Quantum Evaporation of a Naked Singularity

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We investigate here quantum effects in gravitational collapse of a scalar field model which classically leads to a naked singularity. We show that nonperturbative semiclassical modifications near the singularity, based on loop quantum gravity, give rise to a strong outward flux of energy. This leads to the dissolution of the collapsing cloud before the singularity can form. Quantum gravitational effects thus censor naked singularities by avoiding their formation. Further, quantum gravity induced mass flux has a distinct feature which may lead to a novel observable signature in astrophysical bursts.

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Naked singularities are one of the most exotic objects predicted by classical general relativity. Unlike their black hole siblings, they can be in principle directly observed by an external observer. There have been many investigations which show that given the initial density and pressure profiles for a matter cloud, there are classes of collapse evolutions that lead to naked singularity formation (see, e.g., [1] for some recent reviews), subject to an energy condition and astrophysically reasonable equations of state such as dust, perfect fluids, and such others. This has led to extensive debates on their existence, with a popular idea being cosmic censorship conjectures which forbid classical nakedness [2]. Since naked singularities originate in the regime where classical general relativity is expected to be replaced by quantum gravity, it has remained an outstanding problem whether a quantum theory of gravity resolves their formation. Also, with the lack of observable signatures from the Planck regime, naked singularities could in fact be a boon for a quantum theory of gravity. Because, the singularity being visible, any quantum gravitational signature originating in the ultrahigh curvature regime near a classical singularity can in principle be observed, thus providing us a rare test for quantum gravity.

One of the nonperturbative quantizations of gravity is loop quantum gravity [3], whose key predictions include the Bekenstein-Hawking entropy formula [4]. Its application to symmetry reduced mini-superspace quantization of homogeneous spacetimes is called loop quantum cosmology [5], whose success includes resolution of the big bang singularity [6], initial conditions for inflation [7,8], and possible observable signatures in cosmic microwave background radiation [8]. These techniques have also been applied to resolve black hole singularity in a scalar field collapse scenario [9].

Since the dynamics of a generic collapse is very complicated and tools to address such a problem in quantum gravity are still under development, it is useful to work with a simple collapse scenario as of a scalar field. It serves as a good toy model to gain insights on the role of quantum gravity effects at the late stages of gravitational collapse. Existence of naked singularities in these models is well known [10] and one of the simplest setting is to consider an initial configuration of a homogeneous and isotropic scalar field $\Phi = \Phi(t)$ with a potential $V(\Phi)$ [given by Eq. (6)] and the canonical momentum P_Φ. In this case it has been shown that the fate of the singularity being naked or covered depends on the rate of gravitational collapse [11]. For an appropriately chosen potential, formation of trapped surfaces can be avoided even as the collapse progresses, resulting in a naked singularity with an outward energy flux, in principle observable. Since the interior of homogeneous scalar field collapse is described by a Friedmann-Robertson-Walker (FRW) metric, techniques of loop quantum cosmology can be used to investigate the way quantum gravity modifies the collapse.

Let us consider the classical collapse of a homogeneous scalar field $\Phi(t)$ with potential $V(\Phi)$ and the canonical momentum for the marginally bound ($k = 0$) case. The interior metric is given by

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2] \quad (1)$$

with classical energy density and pressure of the scalar field,

$$\rho(t) = \dot{\Phi}^2/2 + V(\Phi), \qquad p(t) = \dot{\Phi}^2/2 - V(\Phi). \quad (2)$$

The dynamical evolution of the system is obtained from the Einstein equations which yield [11]

$$\dot{R}^2 R = F(t, r), \qquad \rho = F_{,r}/\kappa a R^2, \qquad p = -\dot{F}/\kappa R^2 \dot{R}. \quad (3)$$

Here $\kappa = 8\pi G$, and $F(t, r) = (\kappa/3)\rho(t) r^3 a^3$ has the interpretation of the mass function of the collapsing cloud, with $F \geq 0$, and $R(t, r) = ra(t)$ is the area radius of a shell labeled by comoving coordinate r. In a continual collapse the area radius of a shell at a constant value of comoving radius r decreases monotonically. The spacetime region is trapped or otherwise, depending on the value of the mass function. If F is greater (less) than R, the region is trapped (untrapped). The boundary of the trapped region is given by $F = R$.

The collapsing interior can be matched at some suitable boundary $r = r_b$ to a generalized Vaidya exterior geometry, given as [12],

$$ds^2 = -(1 - 2M(r_v, v)/r_v)dv^2 - 2dvdr_v + r_v^2 d\Omega^2. \tag{4}$$

The Israel-Darmois conditions then lead to [11,12] $r_b a(t) = r_v(v)$, $F(t, r_b) = 2M(r_v, v)$, and

$$M(r_v, v)_{,r_v} = F/2r_b a + r_b^2 a\ddot{a}. \tag{5}$$

The form of the potential that leads to a naked singularity is determined as follows. The energy density of the scalar field can be written in a generic form as $\rho = l^{n-4}a^{-n}$, where $n > 0$ and l is a proportionality constant. Using the energy conservation equation, this leads to the pressure $p = [(n-3)/3]l^{(n-4)}a^{-n}$. On substituting Eq. (2) in these we obtain [11]

$$\Phi = -\sqrt{n/\kappa}\ln a, \qquad V(\Phi) = (1 - n/6)l^{n-4}e^{\sqrt{\kappa n}\Phi}. \tag{6}$$

Then it is easily seen that $F/R = (\kappa/3)l^{n-4}a^{2-n}r^2$. Thus in the collapsing phase as $a \longrightarrow 0$, whether or not the trapped surfaces form is determined by the value of n. It is straightforward to check that for $0 < n < 2$, if no trapped surfaces exist initially, then no trapped surfaces would form till the epoch $a(t) = 0$ [11], with $a(t) = (1 - nt/2\sqrt{3})^{2/n}$.

The absence of trapped surfaces is accompanied by a negative pressure implying that for a constant value of the comoving coordinate r, $\dot{F}$ is negative and so the mass contained in the cloud of that radius keeps decreasing. This leads to a classical outward energy flux. As the collapse proceeds, the scale factor vanishes in finite time and physical densities blow up, leading to a naked singularity. Since no trapped surfaces form during collapse, the outward energy flux shall in principle be observable. However, near the singularity when energy density is close to Planckian values, this classical picture has to be modified and we need to investigate the scenario incorporating quantum gravity modifications to the classical dynamics.

Let us hence consider the nonperturbative semiclassical modifications based on loop quantum gravity for the interior. The underlying geometry for the FRW spacetime in loop quantum cosmology is discrete, and both the scale factor and the inverse scale factor operators have discrete eigenvalues [13]. In particular, there exists a critical scale $a_* = \sqrt{j\gamma/3}\,l_P$ below which the eigenvalues of the inverse scale factor become proportional to the positive powers of scale factor. Here $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter [4], l_P is the Planck length, and j is a half-integer free parameter which arises because the inverse scale factor operator is computed by tracing over SU(2) holonomies in an irreducible spin j representation. The value of this parameter is arbitrary and shall be constrained by phenomenological considerations.

The change in behavior of the classical geometrical density ($1/a^3$) for scales $a \lesssim a_*$, can be well approximated by [7]

$$d_j(a) = D(q)a^{-3}, \qquad q := a^2/a_*^2, \qquad a_* := \sqrt{j\gamma/3}\,l_P, \tag{7}$$

with

$$D(q) = (8/77)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] - 11q[(q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4}]\}^6. \tag{8}$$

For $a \ll a_*$, $d_j \propto (a/a_*)^{15}a^{-3}$ and for $a \gg a_*$ it behaves classically with $d_j \approx a^{-3}$. The scale at which transition in the behavior of the geometrical density takes place is determined by the parameter j.

At the fundamental level the dynamics in the loop quantum regime is discrete; however, recent investigations pertaining to the evolution of coherent states have shown that for scales $a_0 = \sqrt{\gamma}\,l_P \lesssim a \lesssim a_* = \sqrt{j\gamma/3}\,l_P$, dynamics can be described by modifications to Friedmann dynamics on a continuous spacetime [14] with the modified matter Hamiltonian

$$\mathcal{H}_\Phi = d_j(a)P_\Phi^2/2 + a^3V(\Phi) \tag{9}$$

and the modified Friedmann equation

$$\dot{a}^2/a^2 = (\kappa/3)[\dot{\Phi}^2/2D + V(\Phi)], \tag{10}$$

which is obtained by the vanishing of the total Hamiltonian constraint and the Hamilton's equations: $\dot{\Phi} = d_j(a)P_\Phi$, $\dot{P}_\Phi = -a^3V_{,\Phi}(\Phi)$ [7]. These also lead to the modified Klein-Gordon equation

$$\ddot{\Phi} + [3\dot{a}/a - \dot{D}(q)/D(q)]\dot{\Phi} + D(q)V_{,\Phi}(\Phi) = 0. \tag{11}$$

Since at classical scales ($a \gg a_*$) $D \approx 1$, the modified dynamical equations reduce to the standard Friedmann dynamical equations. For scales $a \lesssim a_*$, the $\dot{\Phi}$ term acts like a frictional term for a collapsing phase. We note that since semiclassical modifications for inhomogeneous case are still not known, we cannot do a complete quantum analysis of interior and exterior. The exterior is assumed to remain classical. Further, as a continuous spacetime can be approximated till scale factor a_0, the matching of interior and exterior spacetimes remains valid during the semiclassical evolution.

The modified energy density and pressure of the scalar field in the semiclassical regime can be similarly obtained from the eigenvalues of the density operator and using the stress-energy conservation equation [15]

$$\rho_{\rm eff} = d_j(a)\mathcal{H}_\Phi = \dot{\Phi}^2/2 + D(q)V(\Phi) \tag{12}$$

and

$$p_{\rm eff} = \left[1 - \frac{2}{3}\frac{1}{(\dot a/a)}\frac{\dot D(q)}{D(q)}\right]\frac{\dot\Phi^2}{2} - D(q)V(\Phi) - \frac{\dot D(q)}{3(\dot a/a)}V(\Phi). \quad (13)$$

It is then straightforward to check that $p_{\rm eff}$ is generically negative for $a \lesssim a_*$, and for $a \ll a_*$ it becomes very strong. For example, at $a \sim a_0$, $p_{\rm eff} \approx -9\rho_{\rm eff}$. This is much stronger than its classical counterpart $p = [(n-3)/3]\rho$ with $0 < n < 2$. Thus, we expect a strong burst of outward energy flux in the semiclassical regime. Further, for $a \ll a_*$, $D(q) \ll 1$ and the Klein-Gordon equation yields $\dot\Phi \propto a^{12}$. Hence, from Eq. (12), we easily see that the effective density, instead of blowing up, becomes extremely small and remains finite.

The modified mass function of the collapsing cloud can be evaluated using Eqs. (3) and (10),

$$F = (\kappa/3)[d_j^{-1}\dot\Phi^2/2 + a^3V(\Phi)]r^3. \quad (14)$$

In the regime $a \sim a_0$, $d_j^{-1}\dot\Phi^2$ becomes proportional to a^{12}, the potential term becomes negligible and thus the mass function becomes vanishingly small at small scale factors.

The picture emerging from loop quantum modifications to collapse is thus following. (i) Before the area radius of the collapsing shell reaches $R_* = ra_*$ at $t = t_*$, collapse proceeds as per classical dynamics and as smaller scale factors are approached $\dot\Phi$ and the energy density $\rho \propto a^{-n}$ increase. The mass function is proportional to a^{n-3} and (as $0 < n < 2$) it decreases with decreasing scale factor so there is a mass loss to the exterior, which is also understood from existence of negative classical pressure. (ii) As the collapsing cloud reaches R_*, the geometric density classically given by a^{-3}, modifies to d_j and the dynamics is governed by the modified Friedmann and Klein-Gordon equations. The scalar field which experienced antifriction in the classical regime, now experiences friction leading to a decrease of $\dot\Phi$. (iii) The slowing down of Φ decreases the rate of collapse and the formation of singularity is delayed. Eventually, when the scale factor becomes smaller than a_0, this leads to a breakdown of the continuum spacetime approximation and semiclassical dynamics. Discrete quantum geometry emerges at this scale [14] and the dynamics can only be described by quantum difference equation. The naked singularity is thus avoided till the scale factor at which a continuous spacetime exists.

We show the evolution of area radius in time as collapse proceeds in Fig. 1. The semiclassical evolution (solid curve) closely follows classical trajectory (dashed curve) until the time t_*. Within a finite time after t_*, the classical collapse leads to a vanishing R and naked singularity. However, the area radius never vanishes in the loop modified semiclassical dynamics and the naked singularity does not form as long as the continuum spacetime approximation holds. The inset of Fig. 1 shows the evolution of energy density in Planck units. Classical energy density (dashed curve) blows up, whereas it remains finite and in fact decreases in the semiclassical regime.

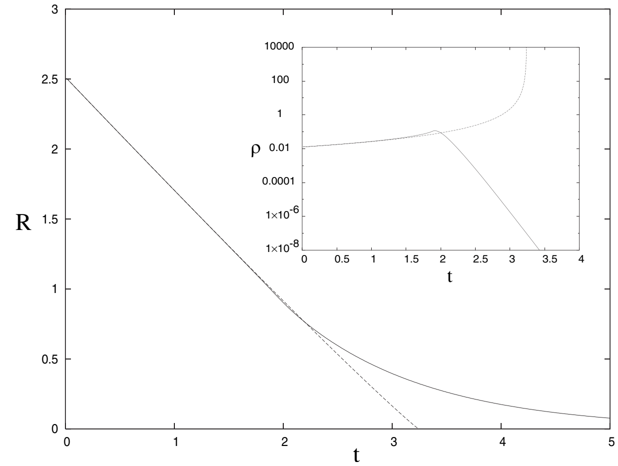

FIG. 1. Evolution of the area radius with time. The classical evolution (dashed curve) leads to naked singularity in finite time whereas in semiclassical evolution (solid curve) it is avoided. Inset: evolution of energy density (in Planck units) with time. The parameters chosen are $n = 1.9$ and $j = 100$.

The phenomena of delay and avoidance of the naked singularity in continuous spacetime is accompanied by a burst of matter to the exterior. If the mass function at scales $a \gg a_*$ is F_i and its difference with mass of the cloud for $a < a_*$ is $\Delta F = F_i - F$, then the mass loss can be computed as

$$\frac{\Delta F}{F(a_i)} = \left[1 - \frac{\rho_{\rm eff} d_j^{-1}}{l^{n-4}a_i^{3-n}}\right]. \quad (15)$$

For $a < a_*$, as the scale factor decreases, the energy density and mass in the interior decrease and the negative pressure strongly increases. This leads to a strong burst of matter. The absence of trapped surfaces enables the quantum gravity induced burst to propagate via the generalized Vaidya exterior to an observer at infinity. The evolution of the mass function is shown in Fig. 2. In the semiclassical regime, $\Delta F/F_i$ approaches unity very rapidly. This feature is independent of the choice of parameter j. The choice of potential causes mass loss to the exterior in classical collapse also, but it is much smaller (and in any case the classical description cannot be trusted at energy densities greater than Planck, when we must consider quantum effects as above).

Interestingly, for a given collapsing configuration, the scale at which the strong outward flux initiates depends on the loop parameter j which controls a_*. If j is large then burst occurs at an earlier area radius and vice versa. The inset of Fig. 2 shows the mass loss ratio for different values of j. For all choices, $\Delta F/F_i \to 1$, but the outgoing flux profile changes. The loop quantum burst has a distinct

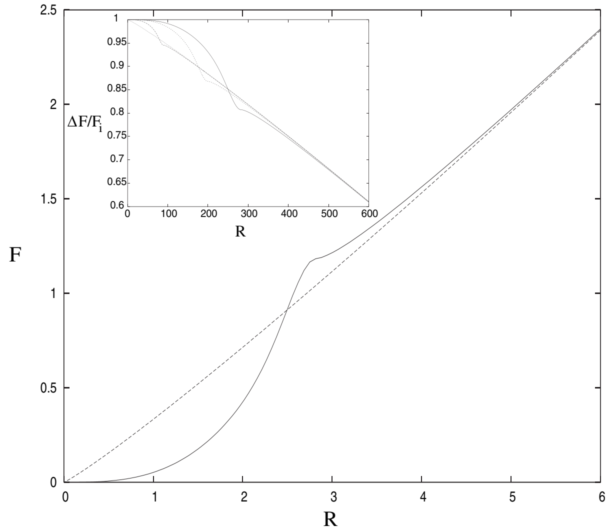

FIG. 2. Evolution of mass function with area radius for the same parameters as in Fig. 1. Loop quantum evolution (solid curve) leads to dissolution of all the mass of the collapsing shell. Dashed curve shows the classical trajectory. Inset: mass loss profile for $j = 10^6$ (outer), $j = 5.0 \times 10^5$ (middle), and $j = 10^5$ (inner).

signature: at $a \sim a_*$ the flux decreases for a short period and then rapidly increases. Since the causal structure of classical spacetime is such that trapped surface formation is avoided, this quantum gravitational signature can be in principle observed by an external observer as a slight dimming and subsequent brightening of the collapsing star. This peculiar phenomena is directly related to the peak in the function $d_j(a)$, and depends solely on the value of parameter j. If we compare this to other phenomenological applications [7–9], this effect could not be masked by the role of other loop quantum parameters in a more general setting. This phenomena is thus a direct probe to measure j and an observer can estimate the loop quantum parameter j by observing the flux profile of the burst based on this mechanism and measuring the variation in luminosity of the collapsing cloud.

During such a burst most of the mass is ejected and this may dissolve the singularity. Thus nonperturbative semiclassical modifications may not allow formation of naked singularity as the collapsing cloud evaporates away due to supernegative pressures in the late regime. It has been demonstrated that these supernegative pressures would exist for arbitrary matter configurations [15], which implies that results obtained here would hold even in a more general setting [16]. Loop quantum effects then imply a quantum gravitational cosmic censorship, alleviating the naked singularity problem. We note that the semiclassical effects do not show that the singularity is absent, it is only avoided until scale factor a_0, below which the semiclassical dynamics and matching may breakdown. If, for a given choice of initial data, semiclassical dynamics is unable to completely dissolve the singularity, the final fate of naked singularity must be decided by using full quantum evolution. Even in such cases we have valuable insights from semiclassical loop quantum effects with the possibility of phenomenologically constraining the j parameter.

In the toy model considered, we showed that the classical outcome and evolution of collapse is radically altered by the nonperturbative modifications to the dynamics. Our considerations are of course within the mini-superspace setting, and the general case of inhomogeneities and anisotropies remains open. However, the possibility of such observable signatures in astrophysical bursts, as originating from the quantum gravity regime near singularity is intriguing, indicating that the gravitational collapse scenario can be used as probes to test quantum gravity models.

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