

Improved Transfer of Quantum Information Using a Local Memory

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We demonstrate that the quantum communication between two parties can be significantly improved if the receiver is allowed to store the received signals in a quantum memory before decoding them. In the limit of an infinite memory, the transfer is perfect. We prove that this scheme allows the transfer of arbitrary multipartite states along Heisenberg chains of spin-1/2 particles with random coupling strengths.

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Suppose you want to send an unknown quantum state to your friend. Which technique should you use? Obviously you cannot just perform a measurement and call him/her, because such a measurement would in general reveal only very limited information about the state. Another possibility would be to send the full physical system of the state, but that is difficult if your state is not implemented in a mobile medium (photons, electrons, etc.) and cannot be converted to such media easily. This is the typical situation one has to face in solid state systems, where quantum information is usually contained in the states of fixed objects such as quantum dots or Josephson junctions. In this case, a *quantum wire* that transports states just like optical fibers transport photons may be used. If local control (gates, measurements) is available all along such a wire, then this state transfer is possible via a series of swap gates or by entanglement swapping followed by teleportation. However, this scenario may be very difficult to realize in practice. Motivated by such experimental restrictions, *permanently coupled* systems without local access were suggested [1,2], but because of dispersion the fidelity of the transfer is in general low. One way of improving this is by engineering specific Hamiltonians [3] or by coupling the system only weakly to the communicating parties [4]. Another approach proposed is to make use of gates at the sender (Alice) and the receiver (Bob) locations and to *encode* the states to be sent to yield perfect state transfer [5–7]. This way the demands on the engineering of the Hamiltonian could be relaxed. In some sense the effort of control and engineering has been shifted to the encoding and decoding by Alice and Bob. Here we would like to go one step further by proposing to make use of even more resources of Bob, i.e., to use his *quantum memory*. We will show that perfect state transfer can be achieved using a single permanently coupled quantum chain if Bob possesses an infinite quantum memory. This is achieved by *swapping* the part of the chain that Bob controls to his memory at equal time intervals. Eventually, the whole quantum information is contained in his memory and can be decoded by unitary operations. Since this happens independently of the initial state of the chain, it is an example

of *homogenization* [8] and *asymptotic completeness* [9]. The crucial difference is that in our system the memory is only interacting with Bob, and the completeness is mediated to the rest of the system through the permanent couplings. We note that with the ideas in [9] it is also possible for Bob to send messages to Alice, using the time-reversed protocol. The main advantage of using a memory is that—opposed to the schemes in [1,3–7]—Alice can send arbitrary multiqubit states, including complex entangled states, with a single usage of the channel. She needs no encoding, all the work is done by Bob. If Bob's memory is only finite, he can still use it to improve the fidelity of the transfer substantially (the fidelity grows exponentially with the size of the memory). The protocol proposed here can be used to improve the performances of the schemes [1,3–7], and it works for a large class of Hamiltonians, including Heisenberg and XY models with arbitrary (also randomly distributed) coupling strengths. Furthermore, the timing of our protocol scales in a reasonable manner with the length of the chain.

Protocol.—Consider a chain of spin-1/2 particles described by a Hamiltonian H which commutes with the total spin component S_z . The chain is assumed to be divided in three portions A (Alice), B (Bob), and C (the remainder of the chain, connecting Alice and Bob) containing, respectively, the first N_A spins of the chain, the last N_B spins, and the intermediate N_C spins, and the total length of the chain is $N = N_A + N_C + N_B$ (see Fig. 1). Bob has access also to a collection of quantum memories M_1, \dots, M_j, \dots , isomorphic with B , i.e., each having dimension equal to the dimension 2^{N_B} of B . Without losing generality it will be useful to represent each of these memories as a noninteracting collection of N_B spins. The protocol goes then as follows. Suppose that at time $t = 0$ Alice prepares her



FIG. 1. Alice and Bob control the spins N_A and N_B interconnected by the spins N_C . At time $j\tau$ Bob performs a swap S_j between his spins and the memory M_j .

spins in the (possibly unknown) input state $|\psi\rangle_A$. The total state of the chain plus memories is then $|\psi000\rangle_{ACBM} \equiv |\psi\rangle_A \otimes |0\rangle_C \otimes |0\rangle_B \otimes |0\rangle_M$, where we assumed C , B , and the memories to be originally in the all-spin-down state (here $|0\rangle_M$ is a compact notation for the product state $|0\rangle_{M_1} \otimes \cdots \otimes |0\rangle_{M_j} \cdots$). To recover Alice message, Bob performs unitary swap operations between the B spins and the memories M_j . In particular, at time $\tau > 0$ Bob performs a swap between the memory M_1 and B ; at time 2τ he performs a swap between M_2 and B ; at time 3τ he performs a swap between M_3 and B ; and so on. Under these hypotheses the state of the whole system after j steps is described by the unitary transformation

$$|\psi000\rangle_{ACBM} \rightarrow W_j |\psi000\rangle_{ACBM}, \quad (1)$$

where W_j is the product of free evolutions of the chain $U \equiv \exp[-\frac{i}{\hbar}H\tau]$ and swap S_j between the memory M_j and B , i.e.,

$$W_j \equiv S_j U S_{j-1} U \cdots S_2 U S_1 U. \quad (2)$$

For simplicity we assumed equal time intervals τ , but the generalization to arbitrary time intervals $\{\tau_i\}_i$ is straightforward. The mapping W_j preserves the total number of excitations in $A + C + B + M$ but tends to decrease the number of excitations in $A + C + B$. In fact, on one hand, the operators U shuffle the spin-up components of the state around the chain $A + C + B$ while, on the other hand, the S_j exchange the state of B with the no-excitation state of the memory M_j . In the limit of large j one expects that eventually this mechanism will provide the transfer of $|\psi\rangle_A$ into Bob memories. To see how this might happen let us consider first the case $N_A, N_B = 1$, where $|\psi\rangle_A$ is a generic superposition of $|0\rangle_A$ and the spin-up state $|1\rangle_A$ of A . In this context one easily verifies that if protocol (1) stops just after the first swap, the state $|\psi\rangle_A$ can be recovered from M_1 with fidelity $\eta_1 = |{}_{ACB}\langle 001|U|100\rangle_{ACB}|^2$ identical to the transfer fidelity of Ref. [1]. If instead protocol (1) runs up to second swap, $|\psi\rangle_A$ can be recovered from the state of the memories $M_1 + M_2$ with fidelity $\eta_2 = \eta_1 + |\sum_{\ell=1}^{N-1} {}_{ACB}\langle 001|U|\ell\rangle_{ACB}\langle \ell|U|100\rangle_{ACB}|^2$, which typically is already higher than the fidelity η_1 (in this expression $|\ell\rangle_{ACB}$ stands for the state of the chain with a single spin-up component in the ℓ th location). Analogously one finds that at the j th step $|\psi\rangle_A$ can be recovered from $M_1 + \cdots + M_j$ with a fidelity η_j which is greater than or equal to the fidelity η_{j-1} of the $(j-1)$ th step. We claim that this a general trend which does not depend on the size of N_A and N_B . In particular, under a quite general hypothesis on H , we will show that in the limit of $j \rightarrow \infty$ the input state $|\psi\rangle_A$ will be transferred to the memories M leaving the chain $A + B + C$ in the no-excitation state $|000\rangle_{ACB}$, i.e.,

$$\lim_{j \rightarrow \infty} W_j |\psi000\rangle_{ACBM} = |000\rangle_{ACB} \otimes |\Phi(\psi)\rangle_M, \quad (3)$$

with $|\Phi(\psi)\rangle_M$ being a state of M which explicitly depends

on the input state $|\psi\rangle_A$ and on τ . If the input state $|\psi\rangle_A$ does not contain excitations, Eq. (3) trivially follows from the fact that for all j the operator W_j maps $|0000\rangle_{ACBM}$ into itself. For $|\psi\rangle_A \neq |0\rangle_A$ instead Eq. (3) requires all the excitations originally present in $A + C + B$ to move in the memory M as j increases. In our protocol, the state of B is set to $|0\rangle_B$ at each step, so for proving Eq. (3) it is sufficient to show that all the excitations leave the subsystem $A + C$. In other words, given the reduced density matrix

$$\sigma_{AC}(j) = \text{Tr}_{BM}[W_j(|\psi000\rangle_{ACBM}\langle\psi000|)W_j^\dagger] \quad (4)$$

of $A + C$ at the j th step of the protocol, Eq. (3) is equivalent to requiring the following identity:

$$\lim_{j \rightarrow \infty} \langle 00|\sigma_{AC}(j)|00\rangle_{AC} = 1. \quad (5)$$

Before proving this result we notice that it implies that Bob can reliably recover Alice's messages by applying a unitary transformation on the memory only (or, alternatively, Bob could feed the memory state into another chain using the time-reversed protocol). In fact, since the W_j are unitary operators, they describe in the limit $j \rightarrow \infty$ a unitary map from A into a subspace M_A of the memory of dimension 2^{N_A} . The explicit form of this map depends upon the unitaries U of Eq. (2) and can be determined by the communicating parties either by knowing the chain Hamiltonian H or by performing a set of measurements prior to the transmission.

Convergence.—We prove Eq. (5) by showing that the probability of having one or more excitations in $A + C$ at the j th step of the protocol converges to zero as $j \rightarrow \infty$. At the beginning of the protocol there are at most N_A excitations in the system. For $1 \leq n \leq N_A$ we are interested in the probability $P_n(j)$ of having n or more excitations in $A + C$ at the j th step of the protocol. This is

$$P_n(j) \equiv \sum_{n'=n}^{N_A} \text{Tr}_{AC}[\Pi_{AC}(n')\sigma_{AC}(j)], \quad (6)$$

where $\sigma_{AC}(j)$ is given by Eq. (4) and $\Pi_{AC}(n')$ are the projectors on the $\binom{N_A+n'}{n'}$ -dimensional Hilbert subspace of $A + C$ formed by the vectors with n' spins up. An inequality for the $P_n(j)$ is obtained by noticing that the total number of excitations in $A + C$ never increases with j : this allows one to upper bound $P_n(j + j_1)$ with the probability $P_{n+1}(j_1)$ of having more than $n + 1$ spins up in $A + C$ at the j_1 th step plus the maximum joint probability $Q_n(j + j_1, j_1)$ of having exactly n spins up at the step j_1 and maintaining them in the next j steps of the protocol, i.e.,

$$P_n(j_1 + j) \leq P_{n+1}(j_1) + Q_n(j_1 + j, j_1). \quad (7)$$

The formal derivation of this rather intuitive expression is straightforward but tedious: we report a sketch of it in [10]. An expression for $Q_n(j_1 + j, j_1)$ follows by noticing that

any state of $A + C + B$ will maintain a constant number of excitations in the chain during the whole protocol if and only if it has no excitations in B when Bob applies the swaps S_j . For instance, consider the state $\sigma_{AC}(j_1)$ of $A + C$ immediately after the j_1 th step. According to the protocol the section B is in $|0\rangle_B$ and the free evolution of the chain in the forthcoming time interval is described by $U(\sigma_{AC}(j_1) \otimes |0\rangle_B\langle 0|)U^\dagger$. The probability of not losing any excitations at step $j_1 + 1$ is then proportional to the probability that this state does not contain excitations in B , i.e.,

$$\begin{aligned} p_1 &= {}_B\langle 0| \text{Tr}_{AC}[U(\sigma_{AC}(j_1) \otimes |0\rangle_B\langle 0|)U^\dagger] |0\rangle_B \\ &= \text{Tr}_{ACB}[T(\sigma_{AC}(j_1) \otimes |0\rangle_B\langle 0|)T^\dagger], \end{aligned} \quad (8)$$

with $T = |0\rangle_B\langle 0|U$. Moreover, if no excitation leaves the chain at the $j_1 + 1$ th step, the state of $A + C + B$ is projected into

$$\tilde{\sigma}_{AC}(j_1 + 1) \otimes |0\rangle_B\langle 0| = \frac{1}{p_1} T(\sigma_{AC}(j_1) \otimes |0\rangle_B\langle 0|)T^\dagger.$$

By iteration the probability that $\tilde{\sigma}_{AC}(j_1 + 1)$ will not loose excitations in the next step of the protocol is $\tilde{p}_2 = \text{Tr}_{ACB}[T(\tilde{\sigma}_{AC}(j_1 + 1) \otimes |0\rangle_B\langle 0|)T^\dagger]$, while the joint probability of not losing excitations in the $(j_1 + 1)$ th and in the $(j_1 + 2)$ th steps is given by $p_2 = p_1\tilde{p}_2$. Analogously the joint probability of not losing excitations in all steps from $j_1 + 1$ up to $j_1 + j$ is equal to

$$p_j = \text{Tr}_{ACB}[T^j(\sigma_{AC}(j_1) \otimes |0\rangle_B\langle 0|)(T^\dagger)^j]. \quad (9)$$

The quantity $Q_n(j_1 + j, j_1)$ can now be computed by assuming $\sigma_{AC}(j_1)$ to have exactly n excitations and maximizing p_j with respect to such a choice, i.e.,

$$\begin{aligned} Q_n(j_1 + j, j_1) &= \max_{|\phi_n\rangle_{AC}} \text{Tr}_{ACB}[T^j(|\phi_n\rangle_{AC}\langle \phi_n| \\ &\quad \otimes |0\rangle_B\langle 0|)(T^\dagger)^j] \\ &= \max_{|\phi_n\rangle_{AC}} \|T^j(|\phi_n 0\rangle_{ACB})\|^2, \end{aligned} \quad (10)$$

where $|\phi_n\rangle_{AC}$ is a generic state of $A + C$ with n excitations and $|\phi_n 0\rangle_{ACB} \equiv |\phi_n\rangle_{AC} \otimes |0\rangle_B$. Notice that by exploiting the convexity of mixed states, the maximization in Eq. (10) has been performed only on pure states. For $n = 0$ it is trivial to see that $Q_0(j_1 + j, j_1) = 1$ for all j and j_1 . We will show now that, for $n \geq 1$ and $j_1 \geq 0$, one has instead

$$\lim_{j \rightarrow \infty} Q_n(j_1 + j, j_1) = 0. \quad (11)$$

Because the operator T conserves the number of excitations, we get

$$\|T^j|\phi_n 0\rangle_{ACB}\|^2 = \|T_n^j|\phi_n 0\rangle_{ACB}\|^2, \quad (12)$$

where $T_n = \Pi_{ACB}(n)T\Pi_{ACB}(n)$ is the restriction of T to the subspace with n excitations. Equation (12) converges to zero for all ϕ_n iff the spectral radius $\rho(T_n)$ of T is smaller than 1 [11]. Since T_n is the product of a projector and a

unitary operator, it is easy to see that this is the case [7] iff there exists no common eigenstate of $|0\rangle_B\langle 0|$ and $U_n = \Pi_{ACB}(n)U\Pi_{ACB}(n)$. Because $U_n = \sum \exp(-iE_n\tau)|E_n\rangle \langle E_n|$, where $|E_n\rangle$ are the eigenstates of the Hamiltonian H with exactly n excitations, it is always possible to find a choice for the interval τ such that Eq. (11) holds, as long as given $n \geq 1$ there are no eigenstates $|E_n\rangle$ of factorizing form with $|0\rangle_B$, i.e.,

$$\nexists |\lambda_n\rangle_{AC}: H|\lambda_n\rangle_{AC} \otimes |0\rangle_B = E|\lambda_n\rangle_{AC} \otimes |0\rangle_B. \quad (13)$$

Under this condition Eq. (7) implies that for any $\delta_1 > 0$ there exists a sufficiently big J_1 such that for all $j > J_1$ one has $P_n(j_1 + j) \leq P_{n+1}(j_1) + \delta_1$. Reiterating this with respect to n one can show that given $\delta > 0$ there is J such that for all $j > J$

$$P_n(j_1 + j) \leq P_{N_A}(j_1) + \delta, \quad (14)$$

where N_A is the maximum number of spin up Alice can introduce in A . From our definitions the quantity $P_{N_A}(j_1)$ is the probability of having N_A spins up in $A + C + B$ at the j_1 th step. This quantity cannot be greater than $Q_{N_A}(j_1, 0)$ of Eq. (10). But, according to Eq. (11), this nullifies in the limit $j_1 \rightarrow \infty$. Therefore for $n \geq 1$ one has $\lim_{j \rightarrow \infty} P_n(j) = 0$, which gives the thesis.

Nearest-neighbor interactions.—The requirement (13) is quite general and does not require any particular constraint on the topology of the system (e.g., it does not need to be a chain). However, in the following we will focus on the special case of linear open chains showing that (13) is always satisfied if they (a) conserve the number of excitations and (b) are connected by nearest-neighbor exchange terms. This includes the randomly coupled chains considered in [6]. Consider in fact one of such chains and assume by contradiction it has an eigenvector $|E_n\rangle$ which falsifies Eq. (13) for some $n \geq 1$. Such an eigenstate can be written as

$$|E_n\rangle = a|\mu_n\rangle_{AC} \otimes |0\rangle_B + b|\bar{\mu}_n\rangle_{AC} \otimes |0\rangle_B, \quad (15)$$

where a and b are complex coefficients and where the spin just before the section B (with position $N_A + N_C$) is in the state $|0\rangle$ for $|\mu_n\rangle_{AC}$ and in the state $|1\rangle$ for $|\bar{\mu}_n\rangle_{AC}$. Since the interaction between this spin and the first spin of section B includes an exchange term, then the action of H on the second term of (15) yields exactly one state which contains an excitation in the sector B which cannot be compensated by the action of H on the first term of (15). But by assumption $|E_n\rangle$ is an eigenstate of H , so we conclude that $b = 0$. This argument can be repeated for the second last spin of section C , the third last spin, and so on, to finally yield $|E_n\rangle = |0\rangle_{ACB}$, as long as all the nearest-neighbor interactions contain exchange parts. This leads to a contradiction for $n \geq 1$.

Time scale.—As we have shown above, the communicating parties can achieve perfect state transfer in the limit of infinite time and an infinitely large memory space.

However, in practice, Bob's resources and time will be limited. If the protocol stops after j operations, how does the fidelity depend on the number of qubits N_A being transferred, and on the total length of the chain? This question is clearly strongly depending on the specific Hamiltonian of the chain. For example, in the case of engineered couplings [3], a *single* swap operation would already suffice. We would like to keep the argument in this section as general as possible to find a rough estimate of the fidelity based on statistical arguments. If the system has some special symmetries, the fidelity may be much higher, as in the case of engineered couplings, or may also be much lower, but in practice these cases are extremely unlikely.

Since the transfer of spin-down components occurs naturally in our model, one may argue that the worst case scenario is when Alice wants to send the state $|11 \cdots 1\rangle_A$. After an initial time T_e that it takes excitations to travel across the chain, we expect that the N_A excitations originally at Alice's site are distributed with an average number of N_A/N excitations per site. On average, Bob's region of the chain should therefore contain $N_B N_A/N$ excitations. Of course the expectation value of the number of excitations is a strongly fluctuating function of time. However, in a slightly modified protocol with optimized swapping times $\{\tau_{ij}\}$, it should be easy to find a swapping time $\tau_1 \in [0, T_e]$ such that after performing the swap operation, there are on average $N_1 = (1 - N_B/N)N_A$ excitations left which remain in the part $A + C$ of the chain. After another time of the order of T_e , they will be spread along the whole chain again, with $N_B N_1/N$ being the average number in Bob's section. More generally, after a time $t \approx jT_e$ the average number of excitations in the system after j swap should be of the order $N_j = (1 - N_B/N)^j N_A$ (we have confirmed this estimate numerically for short Heisenberg spin chains). The fidelity F of the state transfer is lower bounded by the probability of having no excitations in the chain $A + C + B$. For $N_j \leq 1$ we can lower bound this quantity by $1 - N_j$. Thus for large j one has $F \geq 1 - (1 - N_B/N)^j N_A$. Replacing $j \approx t/T_e$ and taking the limit $N \gg 1$, the above inequality shows that the fidelity F can be reached after a time $t \approx NT_e(\ln N_A + |\ln(1 - F)|)/N_B$. In translationally invariant systems the group velocity is typically independent of the length N of the chain. Therefore in these systems T_e is scaling linearly with N [1] and the above equation shows that t scales quadratically with N . A special case of this expression with $N_A = N_B = 1$ and $1 - F$ corresponding to a probability of failure was already considered in the conclusive dual rail schemes [6]. From the above analysis it follows that the size of Bob's region

can make the transfer quicker, and that the time scale depends only logarithmically on the amount of qubits that Alice wants to send. It is therefore more efficient to send many qubits at once rather than repeating the protocol.

Conclusions.—We have shown that the usage of the quantum memory of the receiver can strongly increase the fidelity of quantum state transfer with permanently coupled quantum chains. In the limit of an infinite memory, the transfer is perfect. Furthermore, this scheme allows one to send arbitrary multipartite states rather than just single qubit states.

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