi_{M\mathbf{p}}$ depend on the direction of the momentum $\mathbf{p}$, thus the matrix elements of the derivative $\partial\hat{f}/\partial\mathbf{p}$ in this basis do not coincide with the quantities $\partial f_{MM'}/\partial\mathbf{p}$

$$\left(\frac{\partial \hat{f}}{\partial \mathbf{p}}\right)_{MM'} = \frac{\partial f_{MM'}}{\partial \mathbf{p}} - \frac{i}{\hbar}[\hat{\mathbf{a}}, \hat{f}]_{MM'};$$
$$\mathbf{a}_{MM'} = i\hbar\chi^*_{M\mathbf{p}}\frac{\partial \chi_{M'\mathbf{p}}}{\partial \mathbf{p}}.$$

We see that there appears the commutator of the vector matrix $\hat{\mathbf{a}}$ with $\hat{f}$. Thus for Eq. (3) in the linear response regime ($\mathbf{E} \parallel x$) we obtain

$$eE\cos\phi\frac{\partial f^{(0)}_{MM}}{\partial p}\delta_{MM'} - \frac{i}{2}\frac{\sin\phi}{p}eE[f^{(0)}_{M'M'}(p) - f^{(0)}_{MM}(p)] + \frac{i}{\hbar}[\epsilon_M(p) - \epsilon_{M'}(p)]f_{MM'}(\mathbf{p}) = St[\hat{f}(\mathbf{p})]_{MM'} \quad (4)$$

Here $f^{(0)}_{MM}(p)$ is the equilibrium Fermi function corresponding to the helicity value M. The collision term was derived in many papers, for the Refs. see [3,17], and in the helicity basis has the form

$$St(\hat{f}(\mathbf{p}))_{MM'} = \int\frac{d^2\mathbf{p}_1}{(2\pi\hbar)^2}\sum_{M_1M_1'}\{[\delta(\epsilon_{M_1}(p_1) - \epsilon_M(p)) + \delta(\epsilon_{M_1'}(p_1) - \epsilon_{M'}(p))]K^{MM'}_{M_1M_1'}\cdot f_{M_1M_1'}(\mathbf{p}_1) - \delta(\epsilon_{M_1}(p) - \epsilon_{M_1'}(p_1))[K^{MM_1}_{M_1'M_1'}\cdot f_{M_1M'}(\mathbf{p}) + f_{MM_1}(\mathbf{p})\cdot K^{M_1M'}_{M_1'M_1'}]\}, \quad (5)$$

where the kernel in the Born approximation in the scattering amplitude is

$$K^{MM'}_{M_1M_1'} = \frac{\pi}{\hbar}|U(\mathbf{p} - \mathbf{p}_1)|^2 N d_{MM_1}(\theta)(d_{M'M_1'}(\theta))^*. \quad (6)$$

Here N is the 2D impurity density, $U(\mathbf{p} - \mathbf{p}_1)$ is the Fourier component of the impurity potential. $d_{MM_1}(\theta)$ depends only on the scattering angle $\theta = \phi - \phi_1$. Diagonal components $d_{1/2,1/2} = d_{-1/2,-1/2} = \cos(\theta/2)$, and $d_{1/2,-1/2} = d_{-1/2,1/2} = -i\sin(\theta/2)$. The Born scattering amplitude is given by

$$F^{M\mathbf{p}}_{M_1\mathbf{p}_1} \propto d_{MM_1}(\theta)U(\mathbf{p} - \mathbf{p}_1); \qquad F^{M\mathbf{p}}_{M_1\mathbf{p}_1} = (F^{M_1\mathbf{p}_1}_{M\mathbf{p}})^*. \quad (7)$$

The additional symmetry property (which exists in the addition to the reciprocity theorem) indicated here exists only in the Born approximation [2]. It means the equality of the scattering amplitudes for the direct and reverse processes, which are obtained by interchanging the initial and final momenta without changing their signs.

Smooth scattering potential.—First, consider the mathematically simple case of a smooth scattering potential when the interband transitions are suppressed, which is realized at $m\alpha R/\hbar \gg 1$, R being the radius of impurity. Then, from Eqs. (4) and (5) we obtain

$$eE\frac{\partial f^{(0)}_+}{\partial p} = \frac{2ap}{V_+}f_{++} + \frac{bp}{V_+}(f_{+-} - f_{-+}), \quad (8)$$

$$eE\frac{\partial f^{(0)}_-}{\partial p} = \frac{2ap}{V_-}f_{--} - \frac{bp}{V_-}(f_{+-} - f_{-+}), \quad (9)$$

$$\frac{i}{2}\frac{eE}{p}(f^0_+ - f^0_-) + \frac{i}{\hbar}(\epsilon_+ - \epsilon_-)f_{+-} = c\left(\frac{p}{V_+}f_{++} - \frac{p}{V_-}f_{--}\right) + pd\left(\frac{1}{V_+} + \frac{1}{V_-}\right)f_{+-}, \quad (10)$$

$$\frac{-i}{2}\frac{eE}{p}(f^0_+ - f^0_-) - \frac{i}{\hbar}(\epsilon_+ - \epsilon_-)f_{-+} = -c\left(\frac{p}{V_+}f_{++} - \frac{p}{V_-}f_{--}\right) + pd\left(\frac{1}{V_+} + \frac{1}{V_-}\right)f_{-+}, \quad (11)$$

where $d = a, b = -c, c = -ia, a = -\frac{1}{2}\int d\theta/(2\pi) \times W(\theta)\sin^2\theta$, and quantity $-2ap/V_+$ is equal to the inverse transport scattering time τ, $W(\theta) = N|U(\mathbf{p} - \mathbf{p}_1)|^2/2\hbar^3 \cdot f_+^0(p)$, $f_-^0(p)$ are the equilibrium Fermi functions which correspond to the helicity $\pm$, $V_\pm(p) = p/m \pm \alpha/2$ are the velocity values for a given p for $\pm$ bands. The expressions for the coefficients a, b, c, d are exact but should be used here for $\theta \ll 1$ since we consider small-angle scattering. In Eqs. (8)–(11) the quantities $f_{++}(p)$, $f_{+-}(p)$, $f_{-+}(p)$, and $f_{--}(p)$ depend only on the modulus of $\mathbf{p}$. In deriving these equations we used the following angular dependences of the components of the matrix $\hat{f}(\mathbf{p})$: $f_{++}(\mathbf{p})$, $f_{--}(\mathbf{p}) \propto \cos\phi$ and $f_{+-}(\mathbf{p})$, $f_{-+}(\mathbf{p}) \propto \sin\phi$. Besides, we used the symmetry properties of the matrix $K(\theta)$:

$$K_{++}^{++} = K_{--}^{--}, \qquad K_{-+}^{++} = K_{++}^{-+}, \qquad K_{++}^{+-} = -K_{++}^{-+}, \tag{12}$$

which can be easily proved from Eqs. (6) and (7). The quantities entering Eqs. (8)–(11) have the following relations to the average spin components:

$$\langle S_z \rangle \propto (f_{+-} + f_{-+}), \qquad \langle \vec{S} \cdot \vec{p} \rangle \propto (f_{+-} - f_{-+}), \qquad \langle \vec{S} \cdot \vec{\Omega} \rangle \propto (f_{++} - f_{--}). \tag{13}$$

From the above equations for the quantity of interest we find

$$eE\left(\frac{\partial f_+^0}{\partial p} - \frac{\partial f_-^0}{\partial p}\right) + \frac{eE}{p}(f_+^0 - f_-^0) = -\frac{1}{\hbar}(\epsilon_+ - \epsilon_-) \times (f_{+-} + f_{-+}). \tag{14}$$

This equation is exact for an arbitrary value of $\Delta\tau$, $\Delta = (\epsilon_+ - \epsilon_-) = \alpha p$. Note that Eq. (14) has a clear physical meaning. The second term on the left-hand side was taken into account before [4] and describes the appearance in the electric field of the z component of the spin due to the angular dependence of the wave functions. Exactly this term gives the contribution $-e/8\pi$ after integration in Eq. (2). However, the first term in Eq. (14) describes the change in the distribution functions due to the acceleration along the electric field and cancels exactly the contribution of the second term after integration in Eq. (2) [18]. Hence, $q_{yz} = 0$, i.e., the spin current is zero. Note that the result of Ref. [4] can be obtained if Eq. (11) is subtracted from Eq. (10) and the right-hand side (collision term) is neglected altogether. This is exactly what was done by the authors of Ref. [4] since they solved the equations of motion for the spin totally ignoring the collision term. The solution of collisionless equations gives an incorrect result because the neglected terms give the contribution of the same order (for an arbitrary large τ) as the term which was kept in Ref. [4].

Equation (14) can also be obtained from Eq. (3) if one calculates in the stationary limit the mean value of $\langle \bar{S}_y \rangle = \int d\phi\, \mathrm{Tr}\{\hat{\sigma}_y \hat{f}(\mathbf{p})\}$. Then the contribution from the second term of Eq. (3) gives the left-hand side of Eq. (14) and the right hand side of Eq. (14) follows from the third term [18]. The contribution from the collision term is zero since in the Born approximation (for an impurity of an arbitrary radius) there is no spin relaxation due to rotation of the spin during the collision event itself. It is shown explicitly in Ref. [17] with the use of Eq. (5) for the collision integral.

It should be noted that the collision term, Eqs. (5) and (6), is written in the Born approximation. Therefore, Eqs. (8)–(11) are valid also only in this approximation. Though the situation beyond the Born approximation requires a special investigation [19], some remarks can be made now. Beyond the Born approximation, the quantity $(K_{++}^{+-} + K_{++}^{-+})$ is not zero. Exactly, this quantity is responsible for the generation of the spin flux due to asymmetric scattering when the particle flux flows in the sample [3]. When this quantity is not zero, in Eq. (14) there should appear the term proportional to the first moment of $(K_{++}^{+-} + K_{++}^{-+})$, which means the appearance of q_{yz}. At the moment it is not clear how all non-Born contributions will be canceled when using the exact collision integral [20].

δ-scattering potential.—Here we consider the case of a short range scattering potential when $W(\theta) = W_0$ (constant). Then, due to the elastic scattering, the transitions between the subbands corresponding to the opposite helicities are allowed. From Eqs. (4) and (5) we obtain a system of coupled equations similar to Eqs. (8)–(11) where now the components of spin-density matrix for the values of $p_\pm = p \pm m\alpha$ appear (Fig. 1). These momentum values are obtained from the conditions of the energy conservation, $\epsilon_\pm(p) = \epsilon_\mp(p_\pm)$. To simplify the presentation we will consider only the limiting cases of large and small $\Delta\tau$. When $\hbar/\tau \ll \Delta$ we find that $(f_{+-} - f_{-+})/(f_{+-} + f_{-+}) \simeq (\hbar/\tau)/\Delta \ll 1$. Using this fact, for the z component of the spin we obtain

$$\frac{eE}{p}(f_+^0 - f_-^0) - m\alpha\left(\frac{E_+(p)}{2p + m\alpha} + \frac{E_-(p)}{2p - m\alpha}\right) = -\frac{1}{\hbar}(\epsilon_+ - \epsilon_-)(f_{+-} + f_{-+}), \tag{15}$$

where we introduced the notations: $E_+(p) = eE\partial f_+^0(p)/\partial p$, $E_-(p) = eE\partial f_-^0(p)/\partial p$. Here the ratio between $m\alpha$ and p is arbitrary. Finally, integrating in Eq. (2) between the two Fermi points (see Fig. 1) $p_F^\pm = \mp m\alpha/2 + \sqrt{p_F^2 + (m\alpha)^2/4}$, we again obtain $q_{yz} = 0$. In the opposite case $\Delta = 0$ we immediately see from Eqs. (4) and (5) that $(f_{+-} + f_{-+}) = 0$ and the spin current is zero.

In conclusion, using the spin-density matrix method for the case of the Rashba Hamiltonian we showed that within the Born approximation in the scattering amplitude the intrinsic spin current is zero for an arbitrary ratio of spin splitting and the impurity scattering rate. We argue that the spin current appears only beyond the Born approximation, depends explicitly on the scattering, and corresponds to the well known *extrinsic* spin currents [1,3]. It is shown explicitly that absence of intrinsic spin current is the result of the special symmetry properties of the Born scattering amplitude.

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