Magnetically Tunable Kondo–Aharonov-Bohm Effect in a Triangular Quantum Dot

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The role of discrete orbital symmetry in mesoscopic physics is manifested in a system consisting of three identical quantum dots forming an equilateral triangle. Under a perpendicular magnetic field, this system demonstrates a unique combination of Kondo and Aharonov-Bohm features due to an interplay between continuous [spin-rotation $SU(2)$] and discrete (permutation C_{3v}) symmetries, as well as $U(1)$ gauge invariance. The conductance as a function of magnetic flux displays sharp enhancement or complete suppression depending on contact setups.

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Experimental analysis of the Kondo effect in simple quantum dots (QDs) [1] treats the electron as a local spin-1/2 magnetic moment devoid of orbital degrees of freedom. These are absent also in theoretical discussions of the Kondo effect in composite structures consisting of two or three dots [2–5]. However, orbital effects, which play a crucial role in real metals [6,7], become relevant also in mesoscopic physics, e.g., when a multiple QD is fabricated in a *ring* geometry, having discrete point symmetries. At low temperature it can serve both as a Kondo-scatterer and as a peculiar Aharonov-Bohm (AB) interferometer, since the magnetic flux affects the nature of the QD ground and excited states. A system of three dots forming a triangle has been realized experimentally [8,9]. A recent experiment studies a triangular trimer of Cr ions placed upon a gold surface [10]. The underlying discrete orbital symmetry results in additional degeneracies of the trimer spectrum, which might induce a non-Fermi-liquid (NFL) regime [11].

In the present work, we analyze the physics of tunneling through a triangular triple quantum dot (TTQD) in a magnetic field with one electron shared by its three identical constituents (see Fig. 1). It exhibits an interplay between continuous $SU(2)$ electron spin symmetry, discrete point symmetry C_{3v}, and $U(1)$ gauge invariance of electron states in a magnetic field. The conductance of the device is characterized by an unusual dependence on the magnetic flux Φ through the triangle, displayed by sharp peaks or narrow dips, depending on contact geometry. In a 3-terminal geometry [Fig. 1(a)], sharp peaks arise since the magnetic field induces a symmetry crossover $SU(2) \to SU(4)$. In a 2-terminal geometry [Fig. 1(b)], the Kondo tunneling is modulated by the AB interference, which blocks the source-drain cotunneling amplitude at certain flux values. This Kondo-AB interplay is distinct from the one studied in a mesoscopic AB interferometer with a QD placed on one of its arms [12]. A symmetric TTQD in contact with three metallic leads is described by the Hamiltonian $H = H_d + H_{\rm lead} + H_t$, expressed in terms of dot and lead operators $d_{j\sigma}$, $c_{j\sigma}$, with $j = 1, 2, 3$, and $\sigma = \uparrow, \downarrow$. H_d describes an isolated TTQD,

$$H_d = \epsilon \sum_{j=1}^{3} \sum_{\sigma} d^{\dagger}_{j\sigma} d_{j\sigma} + Q \sum_{j} n_{j\uparrow} n_{j\downarrow} + Q' \sum_{\langle jl \rangle} \sum_{\sigma\sigma'} n_{j\sigma} n_{l\sigma'} + W \sum_{\langle jl \rangle} \sum_{\sigma} (d^{\dagger}_{j\sigma} d_{l\sigma} + \text{H.c.}). \quad (1)$$

Here $n_{j\sigma} = d^{\dagger}_{j\sigma} d_{j\sigma}$, $\langle jl \rangle = \langle 12 \rangle, \langle 23 \rangle, \langle 31 \rangle$, Q and Q' are intradot and interdot charging energies ($Q \gg Q'$), and W is the interdot tunneling amplitude. $H_{\rm lead}$ describes electrons in the respective electrodes,

$$H_{\rm lead} = \sum_{j\mathbf{k}\sigma} \epsilon_{jk} c^{\dagger}_{j\mathbf{k}\sigma} c_{j\mathbf{k}\sigma}, \quad (2)$$

and H_t is the tunneling Hamiltonian,

$$H_t = V \sum_{j\mathbf{k}\sigma} (c^{\dagger}_{j\mathbf{k}\sigma} d_{j\sigma} + \text{H.c.}). \quad (3)$$

The dot energy ϵ is tuned by gate voltage in such a way that the ground-state occupation of the isolated TTQD is $\mathcal{N} = 1$. Consider first a TTQD with three leads and three identical channels [Fig. 1(a)]. Assuming $V \ll W$, the tunnel contact preserves the rotational symmetry of the TTQD, which is thereby imposed on the itinerant electrons in the leads. It is useful to treat the Hamiltonian in the special basis which respects the C_{3v} symmetry, employing an approach widely used in the theory of the Kondo effect in bulk metals [6,13]. The Hamiltonian $H_d + H_{\rm lead}$ is

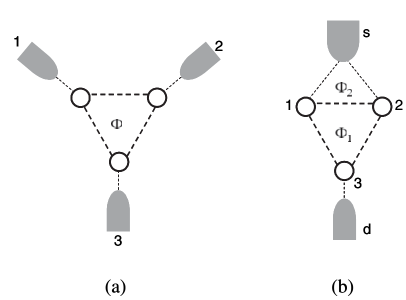

FIG. 1. Triangular triple quantum dot in three-terminal (a) and two-terminal (b) configurations.

diagonal in the basis

$$d^\dagger_{A,\sigma} = (d^\dagger_{1\sigma} + d^\dagger_{2\sigma} + d^\dagger_{3\sigma})/\sqrt{3},$$
$$d^\dagger_{E_\pm,\sigma} = (d^\dagger_{1\sigma} + e^{\pm 2i\varphi} d^\dagger_{2\sigma} + e^{\pm i\varphi} d^\dagger_{3\sigma})/\sqrt{3}; \quad (4)$$

$$c^\dagger_{A,\mathbf{k}\sigma} = (c^\dagger_{1\mathbf{k}\sigma} + c^\dagger_{2\mathbf{k}\sigma} + c^\dagger_{3\mathbf{k}\sigma})/\sqrt{3},$$
$$c^\dagger_{E_\pm,\mathbf{k}\sigma} = (c^\dagger_{1\mathbf{k}\sigma} + e^{\pm 2i\varphi} c^\dagger_{2\mathbf{k}\sigma} + e^{\pm i\varphi} c^\dagger_{3\mathbf{k}\sigma})/\sqrt{3}. \quad (5)$$

Here $\varphi = 2\pi/3$, while A and E form bases for two irreducible representations of the group C_{3v}. The Hamiltonian of the isolated TTQD in this charge sector has six eigenstates $|DA\rangle$, $|DE\rangle$. They correspond to a spin doublet (D) with fully symmetric "orbital" wave function (A) and a quartet doubly degenerate both in spin and orbital quantum numbers (E). The corresponding single electron energies are

$$E_{DA} = \epsilon + 2W, \qquad E_{DE} = \epsilon - W. \quad (6)$$

To describe the orbital effect of an external magnetic field B (perpendicular to the TTQD plane and inducing a flux Φ through the triangle), one rewrites the spectrum as

$$E_{D\Gamma}(p) = \epsilon - 2W \cos\left(p - \frac{\Phi}{3}\right), \quad (7)$$

such that for negative W and for $B = 0$, $p = 0$, $2\pi/3$, $4\pi/3$ correspond, respectively, to $\Gamma = A, E_\pm$. Figure 2 illustrates the evolution of $E_{D\Gamma}(\Phi)$ induced by B. Variation of B between zero and B_0 (the value of B corresponding to the quantum of magnetic flux Φ_0 through the triangle) results in multiple crossing of the levels $E_{D\Gamma}$.

The accidental degeneracy of spin states induced by the magnetic phase Φ introduces new features into the Kondo effect. In the conventional Kondo problem, the effective low-energy exchange Hamiltonian has the form $J\mathbf{S}\cdot\mathbf{s}$, where $\mathbf{S}$ and $\mathbf{s}$ are the spin operators for the dot and lead electrons, respectively [14]. Here, however, the low-energy

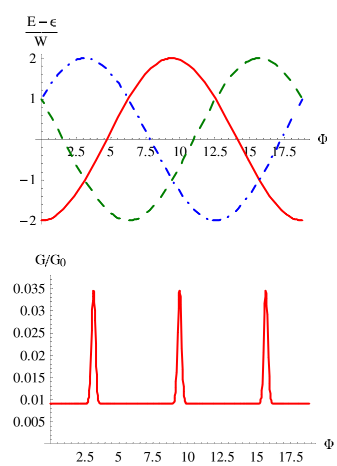

FIG. 2 (color online). Upper panel: Evolution of the energy levels E_A (solid line) and $E_\pm$ (dashed and dash-dotted line, respectively). Lower panel: Corresponding evolution of conductance ($G_0 = \pi e^2/\hbar$).

states of a TTQD form a multiplet characterized by both spin and orbital quantum numbers. The effective exchange interaction reflects the *dynamical symmetry* of the Hamiltonian H_d [3,15]. The corresponding dynamical symmetry group is identified not only by the operators which commute with the Hamiltonian but also by operators inducing transitions between different states of its multiplets. Hence, it is determined by the set of dot energy levels which reside within a given energy interval (its width is related to the Kondo temperature T_K). Since the position of these levels is controlled by the magnetic field, we arrive at a remarkable scenario: Variation of a magnetic field determines the dynamical symmetry of the tunneling device. Generically, the dynamical symmetry group which describes all possible transitions within the set $\{DA, DE_\pm\}$ is $SU(6)$. However, this symmetry is exposed at too high energy scale $\sim W$, while only the low-energy excitations at energy scale $T_K \ll W$ are involved in Kondo tunneling. It is seen from Fig. 2 that the orbital degrees of freedom are mostly quenched, but the ground state becomes doubly degenerate both in spin and orbital channels around $\Phi = (2n+1)\pi$, ($n = 0, \pm 1, \ldots$).

Next we analyze the field dependent Kondo effect variable degeneracy. It is useful to generalize the notion of localized spin operator $S^i = |\sigma\rangle\hat{\tau}_i\langle\sigma'|$ [employing Pauli matrices $\hat{\tau}_i (i = x, y, z)$] to $S^i_{\Gamma\Gamma'} = |\sigma\Gamma\rangle\hat{\tau}_i\langle\sigma'\Gamma'|$, in terms of the eigenvectors (4). Similar generalization applies for the spin operators of the lead electrons: $s^i_{\Gamma\Gamma'} = \sum_{\mathbf{k}\mathbf{k}'} c^\dagger_{\Gamma,\mathbf{k}\sigma}\hat{\tau}_i c_{\Gamma',\mathbf{k}'\sigma'}$. In zero field, $\Phi = 0$, the rotation degrees of freedom are quenched at the low-energy scale. The only vector which is involved in Kondo cotunneling through a TTQD is the spin $\mathbf{S}_{AA} \equiv \mathbf{S}$. Applying the Schrieffer-Wolff (SW) procedure, the effective exchange Hamiltonian reads

$$H_{\rm SW} = J_E(\mathbf{S}\cdot\mathbf{s}_{E_+E_+} + \mathbf{S}\cdot\mathbf{s}_{E_-E_-}) + J_A\mathbf{S}\cdot\mathbf{s}_{AA}. \quad (8)$$

The exchange vertices J_Γ are

$$J_E = -2V^2(\Delta_{Q'}^{-1} - \Delta_Q^{-1})/3,$$
$$J_A = 2V^2(3\Delta_1^{-1} + \Delta_Q^{-1} + 2\Delta_{Q'}^{-1})/3, \quad (9)$$

with $\Delta_1 = \epsilon_F - \epsilon$, $\Delta_Q = \epsilon + Q - \epsilon_F$, and $\Delta_{Q'} = \epsilon + Q' - \epsilon_F$. Note that $J_A > 0$ as in the conventional SW transformation of the Anderson Hamiltonian. On the other hand, $J_E < 0$ due to the inequality $Q \gg Q'$. Thus, two out of three available exchange channels in the Hamiltonian (8) are irrelevant. As a result, the conventional Kondo regime emerges with the doublet DA channel and a Kondo temperature,

$$T_K^{(A)} = D\exp\{-1/j_A\}, \quad (10)$$

where $j_A = \rho_0 J_A$, ρ_0 being the density of electron states in the leads.

At $\Phi = (2n+1)\pi$, when the ground state of a TTQD becomes a spin and orbital doublet, the symmetry of Kondo center is $SU(4)$. This kind of *orbital* degeneracy is different from that of *occupation* degeneracy studied in double quantum dot systems [16]. The 15 generators of $SU(4)$ include four spin vector operators $\mathbf{S}_{E_aE_b}$ with $a, b = \pm$ and one pseudospin vector $\mathcal{T}$ defined as

$$\mathcal{T}^{+} = \sum_{\sigma} |E_{+}, \sigma\rangle\langle E_{-}, \sigma|, \qquad \mathcal{T}^{-} = [\mathcal{T}^{+}]^{\dagger},$$
$$\mathcal{T}^{z} = \frac{1}{2}\sum_{\sigma}(|E_{+}, \sigma\rangle\langle E_{+}, \sigma| - |E_{-}, \sigma\rangle\langle E_{-}, \sigma|). \tag{11}$$

Its counterpart for the lead electrons is $\tau^{+} = \sum_{k\sigma} c^{\dagger}_{E_{+}k\sigma} c_{E_{-}k\sigma}$, $\tau_z = \frac{1}{2}\sum_{k\sigma}(c^{\dagger}_{E_{+}k\sigma}c_{E_{+}k\sigma} - c^{\dagger}_{E_{-}k\sigma}c_{E_{-}k\sigma})$.

The SW Hamiltonian is [17]

$$\begin{aligned} H_{\rm SW} = {} & J_1(\mathbf{S}_{E_+E_+}\cdot \mathbf{s}_{E_+E_+} + \mathbf{S}_{E_-E_-}\cdot \mathbf{s}_{E_-E_-}) \\ & + J_2(\mathbf{S}_{E_+E_+}\cdot \mathbf{s}_{E_-E_-} + \mathbf{S}_{E_-E_-}\cdot \mathbf{s}_{E_+E_+}) \\ & + J_3(\mathbf{S}_{E_+E_+} + \mathbf{S}_{E_-E_-})\cdot \mathbf{s}_{AA} \\ & + J_4(\mathbf{S}_{E_+E_-}\cdot \mathbf{s}_{E_-E_+} + \mathbf{S}_{E_-E_+}\cdot \mathbf{s}_{E_+E_-}) \\ & + J_5(\mathbf{S}_{E_+E_-}\cdot (\mathbf{s}_{AE_-} + \mathbf{s}_{E_+A}) + \text{H.c.}) + J_6\mathcal{T}\cdot \boldsymbol{\tau}, \end{aligned} \tag{12}$$

where the coupling constants are $J_1 = J_4 = J_A$, $J_2 = J_3 = J_5 = J_E$ defined in (9) and $J_6 = V^2(\Delta_1^{-1} + \Delta_{Q'}^{-1})$. Thus, spin and orbital degrees of freedom of a TTQD interlace in the exchange terms. The indirect exchange coupling constants include both diagonal (jj) and nondiagonal (jl) terms describing reflection and transmission cotunneling amplitudes. The interplay between spin and pseudospin channels naturally affects the scaling equations obtained within the framework of Anderson's "poor man scaling" procedure [14]. The system of scaling equations has the following form:

$$\begin{aligned} dj_1/dt &= -[j_1^2 + j_4^2/2 + j_4j_6 + j_5^2/2], \\ dj_2/dt &= -[j_2^2 + j_4^2/2 - j_4j_6 + j_5^2/2], \\ dj_3/dt &= -[j_3^2 + j_5^2], \qquad dj_6/dt = -j_6^2, \\ dj_4/dt &= -[j_4(j_1 + j_2 + j_6) + j_6(j_1 - j_2)], \\ dj_5/dt &= -j_5[j_1 + j_2 + j_3 - j_6]/2. \end{aligned} \tag{13}$$

Here $j_i = \rho_0 J_i$ and $t = \ln(\rho_0 D)$. Analysis of the scaling Eq. (13) shows that the symmetry-breaking vertices j_3 and j_5 are irrelevant, but the vertex j_2, whose initial value is negative, evolves into positive domain and eventually enters the Kondo temperature,

$$T_K^{(E)} = D\exp\{-2/[j_A(1 + \sqrt{2}) + j_E + 2j_6]\}. \tag{14}$$

We see from (14) that both spin and pseudospin exchange constants contribute on an equal footing. Unlike the Kondo Hamiltonian for $\mathcal{N} = 3$ with equal J discussed in Ref. [11], the NFL regime is not realized for $\mathcal{N} = 1$ with $H_{\rm SW}$ (12). The reason of this difference is that starting with the Anderson Hamiltonian with finite Q, Q' in (1), one inevitably obtains the anisotropic SW exchange Hamiltonian for any $\mathcal{N}$. As a result, the interchannel exchange makes the NFL unstable. However, T_K is enhanced due to the inclusion of orbital degrees of freedom, and this enhancement is magnetically tunable. It follows from (7) that the crossover $SU(2) \rightarrow SU(4) \rightarrow SU(2)$ occurs 3 times within the interval $0 < \Phi < 6\pi$ and each level crossing results in enhancement of T_K from (10) to (14) and back [18]. These field-induced effects may be observed by measuring the two-terminal conductance G_{jl} through a TTQD (the third contact is assumed to be passive). Calculation by means of the Keldysh technique (at $T > T_K$) similar to that of Ref. [19] shows sharp maxima in G as a function of magnetic field, following the maxima of T_K (lower panel of Fig. 2).

So far we have studied the influence of the magnetic field on the ground-state symmetry of the TTQD. In a two-lead geometry [Fig. 1(b)] the field B affects the lead-dot hopping phases, thereby inducing an additional AB effect [20]. The symmetry of the device is thereby reduced since it loses two out of three mirror reflection axes. The orbital doublet E splits into two states, but still the ground state is $|DA\rangle$. In a generic situation, the total magnetic flux is the sum of two components $\Phi = \Phi_1 + \Phi_2$. In the chosen gauge, the hopping integrals in Eqs. (1) and (3) are modified as $W \rightarrow W\exp(i\Phi_1/3)$ and $V_{1,2} \rightarrow V_s\exp[\pm i(\Phi_1/6 + \Phi_2/2)]$, and the exchange Hamiltonian now reads

$$H = J_s\mathbf{S}\cdot \mathbf{s}_s + J_d\mathbf{S}\cdot \mathbf{s}_d + J_{sd}\mathbf{S}\cdot (\mathbf{s}_{sd} + \mathbf{s}_{ds}). \tag{15}$$

Cumbersome expressions for the exchange constants $J_s(\Phi_1, \Phi_2)$, $J_d(\Phi_1, \Phi_2)$, and $J_{sd}(\Phi_1, \Phi_2)$ will be presented elsewhere. They depend on the pertinent domain in parameter space of phases $\Phi_{1,2}$. Applying the poor man scaling procedure on the Hamiltonian (15) yields T_K,

$$T_K = D\exp\left\{-\frac{2}{j_s + j_d + \sqrt{(j_s - j_d)^2 + 4j_{sd}^2}}\right\}, \tag{16}$$

and the conductance at $T > T_K$ reads [19]

$$\frac{G}{G_0} = \frac{3}{4}\frac{j_{sd}^2}{(j_s + j_d)^2}\frac{1}{\ln^2(T/T_K)}. \tag{17}$$

The conductance $G(\Phi_1, \Phi_2)$ (17) obeys the Byers-Yang theorem (periodicity in each phase) and the Onsager condition $G(\Phi_1, \Phi_2) = G(-\Phi_1, -\Phi_2)$. We choose to display the conductance along two lines $\Phi_1(\Phi)$, $\Phi_2(\Phi)$ in parameter space of phases, namely, $G(\Phi_1 = \Phi, \Phi_2 = 0)$ and $G(\Phi_1 = \Phi/2, \Phi_2 = \Phi/2)$ (Fig. 3 left and right panels, respectively). The Byers-Yang relation implies respective periods of 2π and 4π in Φ. Experimentally, the magnetic flux is applied on the whole sample as in Fig. 1(b), and the ratio Φ_1/Φ_2 is determined by the specific geometry. Strictly speaking, the conductance is not periodic in the magnetic field unless Φ_1 and Φ_2 are commensurate.

The shapes of the conductance curves presented here are distinct from those pertaining to a mesoscopic AB interferometer with a single correlated QD and a conducting

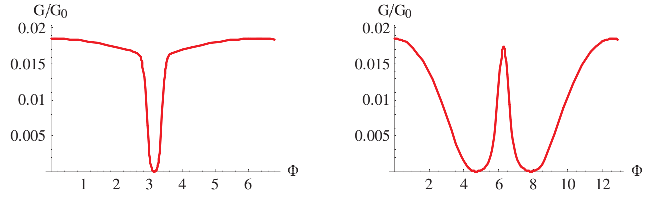

FIG. 3 (color online). Conductance as a function of magnetic field for $\Phi_2 = 0$ (left panel) and $\Phi_1 = \Phi_2 = \Phi/2$ (right panel).

channel [12,21] (termed the Fano-Kondo effect [21]). For example, $G(\Phi)$ in Fig. 3 of Ref. [21] (calculated in the strong coupling regime) has a broad peak at $\Phi = \pi/2$ with $G(\Phi = \pi/2) = 1$. On the other hand, $G(\Phi)$ displayed in Fig. 2 [pertinent to Fig. 1(a) and obtained in the weak coupling regime] is virtually flux independent except near the points $\Phi = (2n + 1)\pi$ (n integer) at which the $SU(4)$ symmetry is realized and G is sharply peaked. The phase dependence is governed here by interference effects on the level spectrum of the TTQD. The three dots share an electron in a coherent state strongly correlated with the lead electrons, and this coherent TTQD *as a whole* is a vital component of the AB interferometer. In the setup of Fig. 1(b), the Kondo cotunneling vanishes identically on the curve $J_{sd}(\Phi_1, \Phi_2) = 0$. The AB oscillations arise as a result of interference between the clockwise and anticlockwise "effective rotations" of TTQD in the tunneling through the {13} and {23} arms of the loop [Fig. 1(b)], provided the dephasing in the leads does not destroy the coherence of tunneling through the two source channels [22]. On the other hand, in the calculations performed on Fano-Kondo interferometers, $G(\Phi)$ remains finite [21]. Another kind of Fano effect due to the renormalization of electron spectrum in the leads induced by the lead-dot tunneling similar to that in chemisorbed atoms [23] is beyond the scope of this Letter.

To conclude, we have shown that spin and orbital degrees of freedom interlace in ring-shaped quantum dots, thereby establishing the analogy with the Coqblin-Schrieffer model in real metals. The orbital degrees of freedom are tunable by an external magnetic field, and this implies a peculiar AB effect, since the magnetic field affects the spectrum and the tunneling amplitudes. The conductance is calculated in the weak coupling regime at $T > T_K$ in three- and two-terminal geometries [Figs. 1(a) and 1(b)]. In the former case it is enhanced due to change of the dynamical symmetry caused by field-induced level crossing (Fig. 2). In the latter case the conductance can be completely suppressed due to destructive AB interference in source-drain cotunneling amplitude (Fig. 3). These results promise an interesting physics at the strong coupling regime as well as in cases of doubly and triply occupied TTQD. It would also be interesting to generalize the present theory for a quadratic QD [24], which possesses rich energy spectrum with multiple accidental degeneracies. Analysis of the TTQD system *without* a magnetic field was recently reported in a conference.

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