# Unconditional Two-Mode Squeezing of Separated Atomic Ensembles

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We propose schemes for the unconditional preparation of a two-mode squeezed state of effective bosonic modes realized in a pair of atomic ensembles interacting collectively with optical cavity and laser fields. The scheme uses Raman transitions between stable atomic ground states and under ideal conditions produces pure entangled states in the steady state. The scheme works both for ensembles confined within a single cavity and for ensembles confined in separate, cascaded cavities.

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Atomic ensembles are currently attracting considerable theoretical and experimental interest from the quantum optics and quantum information communities [1–23]. Collective enhancement of their interaction with electromagnetic fields enables efficient and controllable coupling to (few-photon) nonclassical light fields without the need for strong single-photon single-atom coupling. Given long atomic ground-state coherence lifetimes, they also offer a robust medium for long-lived, high-fidelity storage of quantum states, i.e., for quantum memory. Of particular interest in this context is the preparation of long-lived quantum entangled states of two or more *separate* atomic ensembles [8–16], with the possibility of application to quantum communication protocols such as quantum teleportation [14].

To date, schemes for preparing entangled states of separate atomic ensembles have generally been based on either projective measurements [8–13] and possibly feedback [15,16] or the transfer of quantum statistics from quantum-correlated light fields [13,14]. Here we propose a scheme which requires neither of these; based on a form of quantum reservoir engineering, it is able to produce pure entangled (two-mode squeezed) states of separate atomic ensembles in a *steady state*. Consideration of potential experimental parameters suggests that this scheme is feasible with existing experimental capabilities and could produce high degrees of entanglement on time scales which are orders of magnitude shorter than achievable coherence lifetimes in atomic ensembles [9,24].

Our proposed scheme is illustrated schematically in Fig. 1. Two orthogonal traveling-wave cavity modes (annihilation operators $a$ and $b$) couple to atomic transitions with strengths $g_{ai}$ and $g_{bi}$, respectively, where $i = 1, 2$ denotes the particular atomic ensemble. Classical laser fields, with Rabi frequencies $\{\Omega_{ri}, \Omega_{si}\}$, combine with the cavity fields to drive two distinct Raman transitions between the atomic ground states $|0\rangle_i$ and $|1\rangle_i$ [25]. With a copropagating field geometry as shown in Fig. 1, the (first-order) Doppler effect is eliminated and, provided the light beams are broad in width compared to the ensembles, we can assume a uniform coupling strength of the atoms to each of the fields. The two ensembles are initially prepared via separate optical pumping in different ground states, but for convenience we relabel the ground states in ensemble 2 so that all atoms are initially in state $|0\rangle$ in our theoretical treatment.

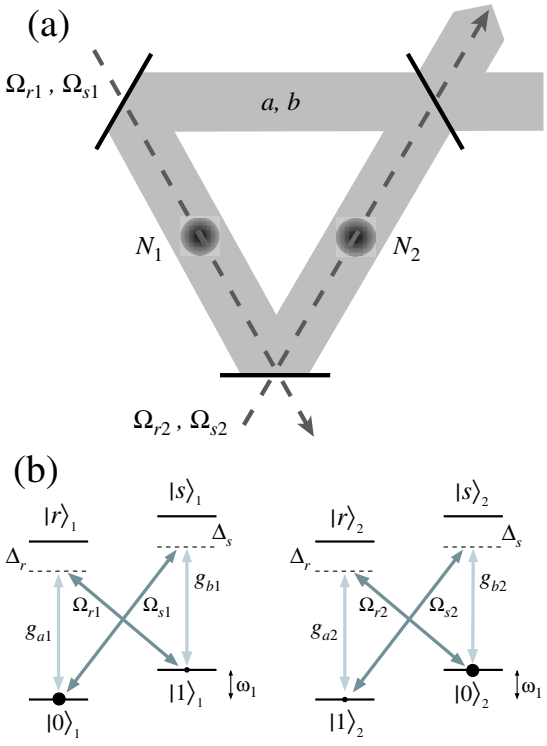


FIG. 1 (color online). (a) Possible ring cavity setup. Ensembles 1 and 2 contain $N_1$ and $N_2$ atoms, respectively. (b) Atomic energy levels and excitation schemes for ensembles 1 (left) and 2 (right). Excited states $|r\rangle$ and $|s\rangle$ can be replaced by a single level, provided the two Raman channels remain distinct.

 

Denoting the laser frequencies by $\omega_{Ls}$ and $\omega_{Lr}$, we consider the case where $\omega_{Ls} - \omega_1 = \omega_{Lr} + \omega_1$ [26], and assuming large detunings $\Delta_r$ and $\Delta_s$ of the fields from the atomic transition frequencies, we perform a standard adiabatic elimination of the atomic excited states and neglect atomic spontaneous emission. Defining collective atomic spin operators by

$$J_{zi} = \frac{1}{2}\sum_{j=1}^{N_i}(|1\rangle\langle 1|_i^j - |0\rangle\langle 0|_i^j), \qquad J_i^- = \sum_{j=1}^{N_i}|0\rangle\langle 1|_i^j, \quad (1)$$

the master equation for the density operator of the total system can then be written (after unitary transformation to an appropriate rotating frame) as

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \kappa_a D[a]\rho + \kappa_b D[b]\rho, \quad (2)$$

where $D[\mathcal{O}]\rho \equiv 2\mathcal{O}\rho\mathcal{O}^\dagger - \mathcal{O}^\dagger\mathcal{O}\rho - \rho\mathcal{O}^\dagger\mathcal{O}$, $\{\kappa_a, \kappa_b\}$ are the cavity field decay rates, and

$$\begin{aligned} H_{\text{eff}} = &\left[\delta_a + \frac{|g_{a1}|^2}{\Delta_r}\left(\frac{N_1}{2} - J_{z1}\right) + \frac{|g_{a2}|^2}{\Delta_r}\left(\frac{N_2}{2} + J_{z2}\right)\right]a^\dagger a \\ &+ \left[\delta_b + \frac{|g_{b1}|^2}{\Delta_s}\left(\frac{N_1}{2} + J_{z1}\right) + \frac{|g_{b2}|^2}{\Delta_s}\left(\frac{N_2}{2} - J_{z2}\right)\right]b^\dagger b \\ &+ \frac{|\Omega_{r1}|^2}{4\Delta_r}\left(\frac{N_1}{2} + J_{z1}\right) + \frac{|\Omega_{s1}|^2}{4\Delta_s}\left(\frac{N_1}{2} - J_{z1}\right) \\ &+ \frac{|\Omega_{r2}|^2}{4\Delta_r}\left(\frac{N_2}{2} + J_{z2}\right) + \frac{|\Omega_{s2}|^2}{4\Delta_s}\left(\frac{N_2}{2} - J_{z2}\right) \\ &+ [a^\dagger(\beta_{r1}J_1^- + \beta_{r2}J_2^+) + \text{H.c.}] \\ &+ [b^\dagger(\beta_{s1}J_1^+ + \beta_{s2}J_2^-) + \text{H.c.}]. \end{aligned} \quad (3)$$

Here, $\delta_{a,b} = \omega_{a,b} - (\omega_{Ls} - \omega_1)$ are detunings of the cavity modes from Raman resonance, and

$$\beta_{ri} = \frac{\Omega_{ri}g_{ai}^*}{2\Delta_r}, \qquad \beta_{si} = \frac{\Omega_{si}g_{bi}^*}{2\Delta_s} \qquad (i = 1, 2) \quad (4)$$

are the Raman transition rates.

In the Holstein-Primakoff representation [27], the collective atomic operators may be associated with harmonic oscillator annihilation and creation operators $c_i$ and $c_i^\dagger$ ($[c_i, c_i^\dagger] = 1$) via $J_i^- = (N_i - c_i^\dagger c_i)^{1/2}c_i$ and $J_{zi} = c_i^\dagger c_i - N_i/2$. For the states that we aim to prepare, the mean number of atoms transferred to the state $|1\rangle$ in each ensemble is expected to be much smaller than the total number of atoms, i.e., $\langle c_i^\dagger c_i\rangle \ll N_i$. The collective atomic operators are thus well approximated by $J_i^- \simeq N_i^{1/2}c_i$ and $J_{zi} \simeq -N_i/2$, and we can reduce $H_{\text{eff}}$ to the form

$$\begin{aligned} H_{\text{eff}} = &\left(\delta_a + \frac{N_1|g_{a1}|^2}{\Delta_r}\right)a^\dagger a + \left(\delta_b + \frac{N_2|g_{b2}|^2}{\Delta_s}\right)b^\dagger b \\ &+ [a^\dagger(\sqrt{N_1}\beta_{r1}c_1 + \sqrt{N_2}\beta_{r2}c_2^\dagger) + \text{H.c.}] \\ &+ [b^\dagger(\sqrt{N_1}\beta_{s1}c_1^\dagger + \sqrt{N_2}\beta_{s2}c_2) + \text{H.c.}], \end{aligned} \quad (5)$$

where we have omitted constant energy terms. With appropriate choices of detunings and/or laser intensities, we assume that the following conditions can be satisfied: (i) $\delta_a + N_1|g_{a1}|^2/\Delta_r = \delta_b + N_2|g_{b2}|^2/\Delta_s = 0$, (ii) $\sqrt{N_1}\beta_{r1} = \sqrt{N_2}\beta_{s2} \equiv \beta$, and (iii) $\sqrt{N_1}\beta_{s1} = \sqrt{N_2}\beta_{r2} \equiv re^{i\theta}\beta$, with $r \in [0, 1]$ real. The effective Hamiltonian thus becomes

$$\begin{aligned} H_{\text{eff}} = &[\beta a^\dagger(c_1 + re^{i\theta}c_2^\dagger) + \text{H.c.}] \\ &+ [\beta b^\dagger(c_2 + re^{i\theta}c_1^\dagger) + \text{H.c.}]. \end{aligned} \quad (6)$$

Consider now a unitary transformation $\tilde{\rho} = S_{12}^+(\epsilon)\rho S_{12}(\epsilon)$ with the two-mode squeezing operator $S_{12}(\epsilon) = \exp(\epsilon^* c_1 c_2 - \epsilon c_1^\dagger c_2^\dagger)$, where $\epsilon = e^{i\theta}\tanh^{-1}(r)$. The master equation for the atom-cavity system becomes

$$\dot{\tilde{\rho}} = -i[\tilde{H}_{\text{eff}}, \tilde{\rho}] + \kappa_a D[a]\tilde{\rho} + \kappa_b D[b]\tilde{\rho}, \quad (7)$$

where

$$\tilde{H}_{\text{eff}} = \sqrt{1 - r^2}[\beta(a^\dagger c_1 + b^\dagger c_2) + \text{H.c.}], \quad (8)$$

which simply describes a system of coupled oscillators. The steady state solution of (7) is the vacuum state for all oscillators. Reversing the unitary transformation, it follows that the steady state of the total system is a pure state, $\rho_{\text{ss}} = |\psi\rangle\langle\psi|_{\text{ss}}$, with

$$|\psi\rangle_{\text{ss}} = \{S_{12}(\epsilon)|0\rangle_1 \otimes |0\rangle_2\} \otimes |0\rangle_a \otimes |0\rangle_b; \quad (9)$$

i.e., the atomic ensembles are prepared in a two-mode squeezed state and the cavity modes in the vacuum state.

The rate at which the state is prepared is determined by the eigenvalues associated with the coupled-oscillator master equation (7), in particular, by the eigenvalue with the smallest nonzero magnitude, which is (taking $\kappa_a = \kappa_b = \kappa$) $\lambda_+ = -(\kappa/2) + [(\kappa/2)^2 - |\beta|^2(1 - r^2)]^{1/2}$. This rate decreases as $r \to 1$, but provided $|\beta|(1 - r^2)^{1/2} \gtrsim \kappa/2$, the time required to reach the steady state will be $\sim 2/\kappa$.

Defining "position" and "momentum" operators for the atomic modes by $X_i = c_i + c_i^\dagger$ and $P_i = -i(c_i - c_i^\dagger)$, respectively, the variances in the sum and difference operators are, for the state (9), given by $V(X_1 \pm X_2) = V(P_1 \mp P_2) = 2\exp[\mp 2\tanh^{-1}(r)]$. Hence, entanglement between the atomic ensembles of the Einstein-Podolsky-Rosen (EPR) type [8,9,28] is generated. Given the stability of the atomic ground states, this entangled state should be long-lived and, using matter-light state-transfer schemes (see, e.g., [1,3–6]), readily recoverable in the form of propagating light pulses in the cavity mode outputs. That is, having prepared the atomic state and switched off all of the laser fields, the fields $\Omega_{r1}$ and $\Omega_{s2}$ could be pulsed on in a suitable fashion at some later time to return all of the atoms to the state $|0\rangle$ and transfer the states of ensembles 1 and 2 to the modes $a$ and $b$, respectively. Alternatively, only one of $\Omega_{r1}$ and $\Omega_{s2}$ might be applied to produce a single light pulse that would be entangled with the atomic ensemble that has not undergone the state-transfer process. This pulse could be used to establish remote quantum

communication, e.g., to teleport the state of a light field to an atomic ensemble.

This scheme is also readily simplified to produce single-mode squeezed states in a single atomic ensemble [19,20]. In particular, for a single cavity-confined ensemble and with $a$ and $b$ chosen to be the same mode, one can realize a dynamics described by a master equation of the form $\dot{\rho} = -i[H_{\rm eff}, \rho] + \kappa_a D[a]\rho$, with

$$H_{\rm eff} = [\beta a^\dagger (c_1 + re^{i\theta} c_1^\dagger) + \text{H.c.}], \tag{10}$$

the steady state solution of which is $\{S_1(\epsilon)|0\rangle_1\} \otimes |0\rangle_a$, where $S_1(\epsilon) = \exp[\epsilon^* c_1^2 - \epsilon (c_1^\dagger)^2]$.

For a potential experimental system and set of parameters, we consider ensembles of $N \sim 10^6$ $^{87}$Rb atoms with the states $|0\rangle$ and $|1\rangle$ corresponding to the ground magnetic states $\{F = 1, m_F = \pm 1\}$. These are coupled via Raman transitions involving circularly polarized ($\sigma^\pm$) cavity modes and laser fields in a ring cavity configuration. An external magnetic field can be used to lift the degeneracy of the $m_F = \pm 1$ states and enable distinct Raman channels between these states [29]. For the single-atom single-photon dipole coupling strength in a ring cavity, we choose $g/(2\pi) \sim 50$ kHz [30,31] and assume laser Rabi frequencies $\Omega/(2\pi) \sim 1$ MHz and atomic excited state detunings $\Delta/(2\pi) \sim 250$ MHz (for simplicity, we omit subscripts from the parameters). These give a Raman transition rate $\beta/(2\pi) \sim 100$ kHz, and for $r = 0.8$ [giving $V(X_1 + X_2) = 0.22$, i.e., a 9.5 dB reduction in the variance [32]], one has $\beta(1 - r^2)^{1/2}/(2\pi) \sim 60$ kHz. Choosing $\kappa/(2\pi) \sim 120$ kHz, the time scale for the state preparation is then $\lambda_+^{-1} \sim 2/\kappa \sim 3$ $\mu$s.

The state preparation dynamics involves only the ''symmetric'' atomic modes represented by $c_{1,2}$; a readout of the atomic quantum memory is accomplished by coupling once more to these modes alone and adiabatically mapping their states onto the readout light fields. Under such circumstances the rate of decoherence of the atomic quantum memory due to atomic spontaneous emission is given by the rate of *single-atom* spontaneous emission [23,33], which is estimated here by $\gamma(\Omega^2/4\Delta^2) \sim 0.02$ kHz, where $\gamma/(2\pi) \sim 6$ MHz is the excited state linewidth for $^{87}$Rb. Hence, spontaneous emission should have a negligible effect on the fidelity of the quantum memory.

Another issue to consider is uncertainty in the atom numbers $N_{1,2}$, which could make it difficult to precisely satisfy the conditions (i)–(iii) for zeroing detunings and fixing the relative Raman transition rates in the two ensembles. If conditions (ii) and (iii) are not satisfied, then the steady state of the system is no longer a pure state. Numerical simulations show, however, that the reduction in the EPR variance is degraded (for $r = 0.8$) by only 1–2 dB for deviations of the ratio $\sqrt{N_2/N_1}\beta_{s2}/\beta_{r1}$ from unity by 10%–15%. We note also that if conditions (ii) and (iii) are not satisfied then the steady states of the cavity modes are no longer the vacuum state and a finite output photon flux is expected. This output flux could in principle be monitored and laser detunings and/or intensities adjusted so as to zero the flux and thereby achieve conditions (ii) and (iii) without exact initial knowledge of $N_{1,2}$.

As mentioned earlier, matter-light state mapping schemes could be applied to transfer the entanglement from one of the ensembles to a propagating light pulse, which could in turn be used to distribute entanglement between distantly separated ensembles. Alternatively, and somewhat remarkably, the scheme described above can in fact be applied to atomic ensembles in separate, *cascaded* optical cavities, as depicted in Fig. 2.

Under precisely the same conditions as were applied in deriving (6), the master equation for the two-cavity system takes the form

$$\dot{\rho} = -i[H_{\rm eff}, \rho] + \mathcal{L}\rho, \tag{11}$$

where now

$$\begin{aligned} H_{\rm eff} = {} & [\beta(a_1^\dagger c_1 + re^{i\theta} a_2^\dagger c_2^\dagger) + \text{H.c.}] \\ & + [\beta(b_2^\dagger c_2 + re^{i\theta} b_1^\dagger c_1^\dagger) + \text{H.c.}], \end{aligned} \tag{12}$$

and the cascaded cavity dynamics are described by [34]

$$\begin{aligned} \mathcal{L}\rho = {} & \kappa D[a_1]\rho + \kappa D[b_1]\rho + \kappa D[a_2]\rho + \kappa D[b_2]\rho \\ & - 2\kappa\sqrt{\eta}([a_2^\dagger, a_1\rho] + [\rho a_1^\dagger, a_2]) \\ & - 2\kappa\sqrt{\eta}([b_2^\dagger, b_1\rho] + [\rho b_1^\dagger, b_2]). \end{aligned} \tag{13}$$

Here $\eta \in [0, 1]$ is the coupling efficiency between the two cavities (assumed to be the same for both modes), and we have assumed the same field decay rate $\kappa$ for all cavity modes.

Solutions to (11) are generally complicated and exhibit correlations between all six modes. A simple solution arises, however, in the limit $\kappa \gg |\beta|$, whereby the cavity modes can be adiabatically eliminated from the dynamics to leave a master equation for the reduced density operator $\rho_a$ of the atomic modes alone [35]. Applying the unitary transformation $\tilde{\rho}_a = S_{12}^+(-\epsilon)\rho_a S_{12}(-\epsilon)$ and assuming

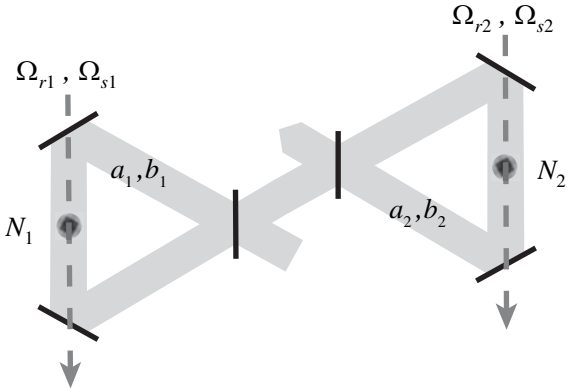


FIG. 2. Possible cascaded ring cavity setup for the preparation of entangled distantly separated atomic ensembles.

ideal intercavity coupling ($\eta = 1$), this master equation reduces to the simple form

$$\dot{\tilde{\rho}}_a = \frac{|\beta|^2(1-r^2)}{\kappa}(D[c_1]\tilde{\rho}_a + D[c_2]\tilde{\rho}_a), \qquad (14)$$

so once again the steady state of the atomic system is a pure two-mode squeezed state $|\psi\rangle_{ss} = S_{12}(-\epsilon)|0\rangle_1 \otimes |0\rangle_2$. This steady state is produced at a rate $\Gamma = |\beta|^2(1-r^2)/\kappa$, which, using parameter values as earlier [$\beta/(2\pi) \sim 100$ kHz, $r = 0.8$], but now with $\kappa/(2\pi) \sim 500$ kHz, takes a characteristic value $\Gamma/(2\pi) \sim 7$ kHz ($\Gamma^{-1} \sim 22$ $\mu$s).

In the presence of coupling loss ($\eta < 1$) the steady state is mixed and the amount of reduction in the EPR variance is limited. In particular, for $\theta = 0$ one finds

$$V(X_1 - X_2) = V(P_1 + P_2) = 2\left(\frac{r^2 - 2r\sqrt{\eta} + 1}{1 - r^2}\right), \qquad (15)$$

which takes a minimum value of $2\sqrt{1-\eta}$ for $r = (1 - \sqrt{1-\eta})/\sqrt{\eta}$. It follows from this result that efficient coupling and transfer between the cavities is essential for generating high degrees of steady state entanglement, although we note that variations on the scheme presented here which utilize single Raman channels and fixed-time evolution may enable reductions in the EPR variance below the value $2\sqrt{1-\eta}$ [36].

In conclusion, we have proposed schemes for the unconditional preparation of EPR-type entangled states of collective atomic modes in physically separated atomic ensembles. These schemes appear within reach of current experiments and expand the range of possibilities for state preparation in atomic ensembles and for remote quantum communication.

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