Proton Reaction Cross Sections as Measures of the Spatial Distibutions of Neutrons in Exotic Nuclei

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Proton and neutron densities from Skyrme-Hartree-Fock calculations of a number of nuclei with masses ranging from 28 to 58 have been used to generate optical potentials for proton elastic scattering. Those potentials, generated by folding the structure functions with effective in-medium nucleon-nucleon (*NN*) interactions, have been used to evaluate proton total reaction cross sections; cross sections that reveal signatures of the structures.

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There is much current interest in the properties of, and reactions with, exotic (radioactive) nuclei. Many can be formed as radioactive ion beams (RIBs) at modern experimental facilities throughout the world, which has spurred interest from the viewpoint of new studies in nuclear physics. However, interest in those systems is more widespread, with the role they play in the quiescent as well as explosive burning processes in stellar systems being one example. All data from reactions must be analyzed with a scattering theory. The most useful of those theories require that all input, other than that of the structure of the nuclei involved in the reaction to be assessed, be preset. Then, with an assumed model structure, as there are no adjustable parameters in such a theory, the results are predictions. Only with such analyses can physical properties of exotic nuclei that may be involved be determined with confidence.

One of the best experiments to consider for the purpose of probing matter densities of exotic nuclei is RIB scattering from hydrogen targets. By inverse kinematics this equates to proton scattering from RIB nuclei as targets. As the external proton interacts more strongly with bound neutrons than it does with bound protons, the scattering is somewhat more, but not exclusively, sensitive to neutron distributions in the nucleus. However, most current research projects with RIBs have been of reactions with complex nuclei as targets, experiments which only probe the long range radial properties of the incident projectile [1,2]. Indeed, breakup occurs readily and that correlates with a strongly absorptive optical potential through most of the nuclear volume. As there are no electron scattering data from (most) radioactive nuclei, RIB scattering from hydrogen targets essentially is the only current sensitive means to probe the character of the ions within the nuclear surface. That proton-nucleus scattering does so probe the nucleus is evident from the results reported in Ref. [3] for the stable nucleus ^{208}Pb (for which there is a corroborating electron scattering form factor), and in Ref. [4] for the radioactive nucleus ^{6}He. That sensitivity can be used to discriminate between competing models of the structure of the nucleus involved, but only when a credible scattering theory is used in the analysis of the scattering data. Defining a credible scattering theory to use, however, is dependent upon the incident energy. For low energies, coupled channel effects involving low-lying discrete states of the nucleus need be considered. How much, and with what additional requirements, has been indicated in recent studies made using a multichannel algebraic scattering theory [5,6]. For energies typically 25 to 250 MeV, a g-folding method [7] of data analysis has proved appropriate [4], as explicit channel coupling is not of prime importance.

As the g-folding method analyses of RIB-hydrogen scattering are highly sensitive to the character of nuclei in and through the nuclear surface, and as the associated relative motion wave functions are required in any distorted wave analysis of other reaction data measured, it is to be hoped that program advisory committees at RIB producing facilities will see much merit to proposals involving elastic scattering from hydrogen, and ascribe greater importance to them than has been the case to date. Such should also be considered very carefully with transfer reaction proposals, especially those involving the transfer of a single nucleon. In any event, a more favorable view of making elastic scattering measurements with RIB's would be most welcome. To extract physics information from any reaction studied invariably requires specification of the relative motion wave functions, so far *only* defined by an analysis of elastic scattering. Relying upon global forms of optical potentials, especially since they are specified from scattering on stable nuclei, is not only fraught with uncertainties but also usually violates the Pauli principle [6].

Theories of ground-state properties of nuclei, stable or radioactive, have been developed with some sophistication. They now give properties, such as root mean square (rms) radii, that are quite consistent with estimates made from experimental studies. With stable nuclei of course there is considerable information, such as extensive spectra (energies and spin parities), gamma decay rates, as well as electron scattering form factors, against which the propriety of the model of structure assumed may be assessed. But that is not the case with most radioactive nuclei. For those that can be made as RIBs, one has essentially only data from their reactions with other nuclei to use in such assessments.

Charge-density distributions and the associated nuclear radii have been calculated previously [8] for the nuclei of interest herein. The Skyrme-Hartree-Fock (SHF) method was used and the resulting wave functions gave form factors in very good agreement with available data from electron scattering. A significant improvement in the agreement is obtained if shell-model occupancies are used. Two forms of the Skyrme interaction were used, the so-called $\mathrm{SkX_{CSB}}$ [9] and SkM* [10] interactions. The $\mathrm{SkX_{CSB}}$ Hamiltonian is based on the SkX Hamiltonian [11] with a charge-symmetry-breaking (CSB) interaction added to account for nuclear displacement energies [9]. The charge densities from all three calculations are very similar and so we will use only those determined from the $\mathrm{SkX_{CSB}}$ model (referred to hereafter as simply "SkX"). There are some small $\leq 5\%$ differences in the interior densities found with these models but they have little effect on scattering results; especially of the total reaction cross sections we consider later. Generally, with this SHF method, good agreement between theory and experiment has been achieved in extensive comparisons of measured nuclear charge-density distributions with calculated values for p-shell, sd-shell, and pf-shell nuclei and some selected magic and semimagic nuclei up to ^{208}Pb.

Root mean square (rms) radii found using the SkX model wave functions for a set of nuclei from ^{28}Si to ^{64}Ni are displayed in Figs. 1 and 2. In the first are shown the proton rms values versus nuclear mass number A. We identify results associated with ^{28}Si, ^{32}S, ^{36}Ar, ^{40}Ca, ^{54}Fe, and ^{58}Ni as the minimal-T set.

The SkX model proton rms radii shown in Fig. 1 are compared with values given by $A^{1/3}$ (dashed curve) and one that best fits the minimal-T set of values portrayed by the solid curve. That solid curve was found from

$$\sqrt{\langle r_\pi^2 \rangle} = [1.074 - 0.0022A]A^{1/3}. \qquad (1)$$

Clearly the isotope sets vary from that optimal curve with, as expected, the proton rms values slowly increasing as neutrons are added to form neighboring isotopes. Those increases are not uniform or even parallel. For the nickel isotopes the increase is quite well matched by the A dependence of the minimal-T curve.

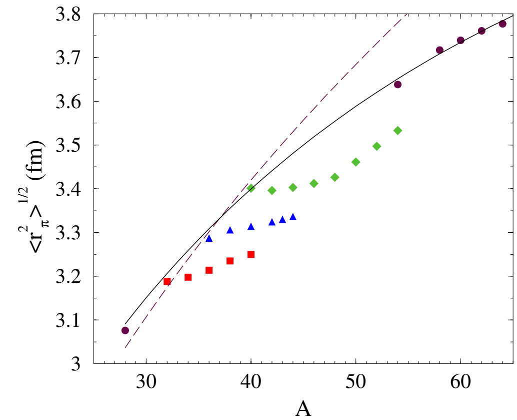

FIG. 1 (color online). The proton rms radii from the SHF models versus mass number. Isotopes of S are given by solid squares, Ar by solid triangles, Ca by solid diamonds; Ni ($A \geq 58$), ^{28}Si, and ^{54}Fe by solid circles.

The SkX model calculations gave rms values for matter radii that are shown in Fig. 2. Those results are compared with the $A^{1/3}$ values (dashed curve) and with the minimal-T curve (solid) derived from

$$\sqrt{\langle r_{\mathrm{mass}}^2 \rangle} = [1.054 - 0.0017A]A^{1/3}. \qquad (2)$$

For this quantity, the increase in neutron rms radii complementing the much slower increase in the proton rms radius makes for a net mass rms radius increase more like the ubiquitous $A^{1/3}$ value. The minimal-T curve indicates that the isotope shifts steadily increase from that curve. In this case it is most evident with the nickel isotope values. Thus the more or less steady values of proton rms radii are compensated by a marked increase in the neutron values. That is clear from the skin thickness, which we define as the difference between the neutron and proton

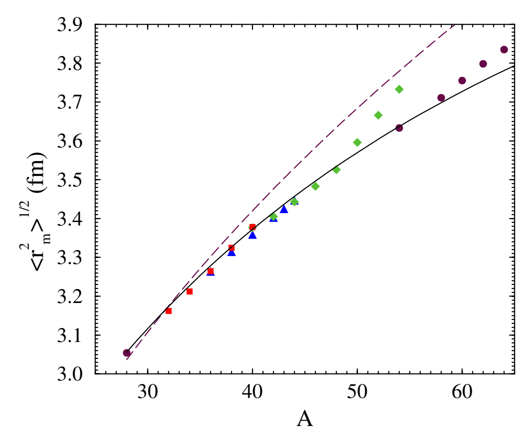

FIG. 2 (color online). The matter rms radii from the SHF models versus mass number. The notation is as in Fig. 1.

rms radii of a nucleus, i.e.,

$$S_n = \sqrt{\langle r_\nu^2 \rangle} - \sqrt{\langle r_\pi^2 \rangle}. \quad (3)$$

The skin thicknesses of the nuclei considered and as given by the SkX model of their structure, are plotted in Fig. 3.

From this model, the minimal-T set of six nuclei all have proton radii larger than the neutron values, while each isotope set has skin thickness values that form almost parallel lines with nuclear mass but with decreasing increments as the charge of the isotope increases.

Good agreement with experiment for electron scattering [8] justifies using the proton and neutron radial wave functions defined by the SHF studies in forming optical model potentials to analyze proton elastic scattering. Specifically, we have formed optical potentials for the elastic scattering of protons with energies of 65 and 200 MeV, energies at which the g-folding method has had many successes already in analyzing scattering data [4,7,12]. That includes predictions of total reaction cross sections for scattering protons from stable nuclei ranging in mass from ^{6}Li to ^{208}Pb, and for energies between 25 and 175 MeV.

To use the g-folding method in data analyses requires specification of an effective (medium modified) NN interaction, single-nucleon bound-state wave functions, and ground-state one-body density matrix elements (OBDME). The latter, mainly, are the shell occupancies of nucleons in the target ground state. The effective NN interactions have been developed from NN g matrices, which are solutions of the Brueckner-Bethe-Goldstone (BBG) equations for infinite nuclear matter [7]. Those we use have been built upon the Bonn-B free NN interaction. The other items required in the method must be defined using specific nucleon-based models of structure for the target nucleus. When all such details are predetermined, and other information has shown them to be credible, the g-folding method gives predictions of observable quantities in very good agreement with measured values. With stable nuclei targets, consistent checks are found when results are interpreted and compared against electron scattering form factors.

The g-folding method is implemented by using the code DWBA98 [13]. A crucial element in that code is explicit evaluation of exchange (knock out) amplitudes in determining total and differential cross sections as well as spin observables.

Herein we illustrate, with two cases, that the g-folding method does successfully predict cross-section and analyzing power data for cases in the mass range of our SHF structures as well as at the two energies (65 and 200 MeV). Much more data at many more energies are to be the subjects of a longer subsequent presentation. In particular, in Fig. 4, the differential cross-section and analyzing power data taken at 65 MeV of proton elastic scattering from ^{40}Ar [14] and at 192 MeV from ^{58}Ni [15] (solid squares), supplemented with data from Ref. [16] (solid circles), are compared with the results found using the g-folding model. These two sets of results are in excellent agreement with the data notwithstanding that predicted analyzing powers drift slightly from measured results as the scattering angle increases.

It is important to note that the results given here are predictions in that the effective NN interactions were predetermined from solutions of BBG equations with the chosen nuclear density of each target defining the effective Fermi momenta of the NN interaction to be used at each radius. Thus there is no parameter to be adjusted and a single calculation of scattering cross sections, etc., was made. The excellent agreement between our predictions and measured data is direct justification that the chosen prescription of the target matter distributions is a credible one.

The prime purpose of a more comprehensive parallel study for a set of isotopes of stable nuclei in the sd and pf shell mass region has been to study the proton total reac-

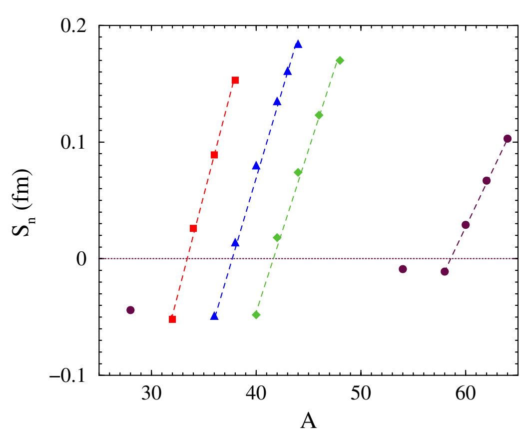

FIG. 3 (color online). The skin thickness S_n from the SHF model structures used. The notation is as defined previously.

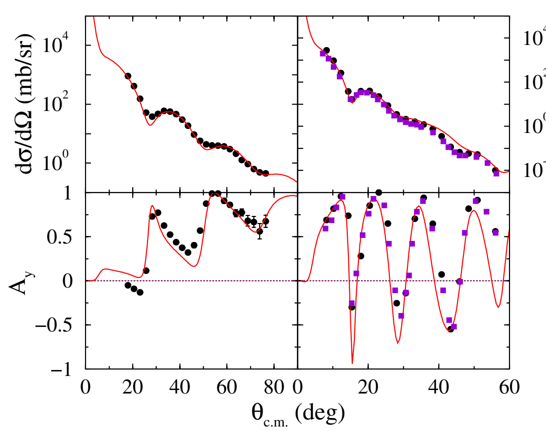

FIG. 4 (color online). The differential cross section (top) and analyzing power (bottom) for the elastic scattering of 65 MeV protons from ^{40}Ar (left) and of 200 MeV protons from ^{58}Ni (right).

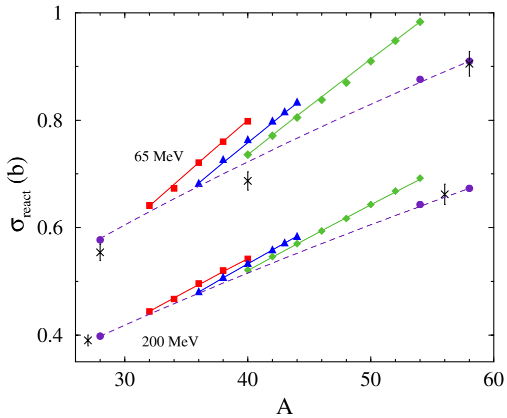

FIG. 5 (color online). Total reaction cross sections for 65 and 200 MeV from g-folding calculations of scattering. The notation is as used in the previous figures, and the measured data [17] are shown by the crosses with error bars (65 MeV: Si, Ca, and Ni; 200 MeV: Al and Fe). The lines are fits found using the Carlson model.

tion cross sections. We included isotopes that are radioactive and, specifically, we considered the reaction cross sections for 65 and 200 MeV protons scattering from isotopes of S, Ar, and Ca, and for scattering from ^{28}Si, ^{54}Fe, and ^{58}Ni. Results are plotted in Fig. 5. Therein some experimental values from Ingemarrson *et al.*. [17] are shown by the crosses with error bars. The lines for each isotope set, and for the minimal-T set of nuclei, are values predicted by the Carlson model [18],

$$\sigma_R = \pi(R_p + r_0 A^{1/3})^2. \tag{4}$$

The values of R_p and r_o required are listed in Table I. The total reaction cross sections of the sets of isotopes all lie on smooth curves defined by the Carlson model as do the minimal-T set of $N = Z$ nuclei, ^{54}Fe, and ^{58}Ni. It is clear that as neutrons are added there is a steady progression in the total reaction cross sections. There is also a Coulomb shift effect that is more pronounced at 65 MeV than at 200 MeV.

For both energies, the total reaction cross sections evaluated for the minimal-T nuclei as well as for each isotope set satisfy the Carlson model very well. The parameter values to fit the minimal-T nuclei properties also agree quite well with the universal set found by Carlson [18]. But the parameter values required to fit the isotope sets are quite different. It is noteworthy that our calculated values tend to be parallel (for each energy), with the 200 MeV results closer to those of the minimal-T set. Also the curves defined from the reaction cross sections for those nuclei are characterized by values of parameters very similar to those assessed by Carlson using a much larger data set. We then anticipate that measurement of total reaction cross sections

TABLE I. Carlson model parameter values that give the curves plotted in Fig. 5.

Set	Energy (MeV)	r_0 (fm)	R_p (fm)
S	65	2.133	−2.255
Ar	65	2.144	−2.416
Ca	65	2.094	−2.322
minimal-T	65	1.301	+0.347
S	200	1.608	−1.346
Ar	200	1.763	−1.913
Ca	200	1.718	−1.806
minimal-T	200	1.281	−0.332

from the scattering of radioactive beams from hydrogen is a means to assess the reliability of the structure assumed.

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