Detecting Topological Order through a Continuous Quantum Phase Transition

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We study a continuous quantum phase transition that breaks a Z_2 symmetry. We show that the transition is described by a new critical point which does not belong to the Ising universality class, despite the presence of well-defined symmetry-breaking order parameter. The new critical point arises since the transition not only breaks the Z_2 symmetry, it also changes the topological or quantum order in the two phases across the transition. We show that the new critical point can be identified in experiments by measuring critical exponents. So measuring critical exponents and identifying new critical points is a way to detect new topological phases and a way to measure topological or quantum orders in those phases.

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For a long time, Landau symmetry-breaking theory [1] was believed to be the theory that describes all possible phases and phase transitions. The Ginzburg-Landau theory [2] based on order parameters and long range order became the standard theory for all kinds of continuous phase transitions.

However, after the discovery of fractional quantum Hall (FQH) effect, [3] people realized that different FQH states all have the same symmetry. So the order in FQH states cannot be described by the Landau's symmetry-breaking theory. The new order is called topological order [4,5]. Topological order is new since it has nothing to do with symmetry-breaking, long range correlation, or local order parameters. None of the usual tools that we used to describe a symmetry-breaking phase applies to topological order. Despite this, topological order is not an empty concept since it can be described by a new set of tools, such as the number of degenerate ground states [6,7], quasiparticle statistics [8], and edge states [5,9,10].

The existence of topological orders has consequences on our understanding of continuous phase transitions. If there exist phases that are not described by symmetry breaking, then it is reasonable to guess that there exist continuous phase transitions that are not described by changes of symmetries and the associated order parameters. Indeed continuous phase transitions exist between two phases with the *same* symmetry [11–15] and between two phases with the *incompatible* [16] symmetries [17]. In this Letter, we will show that even some symmetry-breaking continuous phase transitions are beyond Landau's symmetry-breaking paradigm in the sense that critical properties of the transition are not described by fluctuating symmetry-breaking order parameters and not described by Ginzburg-Landau effective theories. As a result, the critical exponents of those symmetry-breaking transitions are different for those obtained from Ginzburg-Landau theory.

Why do some symmetry-breaking transitions give rise to new class of critical points? One reason is that those transitions not only change the symmetry of the states, they also changes the topological or quantum order in the states. So the appearance of the new critical points, in many cases, implies the appearance of new state of matter with nontrivial topological or quantum orders. It is known that frustrated spin systems on Kagome or pyrochlore lattices contain many different quantum phases. Those different quantum phases in general contain different spin orders as shown by magnetic susceptibility measurements. So one naturally assumes those spin ordered phases are described by symmetry breaking, and the continuous transition between those phases are symmetry-breaking transitions described by Ginzburg-Landau theory. The main message of this Letter is that those $T = 0$ spin ordered phases may contain additional topological orders and represent new states of matter. The additional topological orders can be detected by measuring critical exponents at continuous quantum transition points between those $T = 0$ quantum phases (even when the continuous transitions are symmetry-breaking transitions). If the measured critical exponents do not agree with the those values obtained from Ginzburg-Landau theory, then the transition point will be a new quantum critical point and the two phases separated by the phase transition will contain nontrivial topological orders.

But why is zero temperature important? Can we find new states of matter and new continuous phase transitions at finite temperatures? The answer is yes, but it is more difficult to find new states of matter and new continuous transitions at finite temperatures. This is because most of the known new states of matter are due to string-net condensations [18,19]. String-net condensations and the continuous phase transition between different string-net condensed states only exist at zero temperature. So it is much easier to find new states of matter and new continuous transitions at zero temperature.

To illustrate our point that a continuous $T = 0$ symmetry-breaking phase transition can have a new set of critical exponents beyond Ginzburg-Landau symmetry-breaking theory, we consider a frustrated spin-1/2 model on square lattice: $H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$, where the coupling between the nearest neighbors is J_1 and between the sec-

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ond nearest neighbors is J_2. Previous works showed that for $0.4J_1 \leq J_2 \leq 0.65J_1$ there is a phase without long range magnetic order. Whether this phase is a valence bond solid, or a translational symmetric spin liquid is debated [20–26]. In this Letter we use $SU(2)$ slave-boson theory [27,28] to study the possible spin liquid phases of the above model. We take the large-N limit to control the quantum fluctuations. For large N, where the mean-field results are reliable, we find a continuous phase transition between the two spin liquids at $J_2/J_1 \approx 0.52$ [29,30]. However, for small N, strong quantum fluctuation may drive the system into other phases, such as the valence bond solid phase predicted by Schwinger boson approach [26].

The spin liquid phase for $J_2 < 0.52J_1$ is called $SU(2)$-linear spin liquid (or π-flux phase) [27,28,31] whose low energy effective field theory is a $SU(2)$ gauge theory couple to massless Dirac fermions [30]

$$L = \sum_{i=1}^{N} \bar{\psi}_{ai}(\partial_\mu - i a_\mu^l \tau_{ab}^l)\gamma_\mu \psi_{bi} + \frac{1}{4g^2} f_{\mu\nu}^l f_{\mu\nu}^l, \tag{1}$$

where ψ_{ai} is four-component Dirac fermion field, $a = 1, 2$, $i = 1, 2, \ldots, N$, and $N = 1$. γ^μ, $\mu = 0, 1, 2, 3, 5$ are Dirac matrices and the summation of μ runs through 0, 1, 2. τ^l, $l = 1, 2, 3$ are Pauli matrices and a_μ^l is $SU(2)$ gauge field.

The spin liquid phase for $J_2 > 0.52J_1$ is called chiral spin liquid [29] whose low energy effective field theory is a $SU(2)$ gauge theory couple to fermions with a chiral mass [29,30]

$$L = \sum_{i=1}^{N} \bar{\psi}_{ai}(\partial_\mu - i a_\mu^l \tau_{ab}^l)\gamma_\mu \psi_{bi} + m\bar{\psi}_{ai}(i\gamma^3\gamma^5)\psi_{ai} + \frac{1}{4g^2} f_{\mu\nu}^l f_{\mu\nu}^l. \tag{2}$$

The effective field theory that describes the transition connects (1) and (2) and is given by [30]

$$L = \sum_{i=1}^{N} \bar{\psi}_i(\partial_\mu - i a_\mu^l \tau^l)\gamma_\mu \psi_i + \frac{1}{4g^2} f_{\mu\nu}^l f_{\mu\nu}^l + \sigma\bar{\psi}[i\gamma_3\gamma_5]\psi + \frac{1}{2\rho^2}(\partial_\mu \sigma)^2 + V(\sigma), \tag{3}$$

where $V(\sigma) = V(-\sigma)$.

We note that the $SU(2)$-linear state does not break any symmetry and the chiral spin state breaks the time reversal and parity symmetry. The real σ field is the order parameter of the symmetry breaking. The potential $V(\sigma)$ controls the phase transition (see Fig. 1). $\sigma = 0$ gives rise to the $SU(2)$-linear state and $\sigma \neq 0$ gives rise to the symmetry-breaking chiral spin state. The order parameter σ is related to the following combination of physical spin operators: $\sigma \propto \boldsymbol{S_i} \cdot (\boldsymbol{S_{i+x}} \times \boldsymbol{S_{i+y}})$ [29]

The transition between the $SU(2)$-linear and the chiral spin state is a Z_2 symmetry-breaking transition. So we may

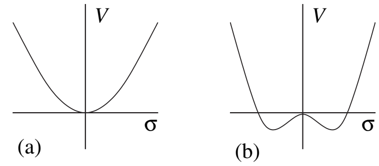

FIG. 1. The behavior of potential $V(\sigma)$ (a) before and (b) after the phase transition from the $SU(2)$-linear phase to the chiral spin phase.

expect the critical point to belong to the universality class of 3D Ising model. For 3D Ising model, the order parameter has scaling dimension $[\sigma]^{\text{Ising}} \approx 0.51$ (or a correlation $\langle \sigma(x)\sigma(0)\rangle = x^{-2[\sigma]^{\text{Ising}}}$) at the critical point. One may conclude that, at the transition point between the $SU(2)$-linear and the chiral spin state, the order parameter $\boldsymbol{W_i} = \boldsymbol{S_i} \cdot (\boldsymbol{S_{i+x}} \times \boldsymbol{S_{i+y}})$ also has the correlation $\langle \boldsymbol{W_i W_j}\rangle \propto |\boldsymbol{i} - \boldsymbol{j}|^{-2[\sigma]^{\text{Ising}}}$. In fact, the above guess is incorrect. For our case, even though the transition breaks a Z_2 symmetry and has a well-defined Z_2 order parameter, the critical point does not belong to the 3D Ising class.

Why does the $SU(2)$-linear state to the chiral spin state transition belong to a new universality class? The reason is that, at the critical point, not only the fluctuations of the order parameter σ give rise to gapless excitations, the fermion field ψ and the $SU(2)$ gauge field a_μ^l also give rise to gapless excitations. Had all gapless excitations come from the order parameter σ, then the transition would be belong to the 3D Ising class. So the key to understand the existence of the new critical point is to understand why ψ and a_μ^l can give rise to gapless excitations.

At first sight, one may expect that both ψ and a_μ^l are gapped due to their interaction. In fact, the self-energy term (see Fig. 2) from the $SU(2)$ gauge interaction, in general, can generate a fermion mass term $\delta m\bar{\psi}_i\psi_i$. Once the fermions are gapped, the $SU(2)$ gauge field is always in confined phase in $1 + 2$ dimensions and the gauge bosons are also gapped. So in order for the new critical point described by the gapless σ, ψ, and a_μ^l field to exist, we must find the reason that protects the gaplessness of ψ and a_μ^l.

To find such a reason, we like to point out that both the $SU(2)$-linear state and the chiral spin state contain nontrivial quantum or topological orders. Such quantum or topological orders are characterized by projective symmetry groups (PSG) which describe symmetry of the effective theory [19,31]. The PSGs of the $SU(2)$-linear state and the

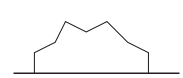

FIG. 2. The fermion self-energy due to the gauge interaction. The solid lines represent the fermion propagator and wiggled line gauge propagator.

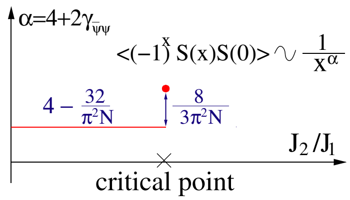

FIG. 3 (color online). Change of scaling dimension of staggered spin-spin correlation function during phase transition. For small J_2/J_1, $SU(2)$-linear phase has massless fermions and massless gauge bosons. At critical point, there is one extra massless σ boson mode. And for large J_2/J_1, chiral spin liquid phase has all modes gapped and only has short range correlation.

chiral spin state are studied in detail and will be described in a forthcoming paper [30]. We find that it is PSG that protects the gaplessness of ψ and a^l_μ [30–32]. In other words, if we regulate the effective field theory (3) in a way that does not break the symmetry described by the PSG, then the self-energy term in Fig. 2 cannot generate fermion mass [30]. With the gapless excitations from σ, ψ, and a^l_μ, the effective field theory (3) describes a new critical point at the transition between the $SU(2)$-linear state and the chiral spin state.

The scaling dimensions of operators are in general difficult to calculate. However, if we assume the number of the fermion fields N to be large (instead of $N = 1$), then they can be calculated systematically in $1/N$ expansion [30]. We find that the order parameter σ has a scaling dimension $[\sigma] = 1 + O(\frac{1}{N})$ at the new critical point, which is different from the scaling dimension for the 3D Ising universality class given by $[\sigma]^{\text{Ising}} = 0.51$. Near the transition, there is a diverging length scale $\xi \propto |t - t_c|^{-\nu}$ where t is a parameter (such as J_2/J_1) that controls the transition. For the new critical point, the coherent length exponent ν is found to be $\nu = 1 + O(\frac{1}{N})$, while for the 3D Ising universality class $\nu^{\text{Ising}} \approx 0.63$.

We can also calculate the staggered spin-spin correlations which are easier to measure. In $SU(2)$-linear phase and at the critical point, spins have algebraic correlations with different exponents. In the chiral spin phase, the spins have short ranged correlation (see Fig. 3).

The scaling dimension of staggered spin-spin correlation function $\langle(-)^x \mathbf{S}(\boldsymbol{x})\mathbf{S}(\mathbf{0})\rangle$ is calculated by the large-N expansion of quantum field theory. In our formalism, one can show that the staggered spin-spin correlation function is just the correlation function of the fermion mass operator $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi\psi(\mathbf{0})\rangle$ in the effective theory Eq. (3). By power counting, the scaling behavior should

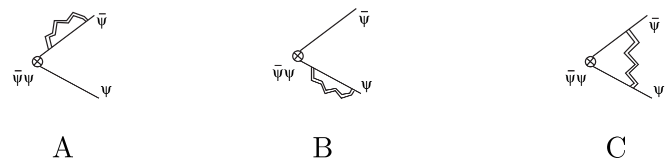

FIG. 5. Gauge dressed three-point correlation function at order of $\frac{1}{N}$.

be $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi\psi(\mathbf{0})\rangle = x^{-4}$, but quantum fluctuation changes it into $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi\psi(\mathbf{0})\rangle = x^{-4-2\gamma_{\bar\psi\psi}}$, where $\gamma_{\bar\psi\psi}$ is called the anomalous dimension of fermion mass operator.

In the following, we will use the spin correlation as an example to demonstrate how various correlations are calculated in the large N limit. It turned out that the easiest way of calculating $\gamma_{\bar\psi\psi}$ is not to calculate $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi\psi(\mathbf{0})\rangle$ directly, but to calculate the correlation function of fermion field ψ: $\langle\psi(\boldsymbol{x})\bar\psi(\mathbf{0})\rangle$, and the three-point correlation function $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi(\boldsymbol{y})\psi(\mathbf{0})\rangle$. Let us first calculate the staggered spin-spin correlation function in $SU(2)$-linear phase, where the low energy effective theory is Eq. (1). We will do our calculations in Landau gauge.

In the large-N limit, the gauge field is strongly screened by fermions. To the leading order of $\frac{1}{N}$, the dressed gauge propagator is shown in Fig. 4. The dressed gauge propagator in Landau gauge is found to be:

$$G^{ab}_{\mu\nu,dr}(k) = \frac{N\delta^{ab}}{16k}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right). \tag{4}$$

The fermion correlation to the first order in $\frac{1}{N}$, is given by (see Fig. 4):

$$S_{dr}(k) = \frac{-ik}{k^2}(1 + \Sigma) \tag{5}$$

$$\begin{aligned} \not{p}\Sigma &= i\int \frac{dq^3}{(2\pi)^3} \frac{\gamma_\mu(-i)(\not{k} + \not{q})\gamma_\nu 3/4}{(k+q)^2} \frac{\times 16}{Nq}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &= -\not{p}\frac{4}{3\pi^2 N}\log\left(\frac{k}{\Lambda}\right). \end{aligned} \tag{6}$$

Therefore, the anomalous dimension of ψ is $\gamma_\psi = -\frac{1}{2} \times \frac{4}{\pi^2 N}$.

Then we look at the dressed three-point correlation function $\langle\bar\psi\psi(\boldsymbol{x})\bar\psi(\boldsymbol{y})\psi(\mathbf{0})$ at order of $\frac{1}{N}$, as shown in Fig. 5. Suppose we fix the momentum of $\bar\psi\psi$ to be $2k$, while $\bar\psi$ and ψ each carry momentum k; then the tree level three-point correlation function will be $G_3(2k, k, k) = \frac{-i\not{k}}{k^2}\frac{-i(-\not{k})}{k^2} = \frac{1}{k^2}$. From the contributions of diagrams in

FIG. 4. Gauge dressed fermion propagator at first order of $\frac{1}{N}$, where the double wiggled line is the dressed gauge propagator in the leading order of large N limit.

FIG. 6. The contribution of σ-boson to fermion propagator at order of $\frac{1}{N}$, where the double dashed line is the dressed σ-boson propagator at leading order.

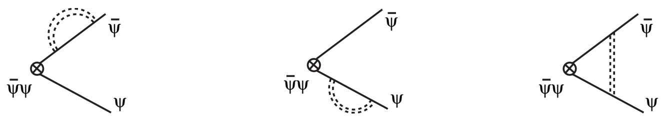

FIG. 7. Contributions of σ-boson to three-point correlation function at order of $\frac{1}{N}$.

Fig. 5, the dressed three-point correlation function is: $G_{3,dr}(2k, k, k) = \frac{1}{k^2}[1 + (A + B + C)\log(\frac{k}{\Lambda})]$, where A, B, C are the contribution from each corresponding diagram. Actually we know that $A + B + C = \gamma_{\bar{\psi}\psi} + 2\gamma_{\psi}$.

It is easy to see that A, B come from the dressed fermion propagator: $A = B = 2\gamma_{\psi}$. New calculations need to be done for vertex correction in C.

$$C \log\left(\frac{k}{\Lambda}\right) = \frac{3/4 \times 16}{N} \int \frac{dq^3}{(2\pi)^3} \frac{\gamma_\mu (\not{q} + \not{k})(\not{q} - \not{k})\gamma_\nu}{(q + k)^2 (q - k)^2 q} \times \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) = -\frac{12}{\pi^2 N} \log\left(\frac{k}{\Lambda}\right). \tag{7}$$

Thus $\gamma_{\bar{\psi}\psi} = A + B + C - 2\gamma_{\psi} = -\frac{16}{\pi^2 N}$.

We can also calculate the spin-spin correlation function at the critical point in a similar fashion. The only difference is that the σ boson becomes massless at critical point and contributes to the anomalous dimension of correlation functions. As shown in Figs. 6 and 7, after similar calculations, we found that at the critical point, $\gamma_{\bar{\psi}\psi} = -\frac{16}{\pi^2 N} + \frac{4}{3\pi^2 N}$, where the second term comes from contribution of massless σ boson.

In this Letter, we study a continuous quantum phase transition that breaks a Z_2 symmetry. Despite being a symmetry-breaking transition with well-defined order parameter, the transition is described by a new critical point which does not belong to the Ising universality class. The new critical point is due to the fact that the transition not only breaks the Z_2 symmetry, it also changes the topological or quantum order in the two phases across the transition. The additional gapless excitations protected by the PSG change the scaling behavior and the universality class of the critical point. So even a symmetry-breaking continuous transition may be described by a new critical point. Thus it is very important to measure critical exponents even for seemingly ordinary symmetry-breaking transition. A new critical point can be identified by measuring critical exponents and confirming the critical exponents to be different from those predicted by Ginzburg-Landau theory. Since a new critical point is usually due to a change in topological or quantum order at the transition, a discovery of new critical points usually implies a discovery of new states of matter with nontrivial topological or quantum orders on the two sides of the transition.

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