

Nonexponential Relaxations in a Two-Dimensional Electron System in Silicon

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(Received 27 September 2005; published 25 January 2006)

The relaxations of conductivity have been studied in a strongly disordered two-dimensional (2D) electron system in Si after excitation far from equilibrium by a rapid change of carrier density n_s at low temperatures T . The dramatic and precise dependence of the relaxations on n_s and T strongly suggests (a) the transition to a glassy phase as $T \rightarrow 0$, and (b) the Coulomb interactions between 2D electrons play a dominant role in the observed out-of-equilibrium dynamics.

DOI: 10.1103/PhysRevLett.96.037403

PACS numbers: 71.30.+h, 71.27.+a, 78.55.Qr

The interplay of strong electronic correlations and disorder is believed to be responsible for a plethora of new phenomena occurring in many complex materials in the region of the metal-insulator transition (MIT) [1]. Even though glassy behavior of electrons is emerging as one of the key concepts, electron or Coulomb glasses [2] in general remain largely unexplored. Slow, nonexponential relaxations characteristic of glassy dynamics have been observed in doped semiconductors [3], strongly disordered InO_x films [4–6], and various granular metals [7], but the regime near the MIT has been even less studied. In many complex systems, the MIT is accompanied by changes in magnetic or structural symmetry, which complicates the situation even further. On the other hand, low-density two-dimensional (2D) electron and hole systems in semiconductor heterostructures, where the MIT has been a subject of great interest and debate [8], represent particularly appealing model systems for studying the effects of interactions and disorder in a controlled and systematic way.

Our recent resistance noise measurements on a 2D electron system (2DES) in Si [9–11] have shown signatures of glassy behavior at densities n_s lower than some well-defined density n_g , such that $n_g > n_c$ (n_c —the critical density for the MIT). In particular, by reducing n_s , it was observed that the dynamics suddenly and dramatically slowed down at n_g , and there was an abrupt change to a correlated statistics, consistent with the hierarchical picture of glassy dynamics. These features were shown to persist even when the 2DES is fully spin polarized [11], indicating that the charge degrees of freedom are responsible for the anomalous noise behavior. Similar results were obtained in both highly disordered samples [9] and those with relatively low disorder [10,11], with $n_g \gtrsim n_c$ in the latter case. In addition, for $n_s < n_g$, the conductivity σ of the 2DES was found to depend on the cooling procedure [9], another characteristic of glassy systems. We note that, even though n_g is in the metallic regime [i.e., $\sigma(T \rightarrow 0) \neq 0$], σ is so small that $k_F l < 1$ (k_F —Fermi wave vector, l —mean free path) [12], which violates the Mott limit for the metallic transport in 2D. Hence, the noise studies [9–11] as well as this work probe the regime of strong disorder that encompasses both the insulating phase and the uncon-

ventional conducting regime where $k_F l < 1$. Such “bad metals” include a variety of strongly correlated materials with unusual properties [13].

Here we report a systematic study of the relaxations (i.e., time dependence) of σ in the same highly disordered 2DES in Si after applying a large perturbation at different n_s and T . The perturbation consists of a large, rapid change of n_s , controlled by the gate voltage V_g . For $n_s < n_g$, we find that, on short enough time scales, $\sigma(t)$ obeys a nonexponential, Ogielski form [14], found in some other systems [15] just above the glass transition. The associated relaxation time τ_{low} diverges as $T \rightarrow 0$, while its density dependence provides strong evidence that the observed out-of-equilibrium behavior is dominated by the Coulomb interactions between 2D electrons. After a sufficiently long time $t \gg \tau_{\text{high}}$, σ relaxes exponentially to its equilibrium value. The corresponding characteristic time $\tau_{\text{high}} \rightarrow \infty$ as $T \rightarrow 0$, suggesting that the glass transition temperature $T_g = 0$. The most peculiar feature of the relaxation is that the system equilibrates only after it first goes *farther away* from equilibrium.

Measurements were carried out on a 2DES in Si metal-oxide-semiconductor field-effect transistors (MOSFETs) with a large amount of disorder (the back-gate bias of -2 V was applied to maximize the 4.2 K peak mobility at ≈ 0.06 $\text{m}^2/\text{V s}$). The sample dimensions $L \times W$ (L —length, W —width) were 1×90 and 2×50 μm^2 . Both samples exhibited the same behavior, and, in general, they were almost identical to the samples used previously in noise measurements [9]. The data obtained on a 2×50 μm^2 sample are presented in detail below. For this sample, $n_s(10^{11} \text{ cm}^{-2}) = 4.31(V_g[\text{V}] - 6.3)$; $n_g(10^{11} \text{ cm}^{-2}) \approx 7.5$, and $n_c(10^{11} \text{ cm}^{-2}) \approx 4.5$, where n_g was determined from noise and n_c from $\sigma(n_s, T)$ measurements on both metallic and insulating sides (see Refs. [9,11]). The samples and the standard ac lock-in technique (typically 13 Hz) that was used to measure σ have been described in more detail elsewhere [9].

The experiment consists of the following procedure. The sample is cooled from 10 K to the measurement temperature T with an initial gate voltage V_g^i . Then, at $t = 0$, the gate voltage is switched rapidly (within 1 s) to a final value

V_g^f , and $\sigma(t, V_g^f, T)$ is measured. In general, the results did not depend on any of the following: initial temperature, as long as it was ≥ 10 K; V_g^i ; the cooling time, which was varied between 30 minutes and 10 hours; the time the sample was kept at 10 K (from 5 minutes to 8 hours); the time the sample spent at the measurement T before V_g was changed (from 5 minutes to 8 hours). Figure 1 shows a typical experimental run with $\sigma(t > 0)$ exhibiting a rapid (< 10 s) initial drop followed by a slower relaxation [16]. After going through a minimum, $\sigma(t)$ increases and approaches a value $\sigma_0(V_g^f, T)$. A subsequent warm-up to 10 K and a cooldown to the same measurement $T = 3.3$ K, while keeping the gate voltage fixed at V_g^f , show that $\sigma_0(V_g^f, T)$ represents the equilibrium conductivity corresponding to the given V_g^f and T . It is interesting that, even though initially it drops to a value close to σ_0 , σ first goes away from equilibrium before it starts approaching σ_0 again.

At the end of the run, the sample is warmed up to 10 K, gate voltage changed back to the same V_g^i , and the experiment is repeated at a different T for the same V_g^f . Figure 2 shows the relaxations of conductivity $\sigma(t, V_g^f, T)$, normalized to the corresponding $\sigma_0(V_g^f, T)$ for different T . The minimum in σ shifts to longer times with decreasing T until, at sufficiently low T , it falls out of the time window of the measurements. The relaxations obviously become slower with decreasing T , but the shape of the relaxation curves remains similar, suggesting that they might be collapsed onto a single scaling function.

First we focus on times just before the minimum in $\sigma(t)$. Figure 3(a) shows that the data can be described by a scaling form $\sigma(t, T)/\sigma_0(T) = a(T)g(t/\tau_{\text{low}}(T))$, such that $a(T) \propto (\tau_{\text{low}})^{-\alpha}$ ($\alpha \approx 0.07$) and $g(t/\tau_{\text{low}}) \sim \exp[-(t/\tau_{\text{low}})^\beta]$ ($\beta = 0.3$). We note that this stretched exponential dependence spans more than 4 orders of magnitude in t/τ_{low} and 2 orders in σ/σ_0 . At shorter times, most easily observable at low T within our experimental time window, the stretched exponential function crosses over to a slower, power-law dependence $\sigma(t, T)/\sigma_0(T) \propto t^{-\alpha}$ [Fig. 3(b)] [17]. In this regime, of course, it is also

possible to collapse the data but, obviously, not in an unambiguous way. Both power-law and stretched exponential relaxations are considered to be typical signatures of glassy behavior and reflect the existence of a broad distribution of relaxation times. In spin glasses, for example, the so-called Ogielski function [14] $g(t) \propto t^{-\alpha} \exp[-(t/\tau(T))^\beta]$ describes the relaxations over a wide range of t and T above T_g [15]. Our results [Figs. 3(a) and 3(b)] suggest that the Ogielski function might be a good candidate to describe also our data over the entire t and T range. Indeed, Fig. 3(c) shows that all the data are consistent with the Ogielski relaxation $\sigma(t, T)/\sigma_0(T) \propto (\tau_{\text{low}})^{-\alpha} (t/\tau_{\text{low}})^{-\alpha} \exp[-(t/\tau_{\text{low}})^\beta]$ over about 25 orders of magnitude in t/τ_{low} . Since it was impractical to perform measurements below 1 K long enough to obtain a substantial overlap between different T curves, we limit our analysis of $\tau_{\text{low}}(T)$ to the $T \geq 1$ K range, where the data collapse well. Even within this relatively narrow range of T , the scaling parameter τ_{low} varies over 7 orders of magnitude [Fig. 3(d)], and the Arrhenius function $1/\tau_{\text{low}} = k_0 \exp(-E_a/T)$ ($k_0 \sim 10 \text{ s}^{-1}$, $E_a \approx 19$ K for $V_g^f = 7.4$ V) provides an excellent fit to the data [18].

The experiment was repeated for different values of V_g^f , allowing us to map out the density dependence of the exponents α and β and of the parameter τ_{low} . The results were obtained either by fitting the data to the appropriate functional form at a fixed T [as in Figs. 3(a) and 3(b)] or from the scaling of the different T data and the fit to the Ogielski function [as in Fig. 3(c)]. Figure 4 shows that the two methods yield very similar results. In particular, we find that α decreases with n_s and vanishes at $n_s(10^{11} \text{ cm}^{-2}) \approx 7.5 \approx n_g$, i.e., exactly where the onset of glassy dynamics was found to take place from the noise measurements [9]. At the same time, the exponent β grows with increasing n_s , indicating that the relaxations become faster, as expected. For all n_s in this range, $1/\tau_{\text{low}} = k_0(n_s) \exp(-E_a/T)$, with $E_a \approx 20$ K independent of n_s ,

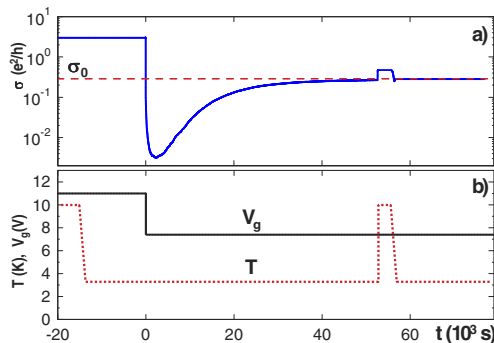


FIG. 1 (color online). (a) $\sigma(t)$ for $V_g^i = 11$ V [$n_s(10^{11} \text{ cm}^{-2}) = 20.26$], $V_g^f = 7.4$ V [$n_s(10^{11} \text{ cm}^{-2}) = 4.74$], and $T = 3.3$ K. (b) Experimental protocol: $V_g(t)$ and $T(t)$.

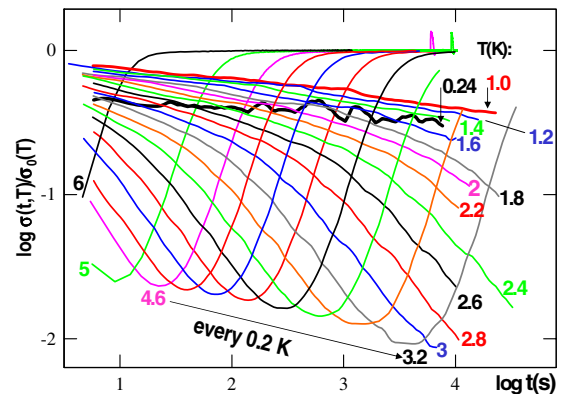


FIG. 2 (color online). $\sigma(t)$ for $V_g^i = 11$ V, $V_g^f = 7.4$ V, and several T , as shown. For $3.2 \leq T$ (K) ≤ 4.2 , T was changed in steps of 0.1 K but some traces have been omitted for clarity. Similarly, the data obtained at $0.24 < T$ (K) < 1 are shown in Fig. 3(b).

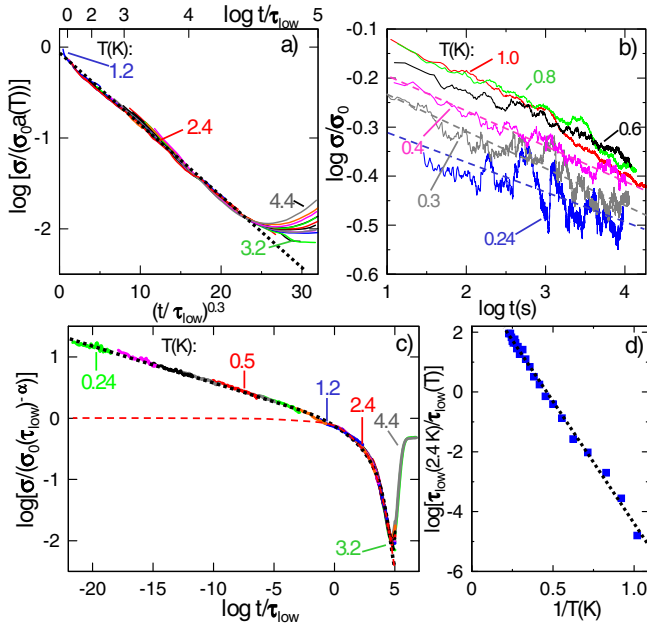


FIG. 3 (color online). (a) Scaling of the Fig. 2 data for t just below the minimum in $\sigma(t)$. The data obey a stretched exponential dependence on t ; the dotted line is a fit. The prefactor $a(T) \propto (\tau_{\text{low}})^{-\alpha}$, $\alpha \approx 0.07$. (b) At short t , relaxations follow a power law $\sigma(t, T)/\sigma_0(T) \propto t^{-\alpha}$ [17]. The dashed lines are fits with slopes $\alpha = 0.07 \pm 0.01$. (c) While the lowest T data clearly deviate from the stretched exponential dependence (dashed line), all the data are consistent with the Ogielski relaxation $\sigma(t, T)/\sigma_0(T) \propto (\tau_{\text{low}})^{-\alpha} (t/\tau_{\text{low}})^{-\alpha} \exp[-(t/\tau_{\text{low}})^\beta]$ (dotted line). The data have been collapsed with respect to the 2.4 K curve. (d) Scaling parameter τ_{low} as a function of T . The dashed line is a fit with a slope equal to an activation energy $E_a \approx 19$ K; $\tau_{\text{low}}(2.4 \text{ K}) \sim 500$ s.

but the prefactor has a very strong density dependence, $k_0(n_s) \propto \exp(-\gamma n_s^{1/2})$ [Fig. 4(b)]. Since the ratio of the Fermi energy to Coulomb energy in 2D systems $1/r_s = E_F/U \propto n_s^{1/2}$ [8,21], our result strongly suggests that Coulomb interactions between 2D electrons play a dominant role in the observed slow dynamics. As n_s is increased

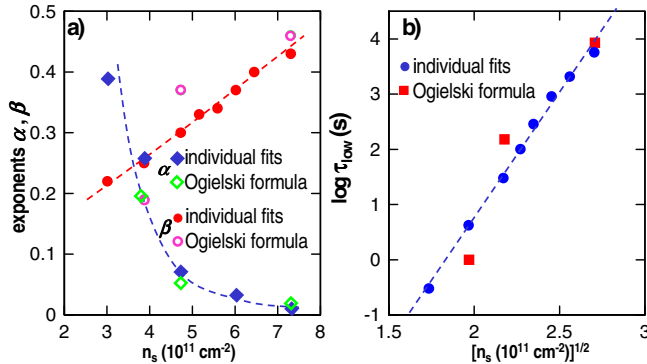


FIG. 4 (color online). (a) The exponents α and β vs n_s . Dashed lines guide the eye. (b) $\log \tau_{\text{low}}$ plotted vs $n_s^{1/2}$. The values of τ_{low} correspond to $T = 3$ K. The dashed line is a fit with a slope $(\log e)\gamma \approx 4.5(10^{-11} \text{ cm}^2)^{1/2}$.

further, beyond n_g , there are still some visible relaxations, albeit with an amplitude σ/σ_0 that is too small to make reliable fits. In fact, the amplitude of the relaxations decreases with n_s and vanishes when $n_s(10^{11} \text{ cm}^{-2}) \sim 30$, where $\sigma_0 \sim 3e^2/h$, i.e., $k_F l \sim 1$. Therefore, all the phenomena discussed in this Letter take place in the regime of strong disorder. We recall that, in general, electron-electron interactions are enhanced in the presence of disorder [22,23].

Finally, at times above the minimum in $\sigma(t)$ (Fig. 2), relaxations at different T approach the corresponding equilibrium values $\sigma_0(T)$. Here all the data can be described by a scaling form $\sigma(t, T)/\sigma_0(T) = f(t/\tau_{\text{high}}(T))$ [Fig. 5(a)], where $f(t/\tau_{\text{high}})$ is a simple exponential function with the characteristic relaxation time τ_{high}/k_1 . For some n_s [see Fig. 5(b)], another slower, simple exponential process also seems to exist with the same T dependence of its characteristic relaxation time τ_{high}/k_2 , but its presence does not seem to depend on n_s in any systematic way. We find that, unlike τ_{low} , τ_{high}/k_1 (and τ_{high}/k_2 , if present) does not depend on n_s in the regime studied. On the other hand, $k_1/\tau_{\text{high}} = (2.7 \times 10^3 \text{ s}^{-1}) \exp(-E_A/T)$, with $E_A \approx 57$ K independent of n_s [Fig. 5(b) inset].

Several striking features of our data stand out.

(1) At long times, the relaxation is exponential with a characteristic time τ_{high}/k_1 that diverges exponentially as $T \rightarrow 0$. This means that, at any finite T , the 2DES will reach equilibrium after a sufficiently long time $t \gg (\tau_{\text{high}}/k_1)$. For example, $\tau_{\text{high}}/k_1 \sim 30$ s at $T = 5$ K but, already at $T = 1$ K, $\tau_{\text{high}}/k_1 \sim 10^{13}$ years, which exceeds the age of the Universe by several orders of magnitude. Therefore, even though, strictly speaking, the system appears to be glassy only at $T = 0$, at low enough T the dynamics is glassy on all experimentally accessible time scales. It is interesting that the recent study of the 2D

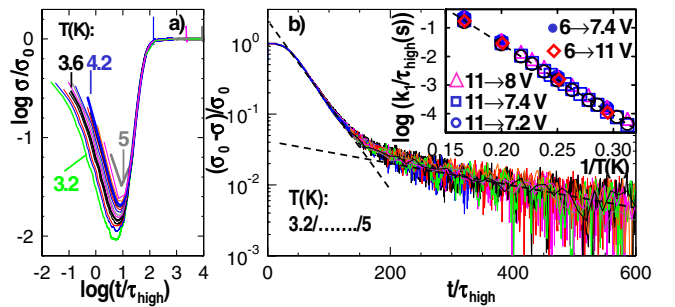


FIG. 5 (color online). (a) Scaling of the Fig. 2 data at long t , i.e., above the minimum in $\sigma(t)$. The data have been collapsed with respect to the 5 K curve. (b) The scaling function, which describes the approach to equilibrium value σ_0 at long times. The dashed lines are fits with slopes $k_1 \approx 2.89 \times 10^{-2}$ and $k_2 \approx 4.37 \times 10^{-3}$ for the faster and slower exponential processes, respectively. (The corresponding relaxation times are τ_{high}/k_1 and τ_{high}/k_2 .) Inset: T dependence of τ_{high}/k_1 for different V_g^i and V_g^f as shown on the plot. The dashed line is a fit with the slope equal to an activation energy $E_A \approx 57$ K.

Coulomb glass model has also found [24] an exponential divergence of the equilibration time as $T \rightarrow 0$, signaling a transition at $T_g = 0$.

(2) In the glassy regime, i.e., on short enough time scales $t < (\tau_{\text{high}}/k_1)$, the relaxations are strongly non-exponential. They obey the Ogielski form $\sigma(t, T)/\sigma_0(T) \propto t^{-\alpha} \exp[-(t/\tau_{\text{low}})^\beta]$, with $0 < \alpha < 0.4$ and $0.2 < \beta < 0.45$, where $\tau_{\text{low}} \propto \exp(\gamma n_s^{1/2}) \exp(E_a/T)$, $E_a \approx 20$ K. Since τ_{low} diverges as $T \rightarrow 0$, the relaxations attain a pure power-law form $\propto t^{-\alpha}$ at $T = 0$. We note that the Ogielski form of the relaxation and the divergence of τ_{low} are consistent with the general scaling arguments [25] near a continuous phase transition occurring at $T_g = 0$. Similar scaling is observed in spin glasses above T_g [15].

The very pronounced and precise dependence of τ_{low} on n_s strongly suggests that Coulomb interactions between 2D electrons play a dominant role in the observed out-of-equilibrium dynamics. In InO_x films, the most extensively studied example of an electron glass [4–6], the dependence of the relaxation time on the carrier density has been interpreted as evidence that the slow dynamics is due to electronic relaxation rather than glassiness of the extrinsic degrees of freedom [5].

(3) The 2DES equilibrates only after it first goes farther away from equilibrium. This overshooting of σ manifests itself as either a minimum [Fig. 1(a)] or a maximum in $\sigma(t)$, depending on the direction of V_g change. For example, if $V_g^i < V_g^f$, $\sigma(t)$ first increases, overshoots σ_0 and reaches a maximum, and then decreases exponentially to σ_0 . An analogous overshooting during relaxation is known to occur in orientational [26] and spin glasses [27] but has not been observed in InO_x films [6]. We note an important difference between our experiment and those on InO_x films. In InO_x , the change in the carrier density due to variations in V_g is typically $\sim 1\%$, and the system remains deep in the insulating state. In our case, n_s is changed up to a factor of 7, and the 2DES may go from the conducting to the insulating regime. Recent numerical work has shown [28] that such a strong perturbation can lead to nontrivial dynamics and, in particular, to a “roundabout” relaxation, where system equilibrates only after it once goes farther away from equilibrium.

In summary, we have studied the relaxations of σ in a strongly disordered 2DES after excitation far from equilibrium by a rapid change of n_s at low T . The data strongly suggest (a) the transition to a glassy phase as $T \rightarrow 0$, and (b) the Coulomb interactions between 2D electrons are primarily responsible for the observed out-of-equilibrium dynamics. Further work is needed to identify the microscopic details of the relaxation phenomena.

We are grateful to I. Raičević for technical assistance and V. Dobrosavljević for useful discussions. This work was supported by NSF Grant No. DMR-0403491 and NHMFL through NSF Cooperative Agreement No. DMR-0084173. D.P. acknowledges the hospitality of the Aspen Center for Physics.

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