

Probing Spacetime Foam with Extragalactic Sources

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Because of quantum fluctuations, spacetime is probably “foamy” on very small scales. We propose to detect this texture of spacetime foam by looking for halo structures in the images of distant quasars. We find that the Very Large Telescope interferometer will be on the verge of being able to probe the fabric of spacetime when it reaches its design performance. Our method also allows us to use spacetime foam physics and physics of computation to infer the existence of dark energy or matter, independent of the evidence from recent cosmological observations.

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Nowadays, many physicists believe that spacetime, like all matter and energy, undergoes quantum fluctuations. These quantum fluctuations make spacetime foamy on small spacetime scales [1,2]. But how large are the fluctuations? How foamy is spacetime? There is no consensus yet on the answers to these questions [2]. But we do expect the foaminess of spacetime to lead to uncertainties in distance (l) measurements given by $\delta l \gtrsim l^{1-\alpha} l_p^\alpha$, where $l_p \equiv (\hbar G/c^3)^{1/2} \sim 10^{-33}$ cm is the minuscule Planck length, the intrinsic length scale characterizing quantum gravity and, hence, relevant to spacetime foam physics. The parameter $\alpha \sim 1$ specifies different spacetime foam models. Because the Planck length is so tiny, spacetime foam effects are naturally exceedingly small. The trick will be to find ways to amplify the minuscule effects [3]. One way is to accumulate effects of spacetime fluctuations over a huge distance. Here we propose using spacetime foam-induced fluctuations in the direction of the wave vector of light from extragalactic point sources to detect spacetime foam. Intriguingly, this method will take us within striking distance of probing the fabric of spacetime once the Very Large Telescope (VLT) interferometer reaches its design performance. Furthermore, as shown below, this method allows us to infer the existence of unconventional (dark) matter or energy [4], independent of the evidence from recent supernovae observations [5,6], observations on the cosmic microwave background anisotropies [7], and recent studies based on local x-ray luminosity of galaxy clusters [8], etc.

Although our discussion is applicable to all the spacetime foam models, for concreteness, we will concentrate on two models corresponding to $\alpha = 2/3, 1/2$. The choice of $\alpha = 2/3$ [9–11] yields a model that turns out to be consistent [12] with black hole physics and the holographic principle [13,14], which, many theoretical physicists believe, correctly governs how densely information can be packed in a region of space via its stipulation that the information content can be encoded on a two-dimensional surface around the region, like a hologram. For good reasons, it has come to be known as the holographic model

[2]. The case $\alpha = 1/2$ corresponds to the random-walk model [15,16], so called because the associated dependence on \sqrt{l} is the hallmark of fluctuations of a random-walk type. There are various ways to derive the two models [17], but here we follow a recent argument [18,19] used to map out the geometry of spacetime. The point is that quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime.

One way to map out the geometry of spacetime is to fill space with clocks, exchanging signals with the other clocks and measuring the signals’ times of arrival. (This is how the global positioning system works.) We can think of this procedure as a special type of computation, one whose purpose is to map out the geometry of spacetime. So imagine using a collection of clocks of total mass M to map out a volume of radius l over the amount time $T = l/c$ it takes light to cross the volume. The total number of elementary events, including the ticks of the clocks and the measurements of signals, that can take place within the volume over this time is constrained by the Margolus-Levitin theorem [20] in quantum computation, which bounds the number of elementary logical operations that can be performed within the volume. Thus, the maximum number of ticks and measurements is given by the energy $E = Mc^2$, times the time T , divided by Planck’s constant. To prevent black hole formation, the total mass M of clocks within a volume of radius l must be less than $lc^2/2G$. Together, these two limits imply that the total number of elementary events (or the number of operations) that can occur in the volume of spacetime is no greater than $lTc/l_p^2 = l^2/l_p^2$. Thus, the total number of “cells” in the volume of spacetime is bounded by the area of the spatial part of the volume. In this Letter, we will ignore multiplicative constants of order 1, content with order-of-magnitude estimates.

To maximize spatial resolution, we can have each clock tick only once in time T . Then each cell in this volume of radius l occupies a space of $l^3/(l^2/l_p^2) = ll_p^2$; i.e., the cells are separated by an average distance of $l^{1/3}l_p^{2/3}$. We interpret this result to mean that the uncertainty δl in the

measurement of any distance l cannot be smaller than $l^{1/3}l_p^{2/3}$ on the average [21]. This yields the holographic model of distance fluctuations.

On the other hand, if we spread the cells out uniformly in both space and time, then the spatial size of each cell is $l^{1/2}l_p^{1/2}$, and the temporal separation of successive ticks of each clock is $l^{1/2}l_p^{1/2}/c$, which is just the time it takes a cell to communicate with a neighboring cell. (As we will show below, this is the accuracy with which ordinary matter maps out spacetime.) This corresponds to the random-walk model of spacetime fluctuations for which $\delta l \sim l^{1/2}l_p^{1/2}$.

To probe spacetime foam with extragalactic sources, we first recall that the phase of an electromagnetic wave through spacetime foam is $kx - \omega t \pm \Delta\phi$, where $\Delta\phi$ is random uncertainty introduced by spacetime fluctuations. This cumulative phase uncertainty is a function of the distance l to the source given by [22] $\Delta\phi \sim 2\pi\Sigma(\delta\lambda/\lambda) \sim 2\pi\delta l/\lambda \sim 2\pi(l_p/\lambda)^\alpha(l/\lambda)^{1-\alpha}$, where λ is the wavelength of the observed light from the source, and we have used the proper cumulative factor [22] $(l/\lambda)^{1-\alpha}$ in the summation Σ (over the l/λ intervals each of length λ , forming the total distance l) to yield $\Sigma\delta\lambda = (l/\lambda)^{1-\alpha}\delta\lambda \sim (l/\lambda)^{1-\alpha}\lambda^{1-\alpha}l_p^\alpha \sim \delta l$. Thus, there is a possibility [23,24] for detecting the Planck-level fluctuations predicted by the various theoretical models (parametrized by α) by using interferometric techniques to search for fringes from point sources in distant objects because of the amplification provided by the factor $(l/\lambda)^{1-\alpha}$.

As discussed in more detail below, when $\Delta\phi \sim \pi$ (unless noted otherwise, we measure angles in radians), the cumulative uncertainty in the wave's phase will have effectively scrambled the wave front sufficiently to prevent the observation of interferometric fringes. It should be noted, however, that intrinsic structure within galaxies (e.g., galaxies in the Hubble Deep Field) can mask spacetime foam effects, thus requiring that test targets be distant point sources such as quasars or ultrabright active galactic nuclei (AGN). Let us therefore consider the case of PKS1413 + 135 [25], an AGN for which the redshift is $z = 0.2467$. With $l \approx 1.2$ Gpc [26] and $\lambda = 1.6 \mu\text{m}$, we [22] find $\Delta\phi \sim 10 \times 2\pi$ and $10^{-9} \times 2\pi$ for the random-walk model and the holographic model of spacetime foam, respectively. Thus, the observation [25] by the Hubble Space Telescope (HST) of an Airy ring for PKS1413 + 135 marginally rules out the random-walk model but fails by 9 orders of magnitude to test the holographic model. See also Ref. [27].

However, it may be possible to test spacetime foam models more stringently than implied above by taking into account the expected scattering from spacetime foam fluctuations, which is directly correlated with phase fluctuations. Consider a two-element interferometer observing an incoming electromagnetic wave whose local wave vector k makes an angle θ with respect to the normal to the interferometer baseline as shown in Fig. 1.

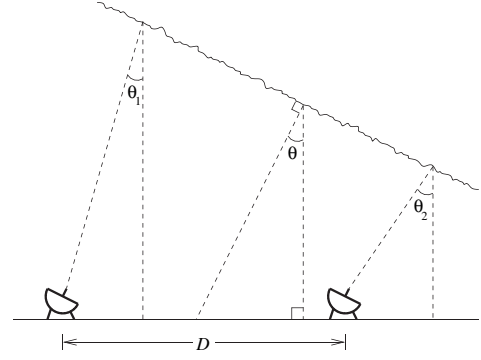


FIG. 1. Interferometer observing an incoming electromagnetic wave from a distant galaxy. The tiny corrugations (greatly exaggerated in the figure) in the wave front are due to spacetime foam-induced fluctuations in phase velocity.

Because of fluctuations in phase velocity, the wave front develops tiny corrugations, in which one portion of the wave front is advanced while another is retarded. The result is that the wave vector acquires a cumulative random uncertainty in direction with an angular spread of the order [28] of $\Sigma \frac{\delta k}{k} = \Sigma \frac{\delta\lambda}{\lambda} \sim \frac{\delta l}{\lambda} \sim \frac{\Delta\phi}{2\pi}$, where we recall that $\Delta\phi \sim 2\pi\delta l/\lambda$. Thus, if the incoming wave has an uncertainty in *both* its phase *and* wave vector direction, the correlated electric field from the two elements is $E = E_0 \exp(i\psi_2/2) + E_0 \exp(-i\psi_1/2)$, where $\psi_{1(2)} = (2\pi D/\lambda) \sin\theta_{1(2)} \pm \Delta\phi$, with D denoting the interferometer baseline length, and $\theta_{1(2)} \sim \theta \pm$ (fluctuations in wave vector direction). Since the fluctuations in wave vector direction are of order $\Delta\phi/2\pi$, arbitrarily setting $\theta_2 = \theta$ and $\theta_1 = \theta - \Delta\phi/2\pi$, we get, for the magnitude of the correlated electric field,

$$|E| \approx 2E_0 \left| \cos\left(\frac{\pi}{2}[2\theta - \Delta\phi/2\pi]\frac{D}{\lambda}\right) \right|, \quad (1)$$

where we have used the fact that θ , $\Delta\phi$, and λ/D are all much less than 1.

For $\Delta\phi = 0$, i.e., nonexistent spacetime foam, the first null occurs when $2\theta = \lambda/D$. But for $\Delta\phi \neq 0$ due to spacetime foam effects, the first null for the halo will occur when $2\theta \sim \lambda/D \pm \Delta\phi/2\pi$. Since $\Delta\phi$ is a random phase fluctuation, the Michelson fringe visibility $V = (P_{\max} - P_{\min})/(P_{\max} + P_{\min})$ calculated from the complex square of Eq. (1) will approximate the normalized visibility of a core-halo structure with a halo angular diameter $2\pi\lambda/D$. That is,

$$V\left(\Delta\phi, \frac{D}{\lambda}\right) = f_c + (1 - f_c) \frac{J_1\left(\frac{\Delta\phi}{2\pi} \frac{D}{\lambda}\right)}{\frac{\Delta\phi}{2\pi} \frac{D}{\lambda}}, \quad (2)$$

where J_1 is the first order Bessel function of the first kind, and we have introduced the parameter $f_c \ll 1$ to account for the possibility of bias in the spacetime foam scattering which may favor the forward direction. Thus, there is a strong reduction in visibility when the argument of J_1 is of order unity, i.e., $\Delta\phi \sim 2\pi\lambda/D$, and spacetime foam theo-

ries can be tested by imaging and interferometry for values of $\Delta\phi$ which are orders of magnitude less than tests based simply on the uncertainty in phase alone.

Again consider the case of PKS1413 + 135. With $D = 2.4$ m for HST, we expect to detect halos if $\Delta\phi \sim 10^{-6} \times 2\pi$ (as compared to $\Delta\phi \sim 10^{-9} \times 2\pi$). Thus, the HST image only fails to test the holographic model by approximately 3 orders of magnitude, rather than the 9 orders of magnitude discussed above for phase scrambling. However, the absence of a spacetime foam-induced halo structure in the HST image of PKS1413 + 135 rules out convincingly the random-walk model. In fact, the scaling relation discussed above indicates that all spacetime foam models with $\alpha \leq 0.6$ are ruled out by this HST observation. We note that the case of a Hubble Deep Field galaxy at $z = 5.34$ considered in Ref. [24] (for which $l \sim 7.7$ Gpc and $\lambda \sim 814$ nm) gives similar bounds (within a factor of 4). At $2 \mu\text{m}$, however, with the maximum VLTI baseline of 140 m, the visibility of fringes from PKS1413 + 135 may be reduced by half because of spacetime foam scattering.

The key for detecting spacetime foam via interferometers, however, relates to the issues of sensitivity and masking. Sensitivity is simply the minimum flux for which interferometer fringes can be detected, $S_{\min} = L/4\pi l^2$, where L is the spectral luminosity of the source in the observational band and l is the distance to the source. For a bright AGN with a spectral luminosity in the near IR of L_{32} (in units of 10^{32} ergs/s Hz) and a sensitivity S_{10} (in units of 10 mJy $\equiv 10^{-25}$ ergs/cm² s Hz), the maximum distance that can be probed for spacetime fluctuations is $l_{\max} \sim 2 \times 10^9 \sqrt{L_{32}/S_{10}}$ (pc).

Masking is any structure of physical size d_{source} that is resolved and, therefore, whose angular size $\beta = d_{\text{source}}/l \geq \Delta\phi$. Note that, since β goes as l^{-1} while $\Delta\phi$ goes as $l^{1/3}$ (for $\alpha = 2/3$), large distances are required to avoid masking. For example, at optical-IR wavelengths the broad emission line region (BELR) of a typical AGN has a physical size d_{BELR} between 0.1 and 1 pc. As a result, avoidance of possible masking (in the optical-IR band) indicates that observations should concentrate on AGN farther than $l_{\min} \sim 1.7 \times 10^8$ parsecs (assuming the upper limit of 1 pc for d_{BELR}). The above arguments then restrict tests to objects such as distant quasar or AGN cores which are highly luminous yet unresolved.

We note in passing that, even though the Very Long Baseline Array has superior resolution and sensibility compared to the VLTI, it is not suitable for searching for spacetime foam scattering because the effects of the quantum foam are inversely proportional to wavelength, which would therefore require a resolution ~ 4 orders of magnitude finer at radio wavelengths. At the opposite wavelength extreme, the shorter x-ray wavelength ($\sim 10^{-8}$ cm) of the Chandra X-ray Observatory might make up for the lack of angular resolution (~ 1 arcsec), provided x-ray quasars can be found which are sufficiently bright to allow for correc-

tions for the broad wings of the Chandra point spread function. We shall explore this possibility in a follow-up paper.

The fact that the random-walk model is ruled out has an interesting cosmological implication as we will now show. We know that our Universe is close to its critical density, about 10^{-9} joule per cubic meter, and the present Hubble horizon of the observable Universe is about 10^{28} cm. Using the Margolus-Levitin theorem, one finds that the Universe (regarded as a computer) can perform up to 10^{106} operations per second. Just as in the discussion of limits to mapping the geometry of spacetime above, the maximum number of elementary operations that can be performed by our Universe in space of the Hubble size R_H over time $T \sim R_H/c$ without undergoing gravitational collapse is just R_H^2/l_p^2 . Therefore, our Universe can have performed no more than 10^{123} operations per second.

We can now use statistical mechanics to calculate the total amount of information that could be stored on conventional matter [18,29], such as atoms and photons, within the horizon. Matter can embody the maximum information when it is converted to energetic and effectively massless particles. In this case, the energy density (which determines the number of operations they can perform) goes as the fourth power of the temperature, and the entropy density (which determines the number of bits they store) goes as the third power of the temperature. It follows that the total number of bits that can be registered using conventional matter is just the number of operations per second raised to the 3/4 power. Therefore, the number of bits currently available is about $(10^{123})^{3/4} \approx 10^{92}$. This yields an average separation of about 10^{-3} cm between neighboring bits, which takes light 10^{-14} sec to cross.

On the other hand, a computation rate of 10^{106} operations per sec with 10^{92} bits means that each bit flips once every 10^{-14} sec. So storing bits in ordinary matter corresponds to a distribution of elementary events in which each bit flips in the same amount of time it takes to communicate with a neighboring bit. It follows that the accuracy to which ordinary matter maps out spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above [18,19] (in the discussion above, replace clocks by bits and the events of the ticking of clocks by the flipping of bits), or in other words, to the random-walk model of spacetime fluctuations.

To the extent that the random-walk model is observationally ruled out, we infer that spacetime can be mapped to a finer spatial resolution than that given by $\delta l \sim l^{1/2} l_p^{1/2}$. This in turn suggests that there must be other kinds of matter or energy with which the Universe can map out its spacetime geometry (to a finer spatial accuracy than that is possible with the use of conventional matter) as it computes. (Here we assume the validity of general relativity

even at cosmological scales.) This line of reasoning strongly hints at the existence of dark matter and dark energy, independent of the evidence from recent cosmological observations [5–8].

In conclusion, we have suggested that we can test spacetime foam models by looking for halo structures in the interferometric fringes induced by fluctuations in the directions of the wave vector of light from extragalactic sources. This method is considerably more powerful than tests based on spacetime foam-induced phase incoherence alone. (For the case of PKS1413 + 135 observed by HST, we obtain an improvement in sensitivity by 6 orders of magnitude.) We have used it to rule out models of spacetime foam with $\alpha \leq 0.6$, including the random-walk model. The holographic model predicts an angular spread in the spacetime foam scattered halo which is still 3 orders of magnitude smaller than the resolution of HST at 1.6 μm . But we note that the VLT and Keck interferometers (plus future space interferometry missions) may be on the verge of testing the holographic model. In particular, the full VLT interferometer, when completed, will have four 8.2 m telescopes with baselines up to ~ 200 m, plus four relocatable 1.8 m auxiliary telescopes, which may reach an angular resolution [30] of 1–2 milliarcsec (~ 5 to 10×10^{-9} radians) with a sensitivity of ~ 50 mJy in the near infrared. This is on the verge of testing the holographic model, with a modest improvement in sensitivity and dynamic range.

We also note that detection of spacetime foam-induced phase fluctuations via interferometry requires $\Delta\phi \sim l^{1-\alpha}/\lambda \sim \lambda/D$. Hence, for the purpose of testing spacetime foam models, we can optimize the performance of the interferometers by observing light of shorter wavelengths from farther extragalactic point sources (subject to sensitivity limits) with larger telescopes and longer interferometer baselines. It also may be noted that tests for spacetime foam effects can, in principle, be carried out without guaranteed time using archived high resolution, deep imaging data on quasars and, possibly, supernovae from existing and upcoming telescopes.

In this Letter, we have also argued that the demise of the random-walk model (coupled with the fact that ordinary matter maps out spacetime only to the accuracy corresponding to the random-walk model) can be used to infer the existence of unconventional energy and/or matter. This shows how spacetime foam physics can even shed light on cosmology—a testimony to the unity of nature.

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