Positive Cross Correlations in a Normal-Conducting Fermionic Beam Splitter

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We investigate a beam-splitter experiment implemented in a normal-conducting fermionic electron gas in the quantum Hall regime. The cross correlations between the current fluctuations in the two exit leads of the three terminal device are found to be negative, zero, or even *positive,* depending on the scattering mechanism within the device. Reversal of the cross correlation sign occurs due to interaction between different edge states and does not reflect the statistics of the fermionic particles which ''antibunch.''

In mesoscopic physics, the investigation of the conductance of small electronic devices is widely used to obtain their transport properties. Additionally, to such timeaveraged measurements, the temporal fluctuations (noise) in the current caused by the granularity of charge and diffraction of the wave-function provide us with important supplementary information about electronic transport [1]. Nonequilibrium noise has been widely explored to determine, for example, the effective charge of carriers [2] or to study the transmission properties of quantum coherent devices such as quantum point contacts [3], diffusive wires [4], or chaotic cavities [5]. Universally, the statistical correlations due to the Pauli exclusion principle are responsible for the negative current-current correlations between different leads in *multiterminal* devices [6,7]. Such negative correlations have been observed in distinct experiments [8–11]. It has also been shown that for a diluted electronic stream obeying classical statistics the negative correlations vanish [10].

In contrast, a positive cross correlation (i.e., ''bunching'') is predicted to occur in devices with ''non-normalconducting'' contacts [12] like hybrid structures which use a superconductor as a current injector: Two entangled electrons, forming a Cooper in the superconductor, are simultaneously emitted into different exit leads, giving rise to a positive correlation [13]. Alternatively, devices with ferromagnetic contacts can show positive cross correlations due to ''opposite spin bunching'' [14] or dynamical spin blockade [15].

In this Letter, we are interested in a discussion by Texier and Büttiker [16] about the occurrence of positive cross correlations in a purely *normal-conducting fermionic* device. As the authors show, the effect is due to current redistribution among different conducting states. We consider this idea here experimentally in a beam-splitter configuration [Fig. 1(a)], where a current *I* injected at contact 1 is split into two equal parts exiting into contacts 2 and 3. Our main result is the observation of positive cross correlations between the two exit contacts for a particular implementation of the beam splitter according to

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Ref. [16]. Although predicted by various theoretical works, such positive cross correlations have not been seen before in mesoscopic devices. We further conclude from our experimental observation that a positive correlation in fermionic systems can be interpreted as a sign of entanglement only if effects such as the one shown here can be ruled out.

Figure 1(b) gives an ''inside view'' of the physical implementation of the beam-splitter configuration used to study the cross correlation sign reversal from negative to positive: A two-dimensional electron gas (2DEG) is exposed to a perpendicular magnetic field so that the current flows in edge states along the border of the device [17,18]. Edge states provide natural fermionic ''beams,'' which are, thanks to their chirality, easily split by a quantum point contact (QPC) into a transmitted and a reflected part [6]. The two (tunable) QPC's in series play different roles: The

FIG. 1. (a),(b) Implementation of the beam splitter in the quantum Hall regime with two edge states (i, ii) $(B = 1.6 T)$. Equilibration between them occurs along the path from *A* to *B* when the current flows into an additional voltage probe 4. (c) Scanning electron microscope image: QPC's are formed by split gates on top of a two-dimensional electron gas.

first one (A) introduces noise within the edge state (s) , while the second one (B) acts as a beam splitter that exits the edge states or parts of them into different contacts. In former experiments, only *one single* spin-degenerated edge state was populated [10]. It was shown that the correlations are always negative as expected for a single stream of fermionic particles showing ''antibunching'' behavior [10,16].

In the following, we consider the case that exactly the last two spin-degenerated Landau levels are fully occupied. Partitioning at *A* with the transmission probability T_A^{ii} gives rise to current fluctuations ΔI^{ii} in the second edge state (ii). Their power spectral density is $\langle (\Delta I^{ii})^2 \rangle_{\omega} = 2G_0 T_A^{ii} (1 T_A^{\text{ii}}/\mu_1$, with μ_1 the electrochemical potential of contact 1 and $G_0 = 2e^2/h$ [19–21]. The first edge state remains noiseless because it is transmitted at *A* with unit probability $(T_A^i \equiv 1)$. Interedge state equilibration introduced via an extra voltage probe 4 redistributes the current fluctuations $\Delta I_{\text{in}}^{\text{ii}}$ in the current $I_{\text{in}}^{\text{ii}}$ incident to the mixing contact 4 between the two outgoing edge states I_{out}^i and I_{out}^{ii} : ΔI_{out}^i $\Delta I_{\text{out}}^{\text{ii}} = \Delta I^{\text{ii}}/2$. Finally, the "beam splitter" *B* separates the two edge states into two different contacts 2 and 3. Since the current fluctuations in both edge states originate from the same scattering process at *A*, the cross correlations are expected to be *positive*. Their power spectral density $\langle \Delta I_2 \Delta I_3 \rangle_{\omega} = \langle \Delta I_{\text{out}}^{\text{i}} \Delta I_{\text{0}}^{\text{ii}}$ divided by the Poissonian value 2*e*j*I*j equals [16]:

$$
\frac{\langle \Delta I_2 \Delta I_3 \rangle_{\omega}}{2e|I|} = \frac{\langle (\Delta I^{ii})^2 \rangle_{\omega}}{8e|I|} = +\frac{1}{4} \frac{T_A^{ii}(1 - T_A^{ii})}{1 + T_A^{ii}}.
$$
 (1)

Here $I = G_0(1 + T_A^{\text{ii}})\mu_1/e$ describes the total current injected at contact 1.

Experimentally, the device illustrated in Fig. 1(b) is implemented in a standard $GaAs/Al_{0.3}Ga_{0.7}As hetero$ structure. The QPC's *A* and *B* are defined by metallic split gates on top of the 2DEG, which forms 60 nm below the surface [Fig. 1(c)]. Two samples with different path lengths *L* (200 and 14 μ m) between the two QPC's have been measured. The solid curve in Fig. 2(a) shows the normalized reflected current I_3/I as a function of the voltage applied to gate *B* with gate *A* open. It is given by $I_3/I =$ $1 - (T_B^i + T_B^i)/2$. For $I_3/I \le 0.5$, we obtain the transmission T_B^{ii} by measuring I_3 ($T_B^{\text{i}} \equiv 1$). The transmission T_A^{ii} is determined similarly.

In order to detect the current-current cross correlations between contacts 2 and 3, the time dependent currents $I_\alpha(t)$ $(\alpha = 2, 3)$ are converted to voltage signals $V_{\alpha}(t)$ by two series resistors $R_{\tilde{\alpha}\alpha} = h/4e^2 + R_{0,\alpha}$ implemented by means of additional Ohmic contacts $\tilde{2}$ and $\tilde{3}$. $R_{0,\alpha}$ denotes the contact resistances of the Ohmic contacts, which is of the order 0.5–3 k Ω . The voltage fluctuations $\Delta V_{\tilde{\alpha}}(t) =$ $\Delta I_{\alpha}(t)R_{\alpha\alpha}$ are measured by two low-noise amplifiers and fed into a spectrum analyzer which calculates the power spectral density. The *RC* damping of the voltage noise due to the finite capacitance of the measurement lines [Fig. 2(b)] and the offset noise S_0 due to the amplifiers

FIG. 2. Current fluctuations due to scattering at *A* are redistributed between the two edge states so that positive cross correlations are observed. Partial scattering at the second point contact *B* reveals the fermionic nature of the edge states and yields a negative correlation. The correlations are zero in the case that no partial scattering occurs at the QPC's. The data (∇) are measured on sample 2. (a) Reflected current at 3 with gate *C* closed. (b) RC damping of the voltage noise.

are obtained from a calibration measurement of the Nyquist noise $4k_B\theta R$ as a function of the bath temperature θ for a given resistance *R*. The noise measurements are performed in a frequency range of 20 to 70 kHz with typical bandwidths of 5 kHz. The measurement frequencies as well as the current bias are chosen such that contributions from $1/f$ noise are negligible. All measurements were performed in a ³He cryostat with a base temperature of 290 mK.

Figure 2 gives the cross correlations $S_{I,23} = \langle \Delta I_2 \Delta I_3 \rangle_{\omega}$ measured on sample 1 for different configurations of gate *A* and *B* and with gate *C* open. In a first measurement, T_A^{ii} equals ≈ 0.5 and the beam splitter *B* is adjusted such that the second edge state is totally reflected $(T_B^{\text{ii}} = 0)$, which corresponds to the configuration shown in Fig. 1(b). For these parameters, we indeed observe a *positive* cross correlation (solid circles). The solid line is the maximal positive cross correlation given by Eq. (1). For comparison, the Poissonian noise $S_0 = 2e|I|$ is given as dotted line. The total offset S_I^0 equals 3.13×10^{-27} A²/Hz. The current noise of the amplifiers gives an offset $S^0_{I,\theta=0}$ of 3.91 \times 10^{-27} A²/Hz, which we obtain from several temperature calibrations. From these two values, the thermal correlations between contacts 2 and 3 can be calculated $S_{I,23}(I =$ $(0) = S_I^0 - S_{I, \theta=0}^0 = -7.9 \times 10^{-28}$ A² s, which turn out to

FIG. 3. (a),(b) Normalized current I_3/I for different transmissions T_A^{ii} . The curves are shifted in *V* for clarity. (a) The edge states equilibrate between *A* and *B* within a floating voltage probe. (b) Gate *C* closed: No equilibration occurs and the currents carried by the two edge states are unequal if the second edge state is only partially transmitted at *A*. (c) I_3/I vs gate *C* with gate *B* closed and $T_A^{\text{ii}} \approx 0.4$. Contact 4 is on ground. (d) A positive cross correlation is observed with gate *C* open which disappears for gate *C* closed

be negative. Thermal correlations are always negative [6,8]. They are not related to the statistics of the charge carriers but occur due to charge conservation. The measured value is in reasonable agreement with the theoretical prediction of $-k_B\theta G_0(3 - T^2) = -8.8 \times 10^{-28}$ A² s for $T_B^{\text{ii}} = 0$, $T_A^{\text{ii}} = T = 0.5$, and $\theta = 290 \text{ mK}$ [16].

For transparencies $T_B^{\text{ii}} > 0$, the second edge state is only partially reflected at the beam splitter. Consequently, the statistical properties of the electrons in the fermionic ''beam'' become apparent, and the cross correlations change sign from positive to negative. The solid line indicates the "full antibunching" of $-2e|I|/8$ for $T_B^{\text{ii}} =$
0.5 and $T_A^{\text{ii}} = 1$ or 0. Naturally, for T_A^{ii} , $T_B^{\text{ii}} \in \{0, 1\}$, the cross correlations are zero, indicating that inelastic scattering between two edge states alone does not introduce any noise in the system.

Next we will discuss what happens if there is no equilibration present. Figure 2(a) gives the reflected current at contact 3 with gate *C closed* for $T_A^{\text{ii}} = 0$ and 1. At the observed plateau, the second edge state is totally reflected. Thus, its height yields a direct measure for the amount of current carried by the edge states [22]. Although for $T_A^{\text{ii}} =$ 0 no current is carried by the second edge state, the plateau at $I_3/I = 0.5$ indicates current redistribution along the path

FIG. 4. Cross correlation for various T_A^{ii} , T_B^{ii} extracted from noise measurements on two different samples (solid symbols: $\overline{AB} \approx 200 \mu \text{m}$, open symbols: $\overline{AB} \approx 14 \mu \text{m}$). (a) Positive cross correlations occur for $T_B^{\text{ii}} = 0$. (b),(c) For $T_A^{\text{ii}} = 0$ or 1, the correlations are always negative.

 $L = \overline{AB}$ from *A* to *B* of length $\simeq 200 \mu$ m. This is in agreement with detailed studies on equilibration lengths in the quantum Hall regime [22,23]. In order to avoid equilibration, we have to consider another device where the path length L (=14 μ m) of the two QPC's is much shorter. Figures 3(a) and 3(b) show the reflected current at contact 3 measured on this second device (sample 2). In Fig. 3(a), the edge states equilibrate in contact 4, like in the first experiment, and the current is redistributed between the two edge states so that the plateau does not change in height. In Fig. 3(b), however, gate *C* is closed and the current I^{II} carried by the second edge state now depends on the transmission T_A^{ii} . The dotted lines correspond to $I^{ii}/I = I_3/I = T_A^{ii}/(1 + T_A^{ii})$. The plateau which appears at the height of the dotted line in Fig. 3(b) thus proves that the two edge states do not equilibrate. This should be noticed in the noise, too. Figure 3(d) presents cross correlation measurements with $T_B^{\text{ii}} = 0$ and $T_A^{\text{ii}} \approx 0.42$. With equilibration in contact 4 (gate $C = 0.0$ V), the correlations are positive (solid circles) and in good agreement with the maximal positive correlation. If gate *C* is closed, the first edge state remains noiseless ($\Delta I^i = \Delta I_2 = 0$) and the correlator $\langle \Delta I^i \Delta I^{ii} \rangle_{\omega} = \langle \Delta I_2 \Delta I_3 \rangle_{\omega}$ vanishes (squares). We thus have a ''knob'' which allows us to turn the positive correlations on and off.

In Fig. 4, we compare the cross correlations $S_{23}/2e|I|$ measured on the two different samples for various parameters T_A^{ii} and T_B^{ii} with theoretical calculations from Ref. [16]. At zero temperature, the correlations between contacts 2 and 3 are described by:

$$
\frac{S_{I,23}}{2e|I|} = -\frac{\sum T_B^n (1 - T_B^n)}{2} + \frac{(\sum T_B^n)(2 - \sum T_B^n)}{4}
$$

$$
\times \frac{\sum T_A^n (1 - T_A^n)}{\sum T_A^n},
$$
(2)

where $n = i$, ii denotes the index of the two edge states. The first term in Eq. (2) describes the negative correlations due to partitioning of the edge states at *B*, whereas the second term gives a positive contribution due to interedge state equilibration. With $T_A^i = T_B^i \equiv 1$, Eq. (2) yields a maxstate equinoration. With $T_A = T_B = 1$, Eq. (2) yields a max-
imal possible positive cross correlation of $(3/4 - 1/\sqrt{2}) \approx$ imal possible positive cross correlation of $(3/4 - 1/\sqrt{2}) \approx$
0.043*S*₀ that occurs for $T_A^{ii} = \sqrt{2} - 1$ with $T_B^{ii} = 0$. In Fig. 4(a), the measured positive correlations are somewhat smaller for sample 1 (black circles). For sample 2, with a smaller path length *L* between the QPC's, the data points (open circles) are rather close to the expected value. Although the QPC *B* is adjusted to a plateau with high precision, the second edge state (ii) might not be reflected perfectly. Already a tiny transmission T_B^{ii} of 2% reduces the maximal positive cross correlation by 23%, illustrated by one of the dashed curves in Fig. 4(a). The positive correlations completely disappear for $T_B^{\text{ii}} > 9\%$. The high sensitivity to any changes from $T_B^{\text{ii}} = 0$, thus, might explain the deviations from the solid curve. The open squares in Fig. 4(a) are the results from sample 2 where gate *C* is closed so that the state-mixing voltage probe 4 is disconnected. Cross correlations larger than $0.043S₀$ could theoretically occur due to additional scattering of the first edge state at *A*. The dashed-dotted curve gives an example for $T_A^i = 0.96$ instead of 1 that would yield a maximal positive correlation of $0.051S_0$.

Figures 4(b) and 4(c) summarize the negative correlations obtained for $T_A^{\text{ii}} = 1$ and 0, respectively. The data points do not exactly agree with the expected values according Eq. (2) (solid curves). The dashed curves denote the changes that would occur due to additional scattering at the first QPC *A*, yielding a small positive contribution to the negative correlations [10]. However, the transmission at *A* equals 0 or 1 (open gate) with quite high precision $(|\Delta T_A^{\text{ii}}| \le 0.03)$, and we think that the deviations observed here are related to nonequal transmissions of the two spinpolarized parts in the second edge state. The dotted lines in Figs. 4(b) and 4(c) are the negative correlations for 20%, 40%, and 100% unequal transmission (from bottom to top). For one spin-polarized edge state totally transmitted and the other totally reflected, the correlations would be zero for $\langle T_B^{\rm ii} \rangle = 0.5$. From the data, we estimate that the differences between the two transmissions are on the order of 20%–40% of *T*ii *B*.

In conclusion, we have observed positive cross correlations in a multiterminal electronic device. These positive correlations occur due to interactions between different current carrying states inside the device and can be switched on and off by means of an external gate voltage, which controls the interaction inside the device.

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