Dissipative Dynamics of Planar *d*-Wave Josephson Junctions

D. V. Khveshchenko

Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599, USA

(Received 15 July 2005; published 18 January 2006)

We study quantum dynamics of and phase transitions in a Josephson junction between two planar *d*-wave superconductors where the processes of both quasiparticle and Cooper pair tunneling give rise to nonlocal dissipative terms in the effective action. By combining a perturbative weak coupling analysis in the charge representation with a variational approach in the phase representation at strong coupling, we ascertain a layout of the junction's phase diagram and discuss the corresponding behaviors.

DOI: 10.1103/PhysRevLett.96.027004 PACS numbers: 85.25.Cp

Quantum dynamics of ultrasmall normal and superconducting (Josephson) junctions has long been a topic of extensive theoretical and experimental studies [1]. However, the bulk of the known theoretical results pertains to the junctions between conventional, fully gapped, *s*-wave superconductors.

The recent interest in the properties of the *d*-wave Josephson junctions has been motivated by the continuing studies of superconducting cuprates and other unconventional, gapless, superconductors [2,3] and also by the recent proposals of utilizing *d*-wave junction-based devices in quantum computing [4].

The microscopic theory of tunnel junctions is based on the imaginary-time effective action formulated in terms of the phase difference across the junction $\phi(\tau) = \phi_R(\tau) - \phi_L(\tau)$, which is conjugate to the accumulated charge $Q(\tau)$ [1]. To second order in the tunneling amplitude, this action takes the form

$$S = \frac{1}{4E_c}\int_0^{1/T}\left(\frac{\partial\phi}{\partial\tau}\right)^2 d\tau - \int_0^{1/T}\int_0^{1/T}\left[\alpha(\tau-\tau')\cos\frac{\phi(\tau)-\phi(\tau')}{2} + \beta(\tau-\tau')\cos\frac{\phi(\tau)+\phi(\tau')}{2}\right]d\tau d\tau', \tag{1}$$

where T is the temperature, and the first (local) term accounts for the charging energy (measured in units of $E_c = e^2/2C$) of a junction with capacitance C.

The (potentially) nonlocal α and β terms represent the processes of quasiparticle and Cooper pair tunneling, respectively. To second order in the tunneling matrix element $t_{k,k'}$, the corresponding integral kernels $\alpha(\tau) = -\sum_{k,k'}|t_{k,k'}|^2 G_L(\tau,k)G_R(-\tau,k')$ and $\beta(\tau) = \sum_{k,k'}|t_{k,k'}|^2 \times F_L(\tau,k)F_R(-\tau,k')$ can be readily found in terms of the Fourier transforms of the normal $[G_{L,R}(i\omega_n,k) = (i\omega_n + \xi_{L,R})(\omega_n^2 + \xi_{L,R}^2 + \Delta_{L,R}^2)^{-1}]$ and anomalous $[F_{L,R}(i\omega_n,k) = \Delta_{L,R}(\omega_n^2 + \xi_{L,R}^2 + \Delta_{L,R}^2)^{-1}]$ quasiparticle Green functions on the left/right bank of the junction [1].

The standard assumption of a generic, momentum-nonconserving, tunneling ($t_{k,k'} = \text{const}$) between two fully gapped *s*-wave superconductors yields an exponentially decaying Copper kernel [$\beta_s(\tau) \propto \exp(-2\Delta_s\tau)$ for $\tau \gg 1/\Delta_s$]. Thus, both constituents of a Cooper pair tunnel almost simultaneously, and the β term in Eq. (1) reduces to the integral $-E_J\int_0^{1/T}\cos\phi(\tau)d\tau$ of the local Josephson energy $E_J = \int_0^{1/T}\beta(\tau)d\tau$.

By contrast, in the case of a junction between two gapless *d*-wave (or any $l \neq 0$-wave, for that matter) superconductors, the assumption of a momentum-independent tunneling results in the kernel $\beta(\tau)$, which, being proportional to the product of two independent angular averages $\sum_k F_{L,R}(\tau,k)$, vanishes identically. However, in the presence of a momentum-conserving [$|t_{k,k'}|^2 \propto \delta(k-k')$] node-to-node tunneling across the junction between two three-dimensional *d*-wave superconductors, both kernels in Eq. (1) demonstrate an algebraic decay $\alpha_{d,3\mathrm{D}} \sim \beta_{d,3\mathrm{D}} \propto 1/\tau^3$, thereby resulting in the super-Ohmic quasiparticle dissipative term in Eq. (1) [2,3]. Moreover, at those relative orientations between the *d*-wave order parameters that allow for the nodal quasiparticles' tunneling directly into the surface-bound zero-energy states (e.g., node-to-antinode), the slower-decaying and, therefore, even more relevant Ohmic quasiparticle tunneling term might appear as well [3,4].

Furthermore, in the case of tunneling between a pair of two-dimensional (planar) *d*-wave superconductors, *both* kernels show the Ohmic decay even in the absence of any zero-energy states [5]

$$\alpha_{d,2\mathrm{D}} = \alpha/\tau^2, \qquad \beta_{d,2\mathrm{D}} = \beta/\tau^2. \tag{2}$$

In the presence of elastic scattering, the power-law behavior of both $\alpha(\tau)$ and $\beta(\tau)$ changes to the exponential decay at time scales in excess of the inverse bulk impurity scattering rate γ, thus effectively restoring the local Josephson energy (see the Erratum in Ref. [5]). However, as suggested by the wealth of transport data, γ turns out to be quite low compared to the maximum gap Δ_d, and, therefore, the action (1) governing the dynamics of $\phi(\tau)$ appears to be essentially nonlocal in the entire interval $\max[1/\Delta_d, 1/E_c] < \tau < \min[1/T, 1/\gamma]$.

In spite of the apparently nonlocal nature of the microscopically derived kernel $\beta(\tau)$, in the previous analyses of the *d*-wave junctions, the Cooper pair tunneling term would be routinely replaced with the conventional (local)

Josephson energy [3,4]. Although such a phenomenological approach might indeed prove justifiable in the three-dimensional high-T_c junctions (as the analysis of the bulk of experimental data seems to suggest [6]), it would obviously fail in the case of tunneling between truly two-dimensional d-wave superconductors [see Eq. (2)].

The resulting anisotropic XY model described by the effective action (1) and (2) resides outside the realm discussed in the literature up to date, and its quantum dynamics has not yet been properly studied (we comment on the earlier work of Ref. [5] in the conclusions). To fill in the gap, in the present Letter we investigate this model by applying a combination of techniques that cover the complementary regimes of weak and strong dissipative couplings.

At weak couplings ($\alpha, \beta \ll 1$), one can proceed with a direct perturbative expansion for the grand partition function in the charge representation

$$Z(Q) = \sum_{n=-\infty}^{\infty} \int_{Q+ne}^{Q+ne} Dq(\tau) \sum_{N=1}^{\infty} \frac{1}{N!} \prod_{i=1}^{N} \int_0^{1/T} d\tau_i^+ \int_0^{1/T} d\tau_i^- \alpha(\tau_i^+ - \tau_i^-) \times \sum_{M=1}^{\infty} \frac{1}{M!} \sum_{q_j=\pm 1, \sum q_j = 0} \prod_{j=1}^{M} \int_0^{1/T} d\tau_j^+ \int_0^{1/T} d\tau_j^- \beta(\tau_j^+ - \tau_j^-) \exp\left(-\frac{1}{4E_c} \int_0^{1/T} q^2(\tau) d\tau\right), \tag{3}$$

where the lower limit in all the integrals is set at $\tau_c \sim 1/E_c$, and the instantaneous value of the total charge of the junction $q(\tau) = Q + ne + e\sum_{i=1}^{N}(\theta(\tau - \tau_i^+) - \theta(\tau - \tau_i^-)) + e\sum_{j=1}^{M} q_j[\theta(\tau - \tau_j^+) + \theta(\tau - \tau_j^-)]$ includes a continuously varying contribution $Q = CV$ induced by an applied external bias V. The sum in (3) is taken over all the trajectories in the charge space $[Q + ne \rightarrow Q + (n \pm 1)e \rightarrow \ldots \rightarrow Q + ne]$ which consist of N pairs of quasiparticle ($Q \rightarrow Q \pm e$ followed by $Q' \rightarrow Q' \mp e$) and M pairs of Cooper pair ($Q \rightarrow Q \pm e$ followed by $Q' \rightarrow Q' \pm e$) tunneling events.

The periodic dependence of the partition function (3) (hence, any physical observable) upon the external charge Q with the period e allows one to restrict its values to the "Brillouin zone" (BZ) $-e/2 \leq Q \leq e/2$ in the charge space [1]. Unlike in the case of the local Josephson energy, there is no room for the "minimum charge $2e$"—periodicity even in the limit $\alpha \rightarrow 0$.

In the absence of tunneling, the ground state becomes degenerate with the first excited one $[Q^2/2C = (Q \pm e)^2/2C]$ only at the BZ boundaries ($Q = \pm e/2$). Provided that the tunneling is weak, the analysis of the perturbative expansion (3) can be readily performed in the vicinity of the degeneracy points where the (renormalized) gap $\Delta_r(Q) = E_1(Q) - E_0(Q)$ between the ground and first excited states remains small compared to E_c. Therefore, close to, e.g., $Q = e/2$, one can safely neglect any transitions between the two lowest energy levels and the rest of the spectrum separated by the energy gap of order E_c, thereby reducing the sum (3) to the trajectories comprised of a sequence of "blips" between the states with the charges Q and $Q - e$ [7].

The resulting two-state problem then becomes amenable to the renormalization group (RG) analysis, akin to that carried out in the context of the Kondo and other quantum spin-1/2 problems. By performing the standard procedure of changing the cutoff in the time integrations in Eq. (3) from τ_c to $\tau_c' > \tau_c$ and integrating over the pairs of opposite blips with separations $\tau_c < |\tau_i^+ - \tau_j^-| < \tau_c'$, one reproduces the renormalized partition function $Z_r(Q)$, which now depends on the effective dissipative couplings α_r, β_r, and $\tilde{\Delta}_r = \Delta_r \tau_c$. The latter obey the RG equations

$$\frac{d\alpha_r}{d\ln\tau_c} = -2\alpha_r(\alpha_r - \beta_r), \qquad \frac{d\beta_r}{d\ln\tau_c} = -2\beta_r(\alpha_r - \beta_r), \qquad \frac{d\ln\tilde{\Delta}_r}{d\ln\tau_c} = (1 - 2\alpha_r) \tag{4}$$

derived under the assumption of small α_r and β_r.

Solving the RG equation (4) and evaluating all the functions at the lowered energy cutoff $1/\tau_c' = \Delta_r$, one obtains renormalized values of the effective couplings

$$\alpha_r = \frac{\alpha\beta_r}{\beta} = \frac{\alpha}{1 + 2\eta \ln E_c/\Delta_r}, \tag{5}$$

where $\eta = \alpha - \beta$, and a self-consistent equation for the renormalized gap

$$\Delta_r = \frac{\Delta_0}{[1 + 2\eta \ln E_c/\Delta_r]^{\alpha/\eta}}, \tag{6}$$

whose bare value is $\Delta(Q) = E_1^{(0)}(Q) - E_0^{(0)}(Q) = E_c(1 - 2Q/e)$.

According to Eq. (5), for $\eta > 0$ the RG trajectory flows towards weak coupling (see Fig. 1) where the invariant charge $\tilde{\Delta}_r$ increases, although the actual gap Δ_r given by Eq. (6) continues to decrease. In this regime, the quantum

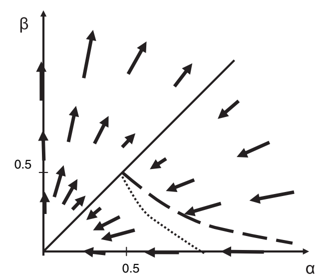

FIG. 1. The renormalization group flow in the anisotropic XY model (arrows). The phase boundary between the insulating and (super)conducting phases is shown for Ohmic (dashed line) and sub-Ohmic (dotted line) dissipations.

phase fluctuations due to the Coulomb blockade destroy the classical Josephson effect, and the junction remains in the insulating state. This conclusion generalizes that drawn in the extensively studied $\beta = 0$ limit where the system possesses the exact XY symmetry and is known to retain its insulating behavior in the entire range of parameters, including arbitrary values of the external charge Q [7].

Nonetheless, the effect of dissipation can still be important, as demonstrated by, e.g., a rounding of the Coulomb staircaselike dependence of the average charge $\langle q(\tau)\rangle$ on Q. Close to the BZ boundary ($\delta Q = Q - e/2 \to 0^-$), one can use Eq. (6) to obtain $\langle q(\tau)\rangle = Q - CdE_0(Q)/dQ = (e/2)(1 + \mathrm{sgn}\delta Q/[1 + 2\eta \ln(e/\delta Q)]^{\alpha/\eta})$. Thus, the dissipation-induced screening of the external charge appears to be enhanced as compared to the case of a normal junction (single-electron box) with $\beta = 0$ [7].

Alongside the reduction of the rate of tunneling from the first excited to the ground state, this effect is manifested by the current-voltage (I-V) characteristics of a voltage-biased junction where the induced dc current $I(V) = \langle q(\tau)\rangle \mathrm{Im}E_1(Q)|_{Q=VC} \approx 2\pi\alpha(V - E_c/e)/[1 + 2\eta \ln(E_c/eV - E_c)]^{1+2(\alpha/\eta)}$ vanishes below a threshold $V_c = E_c/e$. At finite temperatures, the hard Coulomb gap gets partially filled with thermally excited quasiparticle excitations, thus giving rise to a temperature-dependent conductance. By the same token, in a current-biased junction the zero-temperature I-V characteristics should become nonlinear for $I < eE_c$.

Upon approaching the separatrix $\eta = 0$, the RG flow slows down and eventually ceases completely. Along that line, the effective coupling $\alpha_r = \beta_r$ undergoes no renormalization (at least, to first order in the dissipative couplings), while the effective gap demonstrates a power-law dependence $\Delta_r = \Delta_0(\Delta_0/E_c)^{2\alpha/1-2\alpha}$ on its bare value, thus indicating the possibility of a dissipative phase transition at $\alpha = \beta = 1/2$.

Considering that for $\eta = 0$ the symmetry of the system is Ising-like, one might expect this transition to be of the Kosterlitz-Thouless type [7]. Moreover, in light of the constancy of the effective coupling $\alpha = \beta$, this critical behavior appears to be reminiscent of that of a junction with the *local* Josephson energy and *quadratic* [that is, $-\alpha \iint d\tau d\tau' \phi(\tau)\phi(\tau')/|\tau - \tau'|^2$, as opposed to the non-Gaussian quasiparticle tunneling-induced] Ohmic dissipation.

For $\alpha = \beta < 1/2$, the quantum phase fluctuations are strong, the energy bands $E_{0,1}(Q) = E^{(0)}_{0,1}(Q) \mp \Delta_r(Q)/2$ remain nondegenerate, and the junction operates in the insulating (Coulomb blockade-dominated) regime. In contrast, for $\alpha = \beta > 1/2$, the quantum fluctuations are quenched, and the energy bands become progressively more and more degenerate in a finite portion of the BZ which expands from the boundaries ($Q = \pm e/2$) inward as the parameter $\alpha = \beta$ increases. This behavior signals a suppression of the Coulomb blockade and a possible restoration of the classical Josephson effect where the voltage drop across the junction vanishes and the current is determined by a nonzero average value $\langle \phi(\tau)\rangle$ of the phase difference across the junction.

For $\eta < 0$, one finds a runaway RG flow towards strong coupling (see Fig. 1) where both α_r and β_r become of order unity and $\tilde{\Delta}_r$ starts to decrease, regardless of the bare values of the dissipative couplings. Such a behavior suggests that the junction is likely to end up in the (super)-conducting regime for all $\alpha < \beta$.

In the complementary regime of strong coupling ($\eta > 1$), a preliminary insight into the problem can be obtained by virtue of the variational technique where the correlation function of the small (''spin wavelike'') phase fluctuations is sought out in the form

$$\langle |\phi_\omega|^2\rangle = \frac{1}{\omega^2/E_c + g|\omega| + D}. \tag{7}$$

Computing the variational free energy $F = -T\ln\mathcal{Z}_0 + T\langle S - S_0\rangle$ [where the averages such as, e.g., $\langle e^{i\phi(\tau)}\rangle = \exp(-\frac{1}{2}\langle\phi^2(\tau)\rangle)$ are calculated with the use of the quadratic action $S_0(\phi)$, which yields Eq. (7)] and minimizing the result with respect to the parameters g and D, we obtain the equation for the effective dissipative coupling

$$\eta = g^{(g+1/g-1)}(2\beta)^{1/1-g}. \tag{8}$$

Provided that the condition $\alpha - \beta > e\theta(1/2 - \beta) \times \ln(1/2\beta)$ is fulfilled, there is a finite gap due to the XY anisotropy in the two-dimensional space spanned by the unit vector $\mathbf{n} = (\cos\phi/2, \sin\phi/2)$

$$D = E_c(2\beta)^{g/g-1}g^{2/1-g}, \tag{9}$$

where $g \approx \eta^{1-2/\eta}(2\beta)^{1/\eta}$ is the solution of Eq. (8) for $\eta > 0$. For comparison, one finds $g \approx \eta$ throughout nearly the entire domain $\eta < 0$.

A nonzero anisotropy gap D renders the expectation value $\langle\phi^2(\tau)\rangle$ finite, so that the phase ϕ becomes localized, and the real part of the ac conductance $G(\omega) = I(\omega)/V(\omega) = \mathrm{Re}(\omega_n\langle|\phi_{\omega_n}|^2\rangle)^{-1}|_{\omega_n\to -i\omega}$ develops a coherent peak at zero frequency, $G(\omega \to 0) = D\delta(\omega)$, which is indicative of the possible onset of the classical Josephson effect.

Conversely, in the case that Eq. (8) features no solution, the gap D vanishes, the expectation value of the phase fluctuations $\langle\phi^2(\tau)\rangle$ diverges, and the phase remains delocalized. Although it may seem that in the dc limit the real part of the conductance approaches a finite value $G(0) = g$, the continuing downward renormalization of the effective dissipative parameter η [see Eq. (11) below] is expected to eventually bring the system into the insulating regime.

In fact, in the absence of a nontrivial mean field solution, the variational method can be abandoned in favor of the straightforward perturbative approach that reveals a continuous renormalization of the couplings due to the presence of non-Gaussian (quartic and higher order) terms in the action (1). To first order in the small parameter $1/\eta$, one arrives at the RG equations

$$\frac{d\alpha_r}{d\ln\tau_c} = -\frac{\alpha_r}{2\pi^2\eta_r}, \qquad \frac{d\beta}{d\ln\tau_c} = -\frac{\beta_r}{2\pi^2\eta_r}, \tag{10}$$

which demonstrate the before mentioned gradual decrease of the effective dissipative parameter as a function of the lowered energy cutoff $1/\tau_c'$

$$\eta_r = \eta - \frac{1}{2\pi^2}\ln E_c\tau_c'. \tag{11}$$

The steady decrease of both α_r and β_r as well as the approximate constancy of the ratio α_r/β_r agree quite well with the behavior found at weak coupling, and the RG trajectories interpolate smoothly between the two regimes.

Altogether, the above results suggest a tentative phase diagram of Fig. 1 where the insulating behavior sets in within the domain defined by the inequalities $0 \le \beta \le 1/2$ and $\beta \le \alpha \le \beta + e\ln(1/2\beta)$. Upon increasing the value of $\eta > 0$ and/or changing its sign, the insulator gives way to the conducting phase.

In the disordered phase, the correlation function $\langle e^{i\phi(\tau)/2}e^{-i\phi(0)/2}\rangle$ decays algebraically and the insulator-like I-V characteristics show the presence of a hard (at $T = 0$ and $I \to 0$) Coulomb gap. By contrast, in the ordered phase, the system develops an order parameter $\langle e^{i\phi(\tau)/2}\rangle \neq 0$ whose presence indicates that the phase variable is localized in either even ($\phi = 2\pi M$) or odd [$\phi = (2M + 1)\pi$] vacua. This does not, however, constitute a complete phase localization, and, therefore, the standard Josephson could only be observed as a nonequilibrium phenomenon over finite (albeit potentially quite long) observation times, while at still longer times the response would eventually revert to a resistive behavior [1].

Before concluding, we briefly mention another potentially important nonequilibrium effect, such as the excitonic enhancement of the tunneling probability which modifies the exponent in the power-law decaying kernels (2) to $2 - \epsilon$, where ϵ is a function of the (nonuniversal) scattering phase shift [8].

In this sub-Ohmic case, the right-hand side of the RG equation (10) acquires additional terms ($\epsilon\alpha$ and $\epsilon\beta$), and, as a result, there is now a fixed point at $\eta_c = 1/2\pi^2\epsilon$. One would then be led to conclude that, in the domain defined by the relations $\eta_c \le \alpha - \beta \le e\ln(1/2\beta)$, the transition from the insulating to the superconducting phase is preempted by that into a new (conducting, although potentially different from the Josephson-like) state.

Last, if the anisotropic XY model (1) and (2) with the parameter values from Ref. [5] ($\alpha = 2\beta$ and $E_c \sim \Delta_d$) were to be applied to the analysis of a grainy d-wave superconductor, an experimental observation of the apparent Josephson-like response would imply that the dissipative couplings in that sample should indeed be quite strong ($\alpha > \alpha_c \approx 0.85$). Considering that the typical junctions manufactured out of the cuprate superconductors tend to have relatively high conductances [6], the possibility that an average junction between adjacent grains does satisfy this condition is not unrealistic.

Before concluding, it is worth pointing out that the existence of a critical coupling α_c could not be predicted in the framework of the naive gradient expansion of Ref. [5]. For one, a direct application of this technique to the model (1) and (2) gives rise to an undamped correlation function $\langle|\phi_\omega|^2\rangle = (\omega^2/E_c^* + D^*)^{-1}$ with a strongly temperature-dependent renormalized Coulomb energy $E_c^* \sim T$ and a gap $D^* \sim E_c$ [5], in stark contrast with Eq. (7).

In summary, we carried out a microscopic analysis of the anisotropic XY model describing a node-to-node Josephson junction between two planar d-wave superconductors. We found the evidence of a dissipative phase transition and identified a tentative location of the phase boundary in the α-β plane. The corresponding critical behavior differs, in a number of important details, from the previously studied cases of a superconducting junction with the local Josephson energy as well as that of a normal junction (single-electron box). Our specific predictions for such observables as energy spectrum, average excess charge, and I-V characteristics could be tested in Josephson junctions formed by very thin (of a width less than the correlation length) cuprate films.

This research was supported by NSF under Grant No. DMR-0349881 and ARO under Contract No. DAAD19-02-1-0049.

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