Vager and Naaman Reply: In their Comment [1], Hernando and García correctly compared the treatment propose in Ref. [2] to the well-known system of "noninteracting electrons on a circular ring." The Hamiltonian in this case is given by [3]

$$H = \frac{1}{2m}\left(p - \frac{e}{c}A\right)^2 \tag{1}$$

where p is the momentum operator along the path and A is the vector potential along the same path. A is commonly chosen as

$$\bar{A} = \tfrac{1}{2}\bar{H} \times \bar{r} \tag{2}$$

where H is the magnetic field and r originate from the center of the ring of a radius R. Therefore $A = \frac{RH}{2}$ and the angular momentum component along the magnetic field is $\hbar L = pR$.

The Hamiltonian can be written now as

$$H = \frac{\hbar^2 L^2}{2mR^2} + \mu LH + \frac{e^2}{8mc^2}R^2H^2 \tag{3}$$

where μ is the Bohr magneton.

The matrix elements resulting from the last term in (3) are diagonal and constant. Therefore the form of the Hamiltonian used in Ref. [2], which includes only the first two terms of the Hamiltonian shown in (3), has the same spacing between energy levels and the same eigenstates. Therefore, it is justified for a circular ring. [4]

The Hamiltonian shown in (3) can be expressed as

$$H = \frac{\hbar^2}{2mR^2}(L + q)^2 \tag{4}$$

where $q = \frac{\pi R^2 H}{\Phi_0}$ is a measure of the external flux in units of $\Phi_0 = hc/e$.

The eigenstates associated with this Hamiltonian are

$$\psi = \exp[i(M - q)\phi] \equiv \exp[il\phi] \tag{5}$$

where ϕ is the angular variable and M is an eigenvalue of L.

Uniqueness of ψ requires that $l = M - q = \ldots, -2, -1, 0, 1, 2, \ldots$

The ground state angular momentum obeys $|M| = \min|l + q|$ and carries with it a magnetic moment μM which is periodic in q.

Near zero field, $l = 0$ is the ground state and is paramagnetic, namely $\mu M = \mu q$. This happens for fields obeying $-1/2 < q < 1/2$.

In the system described in Ref. [2], due to its small area, paramagnetism can persist till external magnetic fields approaching 1 T. Hence the statement regarding the paramagnetism of the system made in Ref. [2] is justified.

Noninteracting electrons theory as described above predicts magnetization which does not exceed half of a Bohr magneton per participating electron. This prediction is about 2 orders of magnitude too low as compared to the experimental observations [5]. The same discrepancy exists between the theory and experiments performed on the magnetic response of quasi-one-dimensional rings [6]. Therefore, the comment by Hernando and García is correct *vis a vis* the size of the magnetism. Clearly a noninteracting boson model, such as in Ref. [2], cannot explain the size of the giant magnetization observed. Recently, modified theories [7] that assume interacting electrons and disorder partially succeeded in reducing the discrepancy between theory and experiments, but the mystery is not completely solved yet.

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