Decay of Pure Quantum Turbulence in Superfluid ³He-B

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We describe measurements of the decay of pure superfluid turbulence in superfluid ³He-B, in the low temperature regime where the normal fluid density is negligible. We follow the decay of the turbulence generated by a vibrating grid as detected by vibrating wire resonators. Despite the absence of any classical normal fluid dissipation processes, the decay is consistent with turbulence having the classical Kolmogorov energy spectrum and is remarkably similar to that measured in superfluid ⁴He at relatively high temperatures. Further, our results strongly suggest that the decay is governed by the superfluid circulation quantum rather than kinematic viscosity.

In this Letter we present the first quantitative measurements of the decay of turbulence in a pure superfluid system. This is a subject of considerable interest since no conventional dissipation mechanisms are available.

In a classical fluid, turbulence at high Reynolds numbers is characterized by a range of eddy sizes obeying the well-known Kolmogorov spectrum. On large length scales the motion is dissipationless, whereas on small scales viscosity comes into play. Decay of the turbulence proceeds as energy is transferred by nonlinear interactions from the largest nondissipative length scales d (typically the size of the turbulent region) to smaller length scales where the motion is dissipated by viscous forces. The dissipation per unit volume is given by $\rho \nu \omega^2$ where ρ is the fluid density, ν the kinematic viscosity, and ω^2 the mean square vorticity [1]. An interesting question, which has received much theoretical speculation [1], is what happens in a pure superfluid with no viscous interactions?

Conceptually, turbulence in a superfluid is greatly simplified. Superfluids such as He-II and ³He-B are described by macroscopic wave functions with a well defined phase ϕ. The superfluid velocity is determined by gradients of the phase $v_S = (\hbar/m)\nabla\phi$, where m is the mass of the entities constituting the superfluid (the mass of a ⁴He atom for He-II or twice the mass of a ³He atom, $2m_3$, for the Cooper pairs in ³He-B). Consequently, in contrast to classical fluids, superfluid motion is inherently irrotational and vorticity may only be created in the superfluid by the injection of vortex lines. A superfluid vortex is a line defect around which the phase changes by 2π (ignoring here more complex structures such as in ³He-A). The superfluid order parameter is distorted within the relatively narrow core of the vortex where all the circulation is concentrated. The superfluid flows around the core with a velocity, at distance r, given by $v_S = \hbar/mr$, corresponding to a quantized circulation $\kappa = h/m$. Vortex lines are topological defects. They cannot terminate in free space, and therefore must either form loops or terminate on container walls.

Turbulence in a superfluid takes the form of a tangle of vortex lines.

Superfluid hydrodynamics is further simplified by the superfluid component having zero viscosity. At finite temperatures the fluid behaves as a mixture of two fluids, the superfluid condensate component as discussed above and an interpenetrating normal fluid comprising the thermal excitations. The normal fluid component has a finite viscosity and exerts a damping force on the motion of vortex lines via the scattering of thermal excitations, this interaction being known as mutual friction.

To date, studies of superfluid turbulence have largely focussed on He-II at relatively high temperatures. Under these conditions, it is believed that mutual friction effectively couples the turbulent structures in the normal and superfluid components [1]. The ensuing combined turbulence is found to behave in an almost identical manner to that of classical turbulence when generated by a towed grid [2,3]. The decay of grid turbulence observed in He-II can thus be explained quantitatively [2,3] using the classical picture with the conceptually reasonable assumptions that $\omega^2 = (\kappa L)^2$, where L is the length of the vortex line per unit volume, and that the effective kinematic viscosity is $\nu \sim \eta_n/\rho$, where η_n is the normal fluid viscosity and ρ is the total fluid density.

The situation in superfluid ³He should be completely different. The fermionic nature of normal liquid ³He ensures that the liquid is very viscous (comparable to room temperature glycerol). This high normal fluid viscosity means that the normal component can never become turbulent under typical experimental conditions. Further, owing to the interaction via mutual friction, turbulence in the superfluid is also suppressed at high temperatures. Consequently, turbulence in ³He-B is only found at temperatures below $\sim 0.5 T_c$ where the mutual friction has become low enough to decouple the two components, allowing the superfluid to support turbulence independently [4].

At even lower temperatures (below $\sim 0.3 T_c$), both the normal fluid component and mutual friction become ex-

ponentially small, the excitations are too dilute to interact and they become ballistic. In this regime, the whole concept of a normal fluid component breaks down. These are conceptually the simplest conditions for studying turbulence; we effectively have only one incompressible and irrotational fluid component with zero viscosity supporting quantized vortex lines. Here we have a system where the classical decay mechanism *absolutely* cannot operate. So, what happens instead?

Turbulence in superfluid ^{3}He-B can be readily detected at low temperatures via its effect on the quasiparticle dynamics [5]. The dispersion curve $\epsilon(\mathbf{p})$ of these ballistic quasiparticles is tied to the reference frame of the stationary superfluid. The curve thus becomes tilted by the Galilean transformation $\epsilon(\mathbf{p}) \rightarrow \epsilon(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v_S}$ in a superfluid moving with velocity $\mathbf{v_S}$. Consequently, quasiparticles moving along a superflow gradient experience a potential energy barrier and are Andreev reflected if they have insufficient energy to proceed [6]. The Andreev process converts a quasiparticle into a quasihole and vice versa, reversing the group velocity of the excitation but yielding negligible momentum transfer.

The complicated flow field associated with superfluid turbulence acts as a shifting ragged potential for quasiparticles. The net result is that some fraction of incident thermal quasiparticles are Andreev reflected. Quasiparticles may be detected in ^{3}He-B at low temperatures by vibrating wire techniques. The thermal damping of a vibrating wire [7] in ^{3}He-B arises from normal scattering of quasiparticle excitations at the wire surface. A wire immersed in turbulence thus experiences a reduction in damping proportional to the amount of Andreev reflection of incoming thermal excitations caused by the turbulent flow. This effect has been exploited to observe turbulence generated by vibrating wires [5] and vibrating grid [8] resonators at low temperatures. Andreev reflection from turbulence has also been measured directly using ballistic quasiparticle beam techniques [9]. Previous measurements of vortex generation by a vibrating grid have shown that at low grid velocities ballistic vortex rings are emitted [8] and turbulence only forms above a certain critical velocity. Here, we discuss measurements of the decay of turbulence generated from a vibrating grid at higher velocities.

The experimental arrangement is shown in Fig. 1 and is the same as that used for the measurements reported previously [8,10]. The grid is made from a 5.1×2.8 mm mesh of fine copper wires. The wires have an approximately 11 μm square cross section and are spaced 50 μm apart leaving 40 μm square holes. A 125 μm diameter Ta wire is bent into a 5 mm square frame and attached to the inner cell wall of a Lancaster style nuclear cooling stage [11]. The mesh is glued to the Ta wire over thin strips of cigarette paper for electrical insulation.

Facing the grid are two vibrating wire resonators made from 2.5 mm diameter loops of 4.5 μm NbTi wire. The "near" and "far" wires are positioned 1 and 2 mm from the grid, respectively. An additional wire resonator is used
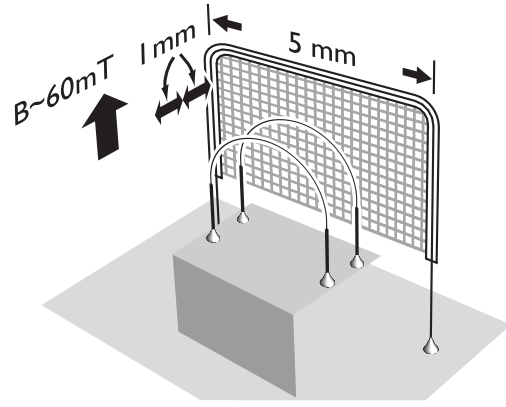

FIG. 1. The arrangement of the grid and associated vorticity detector wires.

as a background thermometer. This wire, not shown in the figure, is located about 4 mm to the side of the grid and enclosed in a mesh cage to ensure that its response is not influenced by any stray turbulence.

The grid is operated similarly to a wire resonator. It is situated in a vertical applied magnetic field and driven by the Lorentz force generated by passing an ac current through the Ta wire. As the grid moves, the Ta wire develops a Faraday voltage proportional to its velocity. The grid resonates at a frequency of $\sim$1300 Hz, determined by the stiffness of the Ta wire and the mass of the grid.

In contrast to a vibrating wire resonator, the grid shows no sign of a pair-breaking critical velocity. In the low temperature limit, the grid's response changes gradually from a linear damping force $F \propto v$ for velocities below around 1 mm/s, to approximately $F \propto v^2$ behavior at higher velocities [10]. The linear response is governed by the intrinsic (vacuum) damping of the resonator motion. The response at high velocities has the form expected for turbulent drag from a classical fluid [10].

Vortices generated by the grid are detected by the two facing vibrating wire resonators as discussed in [8]. Briefly, the two resonators and the thermometer resonator are driven on resonance at relatively low velocity. The resulting induced voltages across the wires are continuously monitored, allowing us to deduce the quasiparticle damping (frequency width of the resonance) $\Delta f_2(T)$ for all three wires. The grid is then driven to some velocity v generating vortex lines (ballistic vortex rings at low velocities; turbulence at higher velocities). This vorticity Andreev reflects some fraction f of quasiparticles approaching a vibrating wire, giving rise to a reduced damping $\Delta f_2(v, T) = (1 - f)\Delta f_2(0, T)$. In practice, significant power is required to drive the grid, resulting in an overall warming of the cell. The damping in the absence of turbulence $\Delta f_2(0, T)$ is therefore inferred from the thermometer wire damping (with no turbulence, the quasiparticle damping on each of the three wires is simply related by a measured constant of proportionality, close to unity). The fractional screening f of quasiparticles due to the surrounding turbulence is thus measured for the two facing wires.

All the measurements discussed below were made at 12 bar and at temperatures below ~$0.2T_c$. At such temperatures the turbulence is found to be insensitive to temperature. This is consistent with previous measurements, both of turbulence generated from vibrating wires [9] and of vortex rings generated from a vibrating grid [8], indicating that we have reached the zero temperature limit for the turbulent dynamics where both the normal fluid fraction and the mutual friction are negligible.

The steady state average values of the fractional screening f are found to increase roughly as v^2. The far wire, 2 mm from the grid, has roughly a factor of 2 less screening than the near wire, 1 mm from the grid, over the entire velocity range. If the variation with distance followed an exponential decay, as found previously for turbulence generated by vibrating wires [12], then this would correspond to a spatial decay length of $d \sim 1.5$ mm.

The approximate vortex line density may be inferred from these measurements using the arguments of [9]. The fraction of quasiparticles Andreev reflected after passing through a homogenous isotropic vortex tangle of line density L and thickness x is given by $f \simeq Lp_F\hbar x/2m_3k_BT$, provided f is small compared to unity. Since in practice the tangle density varies in space, we should strictly integrate an analogous expression over all quasiparticle trajectories incident on the vibrating wire resonators. This is obviously not possible without an accurate knowledge of the spatial dependence of the tangle. We therefore simply use the above expression with $x = d = 1.5$ mm to give an estimated average line density which should be correct to within a factor of order 2.

The transient behavior of the inferred line density after the drive to the grid is turned off is shown in Fig. 2 for the wire nearest to the grid. Data are shown for various initial grid velocities down to 3.5 mm/s. (At lower velocities the recovery is much faster corresponding to ballistic vortex ring production [8].) At late times the data all tend to a single limiting line (line A in the figure).

In Fig. 2 we also show data for turbulent decay from a grid towed at various velocities through He-II [3]. The authors shifted the time axis for each of these curves, but this does not effect the late-time behavior which is fitted by line B [2,3] (see below). The fitted line lies about a factor of 4 higher than our data. The authors were able to explain these observations in some detail on the basis of classical turbulence of the combined normal/superfluid components. The classical cascade process leads to a line density which decays as $L = (d/2\pi\kappa)\sqrt{(27C^3/\nu')}t^{-3/2}$ at late times [2,3], where C is the Kolmogorov constant, expected to be of order unity and d is the characteristic size of the container (which limits the maximum eddy size in the classical theory). Excellent agreement was found with their data using $C \simeq 1.6$ and an effective kinematic viscosity ν' of roughly twice the actual kinematic viscosity, η_n/ρ.

If we take a similar approach and naively use this classical expression for the late-time line density, substi-
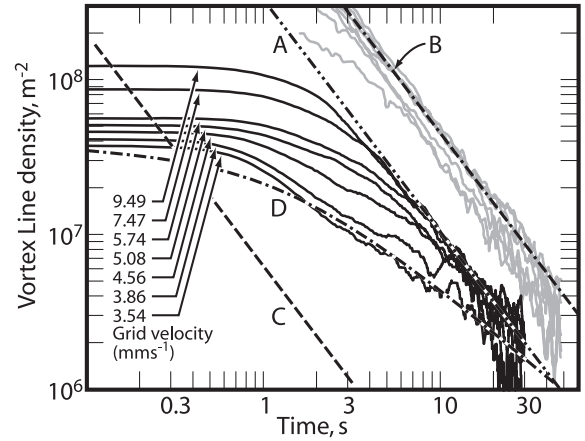

FIG. 2. Solid black curves show the inferred vortex line density as a function of time after cessation of grid motion for various initial grid velocities. Line A is the limiting behavior scaled to our results as discussed in the text. The halftone data is that for a towed grid in superfluid ^{4}He of Skrbek *et al.* [3] with line B showing the late-time limiting behavior. Line C shows the expected behavior for our data assuming the classical dissipation law. Curve D shows the expected behavior for a random tangle in superfluid ^{3}He. See text.

tuting the appropriate numbers for our experiment then we obtain line C in Fig. 2. This line lies much lower than that of ^{4}He partly since dimension d is smaller ($d = 1.5$ mm in our case against $d = 10$ mm for the ^{4}He experiments) but mainly because the normal fluid viscosity [13] is orders of magnitude larger for ^{3}He. It is very clear that our measurements, even though they are similar to those in superfluid ^{4}He, *cannot* be explained by the classical decay mechanism, as we anticipated.

The Kolmogorov energy cascade in classical turbulence is a consequence of dissipation being negligible on large length scales. As suggested by Vinen [1], it seems reasonable to expect that superfluid turbulence as generated by grids will display a similar cascade process owing to the similar absence of large length scale dissipation mechanisms. This expectation is supported by numerical simulations [14], which show evidence of a Kolmogorov-like cascade in pure superfluid turbulence in the absence of any normal fluid component. In other words, for He-II both fluid components have a natural tendency to display the Kolmogorov-like cascade. Therefore, this behavior is likely to occur at arbitrary temperatures, and with the two flows locked together by mutual friction at the higher temperatures. By the same reasoning, one might expect similar behavior for superfluid ^{3}He-B in the low temperature limit. At the higher temperatures, mutual friction will now couple the superfluid turbulence to the highly viscous nonturbulent normal ^{3}He, suppressing turbulence completely at high temperatures, and yielding a different energy spectrum in the intermediate region [15].

At very low temperatures in the superfluid where there are no mutual friction processes, Vinen [1] has argued (on

purely dimensional grounds) that any process leading to loss of vortex line length must depend on the circulation quantum, yielding a dissipation of order $\rho\kappa(\kappa L)^2$. The effective kinematic viscosity in the decay equation should therefore be replaced by a term $\zeta\kappa$ where ζ is a dimensionless constant, presumably of order unity. The line density at late times of the turbulent decay should therefore be described by $L = (d/2\pi\kappa)\sqrt{(27C^3/\zeta\kappa)}t^{-3/2}$.

Since in He-II the kinematic viscosity and the circulation quantum are numerically similar ($\nu \approx 0.1\kappa$), the data of Skrbek *et al.* [3], interpreted above on the basis of the kinematic viscosity, are also consistent with a dissipation based on the quantum expression with $\zeta \approx 0.2$. However, ν and κ are orders of magnitude different in superfluid ^{3}He. If we use the Vinen expression for our data, with $d = 1.5$ mm and $\zeta = 0.2$, then we obtain the expected late-time behavior shown by line A in the figure. (Equivalent to scaling the late-time He-II data by d and κ). The agreement is quite staggering, since not only does the superfluidity in the two systems arise from completely different mechanisms, but both the temperature regimes and normal fluid viscosities differ by many orders of magnitude.

The decay for the lowest grid velocity shown in Fig. 2 appears to show a limiting behavior closer to t^{-1}. A purely random tangle can have only one length scale, that of the intervortex spacing $L^{-1/2}$ and hence no Kolmogorov cascade. In this case the line density is expected to decay by the Vinen equation [1] $\dot{L} = \zeta'\kappa L^2$. Curve D in the figure shows the expected behavior according to this equation with $\zeta' = 0.3$ and an initial line density chosen to match the lowest grid velocity data at the start of the decay. The agreement is fair, suggesting that the Kolmogorov energy cascade might only develop for higher grid velocities (line densities). This is not conclusive however, since the lower grid velocity data could also be made to fit with the full classical model given in [3].

As a final caveat, if the turbulence we generate is inhomogeneous then the observed decay may include a spatial component from the diffusion of the vorticity down a vorticity gradient. However, we can estimate this effect from the computer simulations by Tsubota *et al.* [16] which suggest that inhomogeneous turbulence evolves spatially with a diffusion constant of $\sim 0.1\kappa$. For our experiment this number yields a time scale for diffusion of order $\tau \sim d^2/0.1\kappa \sim 300$ s. This is much longer than the measured decay time and therefore any contribution from diffusion should not be significant. (We also note that turbulence generated in classical fluids by oscillating grids can be quite isotropic under certain conditions [17].)

In conclusion, we have measured the decay of turbulence in superfluid ^{3}He-B generated by a vibrating grid at very low temperatures where there is essentially no normal fluid. The decay is found to be consistent with a classical Kolmogorov-type energy cascade and very similar to that found for turbulence from a towed grid in He-II at high temperatures. This is a remarkable result given that the two

liquids have entirely different mechanisms for superfluidity and that the measurements were performed at opposite ends of the temperature range. In contrast to the He-II case, the decay observed in these measurements *cannot* be explained in terms of a classical decay mechanism (i.e., via a normal fluid viscosity). The measurements strongly indicate that the decay is governed by the circulation quantum, which has a similar magnitude to that of He-II. The questions remaining are: (a) what is the specific microscopic mechanism for the dissipation and (b) how does the superfluid tangle acquire or develop the requisite range of length scales necessary for the Kolmogorov energy cascade to function?

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[1] W. F. Vinen and J. J. Niemela, J. Low Temp. Phys. **128**, 167 (2002).

[2] S. R. Stalp, L. Skrbek, and R. J. Donnelly, Phys. Rev. Lett. **82**, 4831 (1999).

[3] L. Skrbek, J. J. Niemela, and R. J. Donnelly, Phys. Rev. Lett. **85**, 2973 (2000).

[4] A. P. Finne, T. Araki, R. Blaauwgeers, V. B. Eltsov, N. B. Kopnin, M. Krusius, L. Skrbek, M. Tsubota, and G. E. Volovik, Nature (London) **424**, 1022 (2003).

[5] S. N. Fisher, A. J. Hale, A. M. Guénault, and G. R. Pickett, Phys. Rev. Lett. **86**, 244 (2001).

[6] M. P. Enrico, S. N. Fisher, A. M. Guénault, G. R. Pickett, and K. Torizuka, Phys. Rev. Lett. **70**, 1846 (1993).

[7] S. N. Fisher, A. M. Guénault, C. J. Kennedy, and G. R. Pickett, Phys. Rev. Lett. **63**, 2566 (1989).

[8] D. I. Bradley, D. O. Clubb, S. N. Fisher, A. M. Guénault, R. P. Haley, C. J. Matthews, G. R. Pickett, V. Tsepelin, and K. Zaki, Phys. Rev. Lett. **95**, 035302 (2005).

[9] D. I. Bradley, S. N. Fisher, A. M. Guénault, M. R. Lowe, G. R. Pickett, A. Rahm, and R. C. V. Whitehead, Phys. Rev. Lett. **93**, 235302 (2004).

[10] D. I. Bradley, D. O. Clubb, S. N. Fisher, A. M. Guénault, R. P. Haley, C. J. Matthews, and G. R. Pickett, J. Low Temp. Phys. **134**, 381 (2004).

[11] D. I. Bradley, A. M. Guénault, V. Keith, C. J. Kennedy, I. E. Miller, S. G. Mussett, G. R. Pickett, and W. P. Pratt Jr., J. Low Temp. Phys. **57**, 359 (1984).

[12] D. I. Bradley, S. N. Fisher, A. M. Guénault, M. R. Lowe, G. R. Pickett, and A. Rahm, Physica B (Amsterdam) **329**, 104 (2003).

[13] D. C. Carless, H. E. Hall, and J. R. Hook, J. Low Temp. Phys. **50**, 605 (1983).

[14] T. Araki, M. Tsubota, and S. K. Nemirovskii, Phys. Rev. Lett. **89**, 145301 (2002).

[15] G. E. Volovik, J. Low Temp. Phys. **136**, 309 (2004).

[16] M. Tsubota, T. Araki, and W. F. Vinen, Physica B (Amsterdam) **329–333**, 224 (2003).

[17] I. P. D. De Silva and H. J. S. Fernando, Phys. Fluids **6**, 2455 (1994).