Universality in Solar Flare and Earthquake Occurrence

L. de Arcangelis,[1] C. Godano,[2] E. Lippiello,[3,*] and M. Nicodemi[4]

[1]*Department of Information Engineering and CNISM, Second University of Naples, 81031 Aversa (CE), Italy*
[2]*Department of Environmental Sciences and CNISM, Second University of Naples, 81100 Caserta, Italy*
[3]*University of Naples "Federico II," 80125 Napoli, Italy*
[4]*Department of Physical Sciences, University of Naples "Federico II," Coherentia-CNR and INFN, 80125 Napoli, Italy*

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Earthquakes and solar flares are phenomena involving huge and rapid releases of energy characterized by complex temporal occurrence. By analyzing available experimental catalogs, we show that the stochastic processes underlying these apparently different phenomena have universal properties. Namely, both problems exhibit the same distributions of sizes, interoccurrence times, and the same temporal clustering: We find after flare sequences with power law temporal correlations as the Omori law for seismic sequences. The observed universality suggests a common approach to the interpretation of both phenomena in terms of the same driving physical mechanism.

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Solar flares are highly energetic explosions [1] from active regions of the Sun in the form of electromagnetic radiation, particle and plasma flows powered by strong and twisted magnetic fields. Since they cause disturbances on radio signals, satellites, and electric power on the Earth, much interest has been devoted in the past years to space weather forecasts [2,3]. Recent studies have shown that solar flares also affect the Sun's interior, generating seismic waves similar to earthquakes [4]. Actually, despite having different causes, solar flares are similar to earthquakes in many respects, for example, in the impulsive localized release of energy and momentum and their huge fluctuations [5]. The analogy with earthquake occurrence is also supported by the observation of power laws [6–8] in the distribution of flare sizes, $P(s)$, related to the Gutenberg-Richter law for the earthquake magnitude distribution. Various interpretations have been proposed for these power law distributions ranging from magnetohydrodynamics [1,9] to turbulence [10] up to self-organized criticality [11–13]. A better understanding of solar flares and coronal mass ejections from the Sun requires knowledge of the structural details of these events and their occurrence in time. This could greatly improve the prediction of violent space weather and the understanding of the physical processes behind solar events.

Here we present evidence that the same empirical laws widely accepted in seismology also characterize, surprisingly, the size and time occurrence of solar flares. In particular, the same temporal clustering holds both for earthquake, where it is known as the Omori law, and solar flare catalogs: A main flare triggers a sequence of after flares. The evidence of a universal statistical behavior suggests the possibility of a common approach to long term forecasting and raises as well deep questions concerning the nature of the common basic mechanism.

A statistical approach to earthquake catalogs has revealed a scale invariant feature of the phenomenon, as indicated by power law distributions for relevant physical observables [14,15], such as the seismic moment distribution of earthquakes, $P(s) \sim s^{-\alpha}$, where the exponent $\alpha \in [1.6, 1.7]$ is essentially the same in different areas of the world [16]. This relation corresponds to the Gutenberg-Richter law for the distribution of the earthquake magnitude M via the relation $M = 2/3 \log(s) - K$, where K is a constant [17]. It is also observed that earthquakes tend to occur in clusters temporally located after large events: The Omori law states that the number of aftershocks at time t after a main event, $N_{AS}(t)$, decays as a power law $N_{AS}(t) \sim t^{-p}$, with $p \simeq 1$ [18]. Finally, the distribution of intertimes between consecutive earthquakes, $P(\Delta t)$, is not a simple power law but has a nontrivial functional form which, like the other quantities mentioned before, is essentially independent of the geographical region or the magnitude range considered [19]. These observations suggest that $P(\Delta t)$, $N_{AS}(t)$, and $P(s)$ are distinctive features of earthquakes and, thus, fundamental quantities for a probabilistic analysis of the phenomenon characterizing its amplitude and time scales.

In this Letter, we analyze several catalogs of solar flares and compare them with the Southern California catalog for earthquakes. Since emissions at different wavelengths are related to different radiation mechanisms, we present a comparison among solar data from x-ray observations in three different energy ranges and different periods of solar cycle, by using online available flare catalogs. More specifically, soft x-ray data in the 1.5–2.4 and 3.1–24.8 keV ranges are obtained from the Geostationary Operational Environmental Satellite (GOES) systems [20]. We consider $N_e = 21\,567$ events from January 1992 to December 2002 covering the entire 11-year solar cycle. Solar flares in the hard x-ray range (>25 keV) are obtained from the burst and transient source experiment (BATSE) that gives $N_e = 6658$ flares from April 1991 to May 2000 [21]. Finally, $N_e = 1551$ events from January 1990 to July

1992 in the intermediate x-ray (10–30 keV) range are analyzed from the wide angle telescope for cosmic hard x rays (WATCH) experiment [22]. Many earthquake catalogs exist, and the universality of their statistical features has already been established [16,19]. Thus, for clarity, we consider here only the Southern California earthquake catalog [23], which has $N_e = 88\,470$ events with magnitude $M \geq 2$ in the years from 1967 to 2002.

The intertime distribution has been already investigated for both earthquakes [19] and solar flares [24]. The intertime Δt is the time between the start of a flare (or an earthquake) and the start of the next one as reported in the above catalogs. Here, for a catalog with N_e data, we count the number of events $n(\Delta t)$ having intertime between Δt and $\Delta t + \lambda/N_e$, where λ is a constant setting the binning of raw data. This is the statistically relevant quantity to consider [25], since $n(\Delta t)/\lambda \to P(\Delta t)$ in the limit $N_e \to \infty$, and, thus, in the following we refer to $n(\Delta t)$. Here we choose λ such that $\lambda/N_e = 75$ sec for the California catalog and use the same value for all catalogs. Figure 1 shows the intertime distributions $n(\Delta t)$ for the three different solar flares data sets and for the California earthquake catalog. Solar flare data scale one on top of the other to a very good approximation, and, interestingly, they all appear to collapse, within statistical errors, on the same nontrivial distribution function of earthquake intertimes. In particular, this data collapse is obtained without rescaling Δt by any suitable factor: The intertime duration Δt is expressed in the same units (seconds) for all data sets. Thus, Fig. 1 shows that the same intertime distribution function and, surprisingly, even the same time range characterize these apparently different physical processes in the magnitude range of the above catalogs.

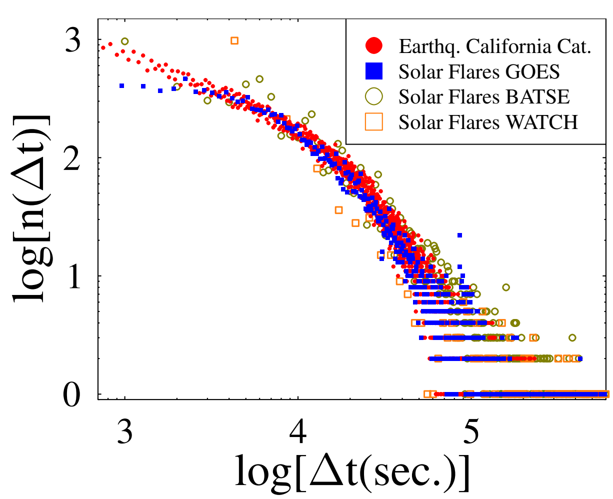

FIG. 1 (color online). The number distribution $n(\Delta t)$ of intertimes Δt between consecutive events in solar flare and earthquake catalogs. Solar data refer to x-ray observations in three different energy ranges covering different periods of the solar cycle: soft x-ray data in the 1.5–2.4 and 3.1–24.8 keV ranges from the GOES catalog (■); hard x-ray (>25 keV) from the BATSE catalog (○); intermediate x-ray (10–30 keV) from the WATCH catalog (□). Earthquake intertimes data are from the California catalog for events with magnitude $M \geq 2$ (●).

The scaling behavior of $n(\Delta t)$ for different solar phases is a widely debated subject [26–28], and a dependence on the solar phase [26] has been observed also in the case of coronal mass ejection [27]. The result for flares has been obtained by taking into account only events with a peak flux larger than 1.4×10^{-6} W m^{-2} (class C1). We have then considered separately data from the GOES catalog corresponding to maximum and minimum solar activity and used the same binning procedure as Fig. 1. In order to take into account the different level of background x-ray flux, we have set different thresholds for different phases: events greater of class C1 in the maximum and class B1 ($10^{-7} \times$ W m^{-2}) in the minimum phase [29]. Figure 2 shows that data from different phases fall on the same universal curve.

The other crucial quantity to be investigated is the distribution of flares sizes, $P(s)$, i.e., the distribution of the flare peak intensity s from the above catalogs. This is compared with the earthquake seismic moments distribution $P(s)$ from the Southern California catalog. In order to normalize the different units and experimental ranges used in each catalog, here we scale the values s of each data set by a given constant amount s_0. Then we calculate the number of events $n(s/s_0)$ with sizes between s/s_0 and $s/s_0 + \lambda/N_e$. Excellent data collapse is observed in Fig. 3 with all data fitted over almost three decades by a power law $n(s/s_0) \sim (s/s_0)^{-\alpha}$, with an exponent $\alpha = 1.65 \pm 0.1$, in agreement with previous results on solar flares [6–8] and earthquakes [16]. Therefore, in analogy to earthquakes, from the above observations it is possible

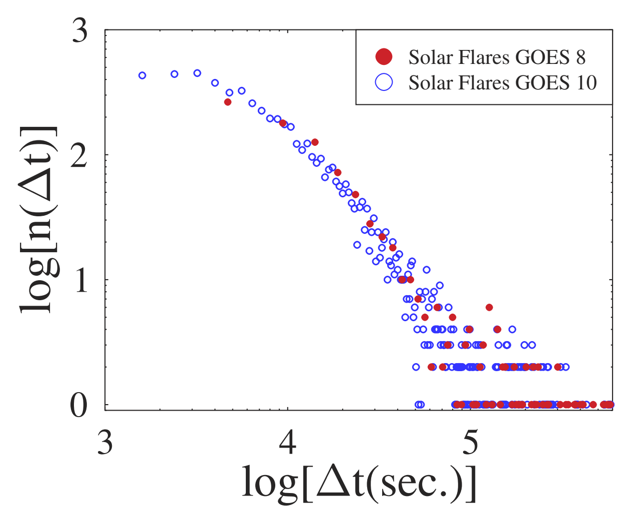

FIG. 2 (color online). The number distribution $n(\Delta t)$ of intertimes Δt between consecutive solar flares for the GOES catalog. Data from GOES8 and GOES10 satellites correspond to the minimum of the solar cycle (from 9/1/1995 to 12/31/1996) and the maximum (from 1/1/2000 to 12/31/2003), respectively.

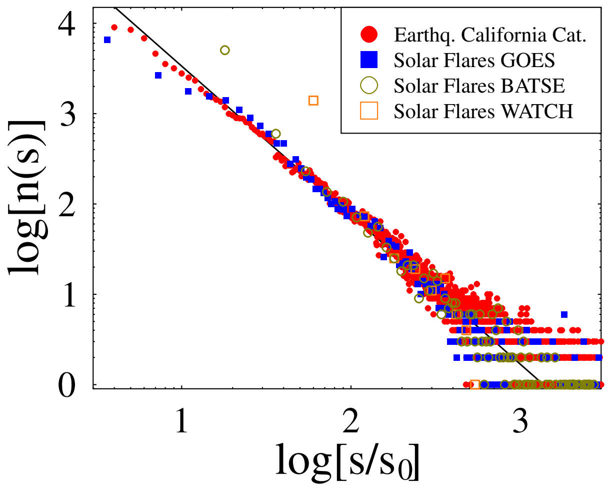

FIG. 3 (color online). The distribution $n(s/s_0)$ of flare peak intensity, from the same catalogs of Fig. 1, and of seismic moments from the California earthquake catalog. We set $\lambda/N_e = 1$ for the California catalog. As for the intertimes of Fig. 1, comparison between the size distributions of earthquakes and solar flares shows very good agreement. The universal distribution is well fitted by a power law with exponent $\alpha = 1.65 \pm 0.1$ (shown as a solid line). $s_0 = 10^{-7}$ W/m^{-2} for GOES, $s_0 = 600$ cmnts/(sec2000 cm^2) for BATSE, $s_0 = 3000$ cnt/bin for WATCH, $s_0 = 30 \times 10^{16}$ Nm for earthquakes.

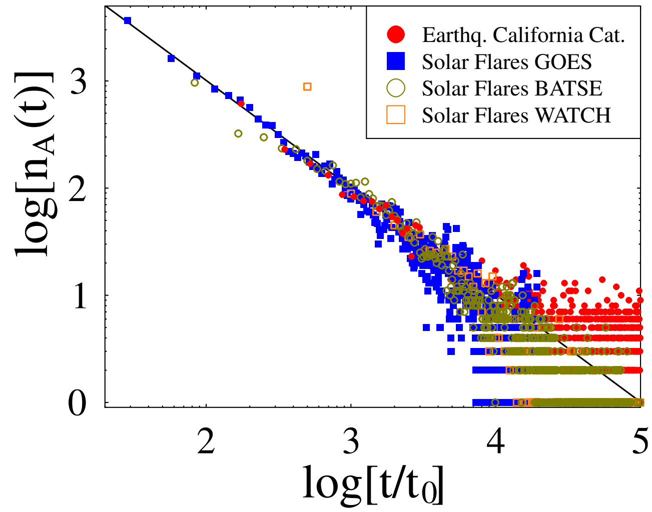

FIG. 4 (color online). The correlation function $n_A(t)$, i.e., the number of after events at time t after a main event for the same catalogs of Fig. 1. To find the best collapse, data from different catalogs are rescaled by a given amount t_0 ($t_0 = 700$ sec for GOES, $t_0 = 60$ sec for BATSE, $t_0 = 1$ sec for WATCH, and $t_0 = 43$ sec for California earthquakes). As for Figs. 1 and 3, the aftershock rate of occurrence for earthquakes and solar flares scale very well. For comparison, we also plot an Omori power law $n_A(t/t_0) \sim t_0/t$ (solid line).

to introduce a Richter scale for flares where their "magnitude" M is defined via the relation: $M(s) = 2/3 \log(s) - K_F$, where K_F is a constant. In terms of the magnitude, the data from flare catalogs are therefore found to follow the Gutenberg-Richter law introduced for earthquakes.

Further evidence of structural similarities in the statistics of the two phenomena is given by the analysis of correlations between events within each of these catalogs. It would be interesting to compare the time correlation between main events and the sequence of their after events, as in the Omori law. We define a "main event" as an event with magnitude $M > M_{\text{main}}$; its "after events" are the following events with $M_{\text{cut}} < M < M_{\text{main}}$, where M_{cut} is a cutoff for small background events. The basic difference with the usual definition used in seismology is that an event with $M < M_{\text{main}}$ considered as an "aftershock" may instead be an independent event totally unrelated to the preceding "main shock." Furthermore, an event with $M > M_{\text{main}}$ considered as a main shock may have been triggered by a previous larger event. Despite these differences, our definition can be straightforwardly applied to flare catalogs, too, and tends to the standard one for large enough M_{main} and M_{cut}: Here we fix $M_{\text{cut}} = M_{\text{main}} - 2.5$. In Fig. 4, we show the number of after events $n_A(t)$, at time t after a main event, for all the mentioned data sources. Interestingly, the time correlation function $n_A(t)$ has the same functional form in all catalogs. A power law $n_A(t) \sim 1/t$ (straight line in Fig. 4), as the Omori law, fits the data. The results are quite robust with respect to changes of the parameter M_{main}, provided that M_{main} is large enough as previously explained. We apply the same procedure to analyze the rate of occurrence of events leading up to a main event and observe that also "foreflares" follow the same power law behavior (Omori law) as foreshocks [30], even if more symmetrical behavior is observed in the flare case.

Figures 1, 3, and 4 indicate that the statistical properties of size and time scales of solar flares (independently of the energy range and the temporal location in the solar cycle of the x-ray radiation) and earthquakes are essentially the same within current statistical accuracy. It is tempting to look at the observed universality in the perspective of the theory of critical phenomena. In the past, analogy between the two phenomena was proposed on the basis of the same theoretical model [31]. Here we follow a completely different approach: We directly compare experimental catalogs, we observe universal behavior, and, therefore, we propose the presence of a common driving physical mechanism. Most earthquakes occur where the elastic energy builds up owing to relative motion of tectonic plates. Schematically, as the friction locks the sliding margins of the plates, energy load increases. When elastic stress overcomes the threshold of frictional resistance, it is relaxed, causing the occurrence of an earthquake. This "stick-slip" behavior redistributes the stress-energy field in the crust, generating new earthquakes where and when the local slipping threshold is exceeded. A quantitative prediction on the aftershocks decay cannot be derived by simple stress transfer but can be obtained in terms of a state variable constitutive formulation, where the rate of earthquakes results from the applied stressing history [32]. This formulation gives account for long-range correlations between earthquakes affecting the shape of the whole intertime distribution.

The universal scaling of Figs. 1–4 suggests a similar physical mechanism at the basis of solar flare occurrence. Flares and x-ray jets arise in active solar regions where magnetic flux emerges from the solar interior and interacts with an ambient magnetic field. These interactions are thought to occur in electric current sheets separating regions of opposite magnetic polarity. The dynamics and energetics of these sheets are governed by a complex magnetic field structure [33]. Opposite fluxes lead to rearrangement of field lines, building magnetic stress up to a breaking point, where magnetic energy is released in a flare via magnetic reconnections. The observed temporal clustering of Fig. 4 shows that the rate of flare occurrence decreases in time as a power law after a main flare. Since the same behavior is found for seismic sequences, here we propose that the mechanism at the basis of seismic energy redistribution can be responsible for after flare occurrence. In particular, magnetic stress transfer in the Solar Corona plays the role of elastic stress redistribution on the Earth's crust. As a consequence, the state-rate formulation [32] can be generalized to solar flares, namely, the flare triggering depends on the entire history of magnetic stresses. Beyond issues of fundamental science, the present results can also have very practical consequences, for instance, to improve the prediction of violent space weather by applying established methods of seismic forecasting [34].

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*Present address: Department of Physics "E. R. Caianiello," University of Salerno, 84081 Baronissi (SA), Italy.

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