Out-of-Equilibrium Dynamics of the Vortex Glass in Superconductors

Sebastian Bustingorry,[1] Leticia F. Cugliandolo,[2] and Daniel Domínguez[1]
[1]*Centro Atómico Bariloche, 8400 San Carlos de Bariloche, Río Negro, Argentina*
[2]*LPTHE, 4 Place Jussieu, 75252 Paris Cedex 05, France*
and LPT Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05 France
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We study the relaxational dynamics of flux lines in high-temperature superconductors with random pinning using Langevin dynamics. At high temperatures the dynamics is stationary and the fluctuation dissipation theorem (FDT) holds. At low temperatures the system does not equilibrate with its thermal bath: a simple multiplicative aging is found, the FDT is violated, and we find that an effective temperature characterizes the slow modes of the system. The generic features of the evolution—scaling laws—are dictated by those of the single elastic line in a random environment.

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The effect of quenched disorder on the vortex phase diagram of superconductors has attracted much interest since the discovery of high T_c cuprates. Since the long-range order of the vortex lattice is destroyed by any small amount of disorder [1], an amorphous "vortex glass" (VG) phase was predicted [2]. While it is currently understood that there is a "Bragg glass" phase for weak disorder [3], there is still some controversy about the nature of the amorphous phase at strong disorder. Experimentally, one finds an irreversibility line (IRL) below which the magnetization is irreversible and the linear resistivity drops to nearly zero [4]. The original proposal that the IRL signals a thermodynamic continuous transition from the vortex liquid (VL) to a VG with zero linear resistivity [2] was initially supported by experiments in which a scaling of current-voltage (IV) curves was found [5] and by simulations in randomly frustrated 3D XY models with a finite temperature transition [6]. However, when magnetic screening is included in these 3D XY models, the transition disappears [7]. Moreover, recent experiments showed that the IV curves do not scale as expected [8].

One possible scenario for the IRL is that at low temperatures experiments occur out of equilibrium. Simulations of the London-Langevin model for vortices (which includes screening) showed that below a crossover temperature, where the VG type of criticality is arrested, the time scales grow very quickly, similar to a Volger-Fulcher form [9] as in structural glasses [10]. Therefore, it is important to compare the dynamic behavior of the vortex system with that of structural glasses. In this Letter we apply the analysis of correlation and response functions used in the study of relaxation in structural glasses [11] to analyze the VG phase in a simulation of a London-Langevin model similar to the one studied in Refs. [9,12].

We consider 3D elastic flux lines in a superconductor where the ith line has coordinates $\mathbf{r}_i(z) = [x_i(z), y_i(z)]$ with z the vertical direction along the magnetic field B. The superconductor has anisotropy $\epsilon = \xi_c/\xi_{ab} = \lambda_{ab}/\lambda_c$, with ξ_{ab}, ξ_c the coherence lengths and λ_{ab}, λ_c the penetration depths; the axes are such that $c \parallel z$ and $ab \parallel \mathbf{r}_i$. Assuming that $\mathbf{r}_i(z)$ varies slowly with z, the Hamiltonian for a London model of elastic flux lines is

$$\mathcal{H} = \sum_z \Big[\sum_i U_l(\Delta\mathbf{r}_{iz}) + U_d(\mathbf{r}_{iz}) + \sum_{i<j} U_{\rm in}(\mathbf{r}_{jz} - \mathbf{r}_{iz}) \Big].$$

Having discretized z in units of d_z, $\mathbf{r}_{iz} \equiv \mathbf{r}_i(z)$ represents the 2D position of a line element in the plane z. The interaction energy between line elements in the plane z is approximated as $U_{in} = 2\epsilon_0 d_z K_0(r/\lambda_{ab})$ with $\epsilon_0 = (\Phi_0/4\pi\lambda_{ab})^2$ [9,12,13]. The elastic line energy of the ith vortex is $U_l = \frac{1}{2}c_l(\partial_z \mathbf{r}_i)^2 dz$, which is discretized as $U_l = \frac{1}{2}c_l(\frac{\Delta\mathbf{r}}{d_z})^2 d_z$, with $\Delta\mathbf{r}_{iz} = \mathbf{r}_{i,z+1} - \mathbf{r}_{i,z}$. A natural choice for d_z is the distance between CuO planes, in which case one can account for the effect of the Josephson coupling [13] using $U_l = 2c_l\lambda_J \frac{|\Delta\mathbf{r}|}{d_z}$ for $|\Delta\mathbf{r}| > 2\lambda_J$ (keeping the previous expression for $|\Delta\mathbf{r}| < 2\lambda_J$), where $\lambda_J = d_z/\epsilon$ is the Josephson length. The quenched disorder potential due to impurities is $U_d(\mathbf{r}) = \int d^2\mathbf{r}' u(\mathbf{r}')p(|\mathbf{r} - \mathbf{r}'|)$, where $p(r) = 2\xi_{ab}^2/(r^2 + 2\xi_{ab}^2)$ and $\langle u(\mathbf{r},z)u(\mathbf{r}',z')\rangle = \gamma\delta(\mathbf{r} - \mathbf{r}')\delta_{zz'}$ defines the disorder strength γ [12,14]. We model the dynamics with the Langevin equation

$$\eta \frac{\partial \mathbf{r}_{iz}(t)}{\partial t} = -\frac{\delta\mathcal{H}[\{\mathbf{r}_{lz}(t)\}]}{\delta\mathbf{r}_{iz}} + \mathbf{f}_{iz}^T(t),$$

where η is the Bardeen-Stephen friction coefficient. The thermal force $\mathbf{f}_{iz}^T(t)$ satisfies $\langle f_{iz,\mu}^T(t)\rangle = 0$, and $\langle f_{iz,\mu}^T(t) f_{i'z',\mu}^T(t')\rangle = 2\eta k_B T\delta(t - t')\delta_{zz'}\delta_{ii'}\delta_{\mu\mu'}$, where $\mu, \mu' = x, y$ and T is the thermal bath temperature.

The above model gives a good quantitative description of the vortex physics of moderately anisotropic high-T_c superconductors like $YBa_2Cu_3O_{7-\delta}$ [12–15]. We therefore choose parameter values corresponding to $YBa_2Cu_3O_{7-\delta}$: $\epsilon = 1/5$, $\lambda_{ab}/\xi_{ab} = 100$, and $\lambda_J/\xi_{ab} = 16$; and we use $c_l = \epsilon^2\epsilon_0 2[1 + \ln(\lambda_{ab}/d_z)]/\pi$ [13]. The strength of disorder is set to $\gamma = 10^{-5}$, for which case we find that above $B_{cr} \sim 0.002H_{c2}$ the Bragg peaks disappear and the flux lines are frozen in a highly amorphous structure at low temperatures. We therefore choose to study the case with $B = 0.01H_{c2} \gg B_{\rm cr}$, which is deep within the VG regime at low T. Time is normalized by $t_0 = \xi_{ab}^2 \eta/\epsilon_0$, length by the vortex lattice parameter $a_0 = [2\Phi_0/(\sqrt{3}B)]^{1/2}$, energy by $\epsilon_0 d_z$, and temperature by $\epsilon_0 d_z/k_B$. We simulate $N = 56$ vortices in a box of size $7a_0 \times 8a_0\sqrt{3}/2$ with periodic boundary conditions for the in-plane coordinates. The z direction is discretized in $L = 50$ planes with free boundary conditions. Averages are performed over 10 realizations of the disorder.

In order to study the out-of-equilibrium dynamics we use the following protocol. First we equilibrate the system at an initial high temperature well inside the VL ($T_i = 0.3$) evolving during $t = 10^4$ steps. Then we quench the system to a low temperature T, where the time count is set to zero. Starting with this far from equilibrium initial condition, the system is evolved during a waiting time t_w, after which the quantities of interest are measured. In general, one defines two-time correlation or response functions $C(t, t_w)$. When the system reaches equilibrium, these quantities become independent of t_w and depend only on the difference $\tau = t - t_w$. If the system is not able to reach equilibrium within the observation time window, $C(t, t_w)$ depends on the 2 times. In particular, if the decay gets slower for longer t_w's we say that the system "ages." We study the following: (a) The dynamic wandering [16], $W(z, t, t_w)$, defined by

$$NW(z,t,t_w) = \sum_i \langle |[\mathbf{r}_{iz}(t) - \mathbf{r}_{i0}(t)] - [\mathbf{r}_{iz}(t_w) - \mathbf{r}_{i0}(t_w)]|^2 \rangle,$$

which measures how the displacement of the vortex segment in the zth plane with respect to the bottom plane correlates between t and t_w ($\langle \cdots \rangle$ means average over thermal noise and disorder). (b) The mean square displacement (MSD) in the planes,

$$B(t, t_w) = \frac{1}{LN} \sum_{iz} \left\langle \left[x_{iz}(t) - x_{iz}(t_w) \right]^2 \right\rangle.$$

First, we analyze the single, noninteracting, flux line without disorder. In Figs. 1(a) and 1(b) we show the dynamic wandering and the MSD, respectively, when the line is quenched to $T = 0.06$. In this case the flux line reaches equilibrium in a short time: both correlation functions are independent of t_w and depend only on $\tau = t - t_w$ (data for several t_w overlap in the plots). From the behavior of W, we define a set of characteristic times t_z, such that when $\tau = t - t_w > t_z$, $W(z, \tau \gg t_z) \sim z^{2\zeta}$ saturates to a constant value. Here ζ is the roughness exponent given by thermal fluctuations, i.e., $\zeta = \zeta_T = 1/2$ [14]. t_z is the time needed for the line element in the zth plane to feel the elastic interaction with the line element in the plane $z = 0$. In Fig. 1 we show the times scales t_1 for neighboring planes separation and t_L for full system size separation. We found that for $\tau < t_1$, $W(z, \tau) \sim \tau$ for all z. For times $\tau > t_L$, $W(L, \tau)$ saturates. For times $t_1 < \tau < t_z$ the dynamic wandering shows an intermediate regime between these two extreme behaviors. In Fig. 1(b) the MSD is shown and the same regimes are identified. First, for $\tau < t_1$ the MSD follows the 2D diffusion of individual line elements ("pancakes"), $B(\tau) \sim \tau$, before the elastic interplane interaction becomes relevant. Second, for $t_1 < \tau < t_L$, flux line thermal relaxation is observed, characterized by sublinear diffusion $B(\tau) \sim \tau^\alpha$, with $\alpha = 1/2$ [13,14]. Third, for $\tau > t_L$, we observe diffusion of the center of mass of the flux line $B(\tau) \sim \tau$. t_L is the time scale above which finite size effects dominate. The time regime we wish to study is the one without finite size effects, $\tau < t_L$ ($\tau \lesssim 10^4$ for $L = 50$).

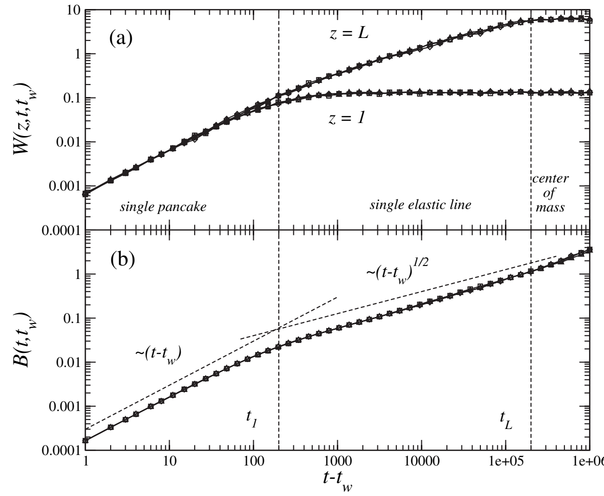

FIG. 1. (a) Dynamic wandering and (b) mean square displacement for elastic lines with no in-plane interaction and without disorder, after the quench to $T = 0.06$. The elastic correlation times t_1 and t_L separate three regimes (single pancake, elastic line, and center of mass diffusion). The behaviors $B \sim (t - t_w)$ and $B \sim (t - t_w)^{1/2}$ are highlighted.

Second, we analyze the evolution of interacting lines in the presence of disorder. When we quench the system to relatively high temperatures ($T > 0.2$), it reaches equilibrium, $W(t, t_w)$ and $B(t, t_w)$ are independent of t_w, and they behave as in Fig. 1 for noninteracting flux lines, meaning that we are clearly within the VL. Below a crossover temperature $T_g \approx 0.18$ the system is no longer able to equilibrate, and the correlation functions depend on t_w. The MSD at $T = 0.02$ is shown in Fig. 2(a). For $\tau < t_1$, the dynamics is governed by single pancake fluctuations and the MSD is independent of t_w. For $\tau > t_1$, there is "aging," the longer the waiting time t_w, the slower the relaxation of the flux lines [17].

In order to study the modifications of the fluctuation dissipation theorem (FDT) in the out-of-equilibrium regime, a response function should be measured. To this end, a random force of the form $\mathbf{f}_{iz} = \delta s_{iz} \hat{\mathbf{x}}$ is switched on at a time t_w on a replica of the system, where δ is the intensity of the perturbation, and $s_{iz} = \pm 1$ with equal probability [18,19]. The integrated response is

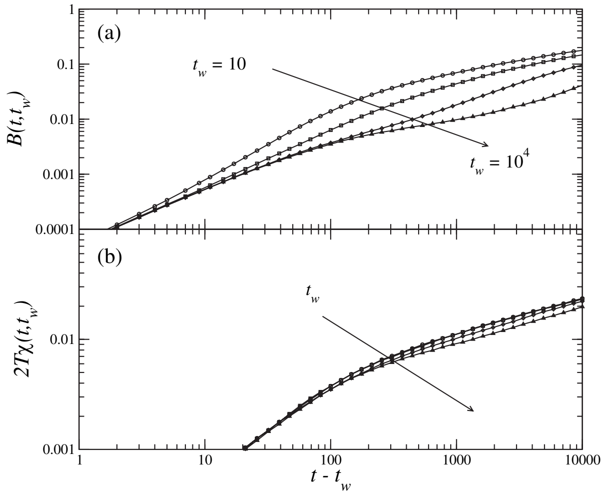

FIG. 2. (a) Mean square displacement and (b) integrated response for the VG at $T = 0.02$. Different waiting times $t_w = 10, 10^2, 10^3$, and 10^4 are shown, from top to bottom.

$$\chi(t, t_w) = \frac{1}{LN\delta} \sum_{lz} \left\langle s_{iz} \left[x_{iz}^{\delta}(t) - x_{iz}(t) \right] \right\rangle,$$

where x_{iz}^{δ} and x_{iz} correspond to the position evaluated in two replicas of the system, with and without the perturbation. In equilibrium, FDT implies $2T\chi(\tau) = B(\tau)$. In Fig. 2(b), we show $2T\chi(t, t_w)$ at $T = 0.02$. χ depends on t_w showing aging. It is also clear that for long τ and long t_w $2T\chi$ is not proportional to B, violating FDT. This type of behavior, aging in B and χ and violation of FDT for long t_w and τ, is observed at all $T < T_g$.

Typically, a correlation function $C(t, t_w)$ in structural glasses has an additive scaling form $C(t, t_w) = C_{\rm eq}(t - t_w) + C_{\rm ag}[h(t)/h(t_w)]$, with, very often, $h(t) = t$ (known as simple aging) [11,20]. This does not hold in the vortex problem. Instead, we found a "multiplicative" scaling similar to the one proposed for the low temperature behavior of a directed polymer in random media [21] and the out-of-equilibrium critical dynamics of the 2D *XY* model [22]. Following Yoshino [21] we tried the scaling form $B(t, t_w) = \tilde{B}(\tilde{t})t_w^{\alpha}$ and $2T\chi(t, t_w) = \tilde{\chi}(\tilde{t})t_w^{\alpha}$, with $\tilde{t} = t/t_w$ and $\tilde{B}$ and $\tilde{\chi}$ given by

$$\tilde{B}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \ll 1, \\ c_2(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \gg 1, \end{cases}$$

$$\tilde{\chi}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \ll 1, \\ y(T)c_2(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \gg 1, \end{cases}$$

where c_1, c_2, and y are temperature dependent coefficients [23]. $y(T)$ measures the modification of FDT [20] $2T\chi(t, t_w) = y(T)B(t, t_w)$ [or $\tilde{\chi} = y(T)\tilde{B}$], and an effective temperature [24] is defined by $T_{\rm eff} = T/y$.

In Fig. 3(a) we show the scaled $\tilde{B}$ and $\tilde{\chi}$ at $T = 0.02$. The data for different t_w fall on two master curves. $\tilde{B}$ and $\tilde{\chi}$ coincide for $\tilde{t} \ll 1$, which means that FDT holds, while for longer times ($\tilde{t} \gg 1$) $y(T) < 1$ signals the violation of FDT. In Fig. 3(b) a parametric plot of the scaled $\tilde{B}$ and $\tilde{\chi}$ is shown; for $\tilde{t} \ll 1$ it has a slope equal to one, corresponding to the FDT regime (see an enlargement in the inset), while for longer times the slope is $y = 0.111$. From this kind of plot the temperature dependent dynamic exponent $\alpha(T)$ and the parameter $y(T)$ in Fig. 4 are obtained. At high temperatures $\alpha = 1/2$ as for single flux lines. The low temperature phase has a lower dynamic exponent [$\alpha(T) < 0.4$], implying a much slower relaxation. In Fig. 4(a) we see that $\alpha(T)$ depends weakly with T and that it decreases with increasing disorder strength within the glassy regime. In Fig. 4(b) we show $y(T) = T/T_{\rm eff}$; it is well described by a linear form implying an effective temperature $T_{\rm eff}$ that is independent of T. A linear fit yields $T_{\rm eff} = 0.175$. It is remarkable that this value is very near the crossover temperature $T_g \approx 0.18$ below which aging is observed. A similar result is observed in structural glasses: $T_{\rm eff} \approx T_g$ (as in a random energy model scenario [18]).

To further check a "fragile glass transition scenario" [10], we estimated a characteristic relaxation time by evaluating $C(t, t_w) = \exp[-B(t, t_w)]$. In the VL we expect $C(\tau) \sim \exp[-(\tau/t_r)^{\alpha}]$. For single flux lines the T dependence of t_r should be $t_r \propto 1/T^2$ [13,14]. We analyzed this correlation function for different T, fitting the corresponding stretched exponential with $\alpha = 1/2$ and estimating the relaxation time $t_r(T)$ that is plotted in the inset of Fig. 4(b). In the VL we obtained $t_r \propto 1/T^2$, as

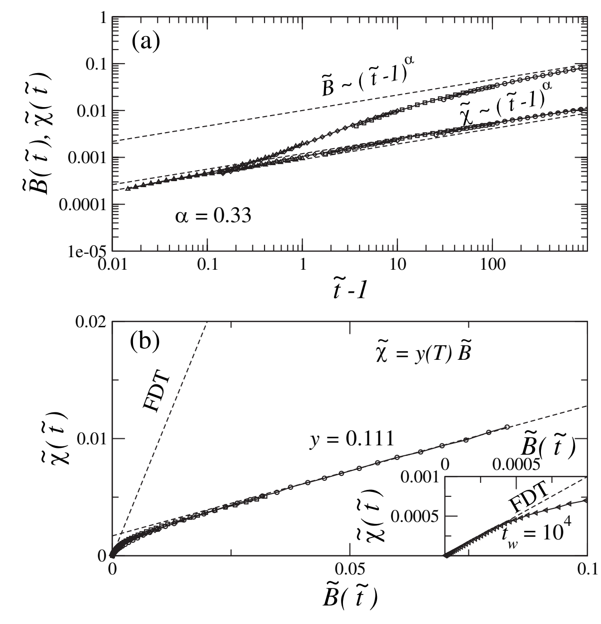

FIG. 3. (a) Multiplicative scaling for the mean square displacement and the integrated response function corresponding to the data in Fig. 2. $T = 0.02$, and the value of α is quoted. (b) The parametric plot $\tilde{\chi}(\tilde{B})$ for the same data. FDT holds at short rescaled times while a violation of FDT with $y = 0.111$ holds at longer times. The inset shows the short rescaled time FDT regime for $t_w = 10^4$.

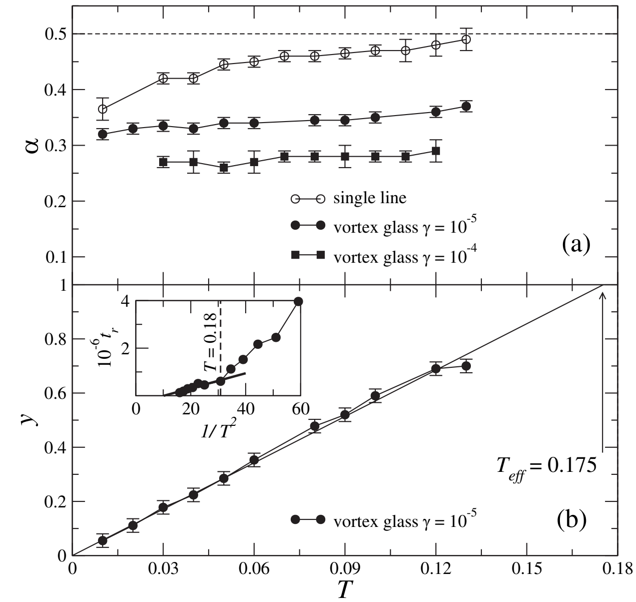

FIG. 4. (a) Temperature dependence of the dynamic exponent. The equilibrium value for single elastic lines, $\alpha = 1/2$, is highlighted. (b) The slope $y(T)$. The data are fitted linearly with $y = T/T_{\rm eff}$, and the values of $T_{\rm eff}$ are indicated. Inset shows the relaxation time at the VL-VG transition.

expected. Below $T_g \approx 0.18$ the system does not equilibrate, but we can extract a "t_r" by fitting $C(t, t_w)$ for the largest t_w with the same exponential form. The obtained t_r increases very rapidly below T_g, reflecting that the system falls out of equilibrium in this aging regime.

Finally, we performed the same analysis for single flux lines in the presence of disorder. For high temperatures we found no aging and $\alpha \approx 1/2$, with important finite size effects (cf. Fig. 1). Below a crossover temperature we observe similar aging to the one shown in Fig. 2. The multiplicative scaling and the same definitions of $\tilde{B}$ and $\tilde{\chi}$ describe the data very accurately, the only differences being that $\alpha(T)$ takes larger values that are closer to $\alpha = 1/2$ than in the VG [see Fig. 4(a)], and that $T_{\rm eff}$ takes a smaller value, $T_{\rm eff} = 0.156$, than in the VG.

The results described in the last paragraph imply that, rather surprisingly, the out-of-equilibrium slow dynamics of the VG is dominated by the relaxation of the elastic lines along the z direction. Aging follows a multiplicative scaling as for single lines, but with a smaller exponent $\alpha(T)$. Do vortices freeze like a structural (fragile) glass? Yes and no. Yes, because we found aging and a simple violation of FDT with an effective temperature $T_{\rm eff}$ that is independent of T within our numerical accuracy. No, because aging follows a multiplicative scaling, similar to polymers in random media and, more precisely, critical systems like the 2D *XY* model [22]. Longer times are needed to decide whether this "critical" scaling holds in the limit $t_w \to \infty$ and to grasp the nature of the low-T phase. Our results suggest to do experiments with a fast quench to low T: with relaxational measurements of transport or magnetic properties one could study the aging regime, while with voltage or flux noise measurements one could test how the FDT is violated.

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