Interaction of Light Filaments Generated by Femtosecond Laser Pulses in Air

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The interaction of two light filaments propagating in air is simulated. Simulations show that the interaction of the two light filaments displays interesting features such as attraction, fusion, repulsion, and spiral propagation, depending on the relative phase shift and the crossing angle between them. A long and stable channel can be formed by fusing two in-phase light filaments. The channel becomes unstable with the increase of the crossing angle and phase shift. The interaction of two light filaments in different planes is studied and the spiral propagation is observed.

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The self-guided propagation of an ultrashort intense laser beam in air attracts great attention not only due to the fundamental process involved in this highly nonlinear phenomenon, but also due to the potential applications to lightning control and remote sensing [1,2]. If the peak power of the laser beam is many times higher than the self-focusing threshold, the laser beam breaks up into several filaments in space and several subpulses in time [3–13]. The mechanism of multiple-filament (MF) patterns was explained by the modulation instability of the laser beam [14] and the optical turbulence model [8]. Recent studies show that the MF pattern is induced by interplay between stable filaments formed by the fluctuations of a laser beam and small-scale light filaments nucleated randomly [7,9]. In addition, the impacts of the initial fluctuations of the laser beam on the interaction dynamics of filaments have been discussed in Ref. [10]. The MF pattern induced by small perturbations of a laser beam cannot be predicted. However, the MF pattern can be controlled by imposing strong field gradients or phase distortions in the laser beam profile [11–13]. As seen above, the previous studies are mainly about the influence of the initial beam on the evolution of filaments. In fact, the evolution of MF patterns can be regarded as the propagation of a group of interacting "light bullets" (small-scale light filaments); thus, the study of the interaction dynamics of light bullets is critical in understanding the characteristics of the propagation of an ultrashort laser beam in air.

In this Letter we present simulations on the interaction between two light bullets using a (3D + 1) model, rather than the (2D + 1) model in Refs. [7,9] in order to illustrate the temporal dynamics of the interaction. The effects of their relative phase shift, crossing angle, and the initial position on the bullets interaction are considered. The attraction, fusion, repulsion, and collision are observed in the simulations. The stability of the channel formed by two interacting light bullets strongly depends on the relative phase shift and the crossing angle between the two light bullets.

The propagation of a femtosecond laser beam in air can be described by an extended nonlinear Schrödinger (NLS) equation coupled with electron density of plasma due to multiphoton ionization. The NLS equation describes a slowly varying envelope of a linearly polarized laser electric field in the frame moving with the laser beam. The coupled equations can be written as

$$2i\frac{\partial E}{\partial z} + \frac{1}{k_0}\Delta_\perp E - k''\frac{\partial^2 E}{\partial t^2} + k_0 n_2 |E|^2 E - k_0\frac{\omega_{pe}^2(\rho)}{\omega_0^2}E + i\beta^{(K)}|E|^{2K-2}E = 0, \quad (1)$$

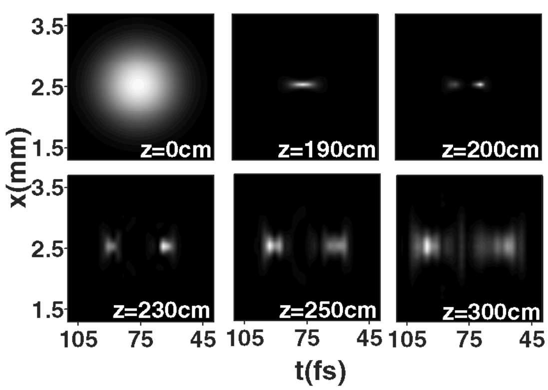
$$\frac{\partial \rho}{\partial t} = \frac{\beta^{(K)}}{K\hbar\omega_0}|E|^{2K}\left(1 - \frac{\rho}{\rho_{at}}\right), \quad (2)$$

where z refers to the propagation distance, $k_0 = 2\pi/\lambda_0$ is the central wave number, and $\lambda_0 = 800$ nm is the central wavelength of the laser beam. Here the Laplacian operator $\Delta_\perp$ describes the beam transverse diffraction, and the remaining terms account for the group velocity dispersion with the coefficient of $k'' = 0.2\ \mathrm{fs}^2/\mathrm{cm}$, the self-focusing effect due to the Kerr response of air with the nonlinear index of refraction of $n_2 = 3.2 \times 10^{-19}\ \mathrm{cm}^2/\mathrm{W}$, the defocusing effect resulting from the multiphoton ionization with the coefficient of $\beta^{(K)} \simeq 1.27 \times 10^{-126}\ \mathrm{cm}^{17}/\mathrm{W}^9$ for the number of photons $K = 10$ [5]. The plasma frequency is $\omega_{pe} = (q_e^2\rho/m_e\varepsilon_0)^{1/2}$ (q_e, m_e, and ρ are the electron charge, mass, and density, respectively), and the density of neutral atoms is $\rho_{\mathrm{at}} = 2.7 \times 10^{19}\ \mathrm{cm}^{-3}$ in Eq. (2).

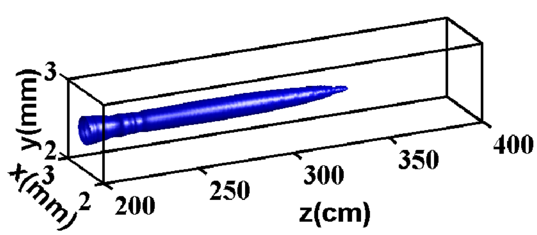
We use a Gaussian pulse with a peak power slightly higher than the self-focusing threshold to generate light bullets. The envelope of the pulse can be written as $A(x, y, z, t)|_{z=0} = A_0 \exp(-(x^2 + y^2)/r_0^2 - t^2/\tau_0^2)$, the energy of the pulse is $E_{\mathrm{in}} = 0.2$ mJ. The peak power of the pulse is $P_{\mathrm{in}} = 6.3GW \approx 2P_{\mathrm{cr}}$, where P_{cr} is the self-focusing threshold. The pulse exhibits a transverse waist (FWHM) $w_0 = \sqrt{2\ln 2}r_0 = 1$ mm and a temporal duration (FWHM) $\Delta T = \sqrt{2\ln 2}\tau_0 = 30$ fs and is collimated along the z direction. Figure 1 shows the spatiotemporal intensity distributions of the laser pulse at $y = 0$. From Fig. 1(a) we can see that the pulse self-focuses at the distance

$z = 190$ cm, and then breaks into two light bullets at $z = 200$ cm. Here one bullet exhibits a 4 fs temporal duration and a 57 μm transverse waist, and the other exhibits a 5.3 fs temporal duration and a 100 μm transverse waist. The peak intensities of the two light bullets are 2.05×10^{13} W/cm^2 (left) and 5.75×10^{13} W/cm^2 (right), respectively; and the electron density generated by multiphoton ionization is 4.9×10^{14} cm^{-3} in the channel. Figure 1(b) shows the isosurface of the energy fluence of the laser pulse in the region from 200 cm to 400 cm. The fluence here is normalized by the on-axis fluence at $z = 0$ cm. The channel, formed by two light bullets, can stably propagate over 1 m distance, covering several tens of Rayleigh length.

We now study the interaction of two parallel light bullets. The envelope of each light bullet is taken from the one with higher peak intensity at $z = 200$ cm in Fig. 1. First, we study the effect of the relative phase shift between two light bullets on the interaction process. Therefore, the beam profile can be written as $A(x, y, z, t)|_{z=0} = A_b(x - \frac{a}{2}, y, t)\exp(i\phi) + A_b(x + \frac{a}{2}, y, t)$. Here, A_b is the envelope of the more intense (right) light bullet at $z = 200$ cm in Fig. 1(a), a represents the initial distance between light bullets and ϕ refers to the phase shift. Figure 2 illustrates the interaction of two parallel bullets with a separation $a = 0.5$ mm and different phase shifts, $\phi = k\pi$, $k = 0$ [Fig. 2(a) and 2(b)], 0.25 [Fig. 2(c)], 0.5 [Fig. 2(d)], 0.75 [Fig. 2(e)], 1 [Fig. 2(f)]. It is shown that the phase shift plays an important role in the interaction of light bullets. The mechanism is similar to the interaction of solitons [9,15]. When the two light bullets are in phase, they firstly disperse and evacuate energy to the background reservoir. During this process, the peak intensities of the two light bullets decrease. This reduces the energy loss caused by the multiphoton ionization, which is proportional to I^K. Because the two light bullets interfere constructively, the intensity in the overlapping region becomes larger. This leads to an increase of the refractive index in the center due to the Kerr effect and the background energy is attracted towards the center resulting in a new light bullet, as shown in Fig. 2(a). It is shown that the two in-phase light bullets attract each other and fuse into one channel. The fusion process prolongs the propagating distance and enhances the channel stability compared with that of a single light bullet. From Fig. 2(b), we can see that the channel formed by the two in-phase light bullets can extend over a distance more than 2 m. In order to compare with the stability of a channel formed by a single bullet with more energy, we



(a)



(b)

FIG. 1 (color online). (a) The spatiotemporal intensity distribution formed by an input beam with a 0.2 mJ energy. (b) The energy fluence distribution (fluence$_{\rm iso}$ = 1.65) of the laser beam with the propagation distance z under the same condition as (a).

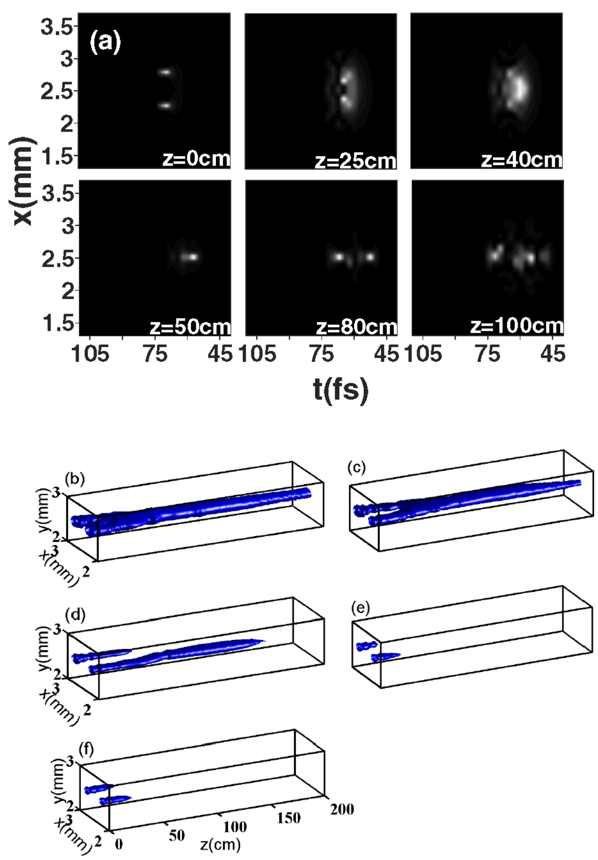


FIG. 2 (color online). (a) The spatiotemporal intensity distribution of two interacting bullets which are in parallel and in-phase. (b), (c), (d), (e), (f) The energy fluence distribution (fluence$_{\rm iso}$ = 1.65) of two parallel bullets with a separating distance $a = 0.5$ mm and relative phase shift $\phi = k\pi$, $k = 0$ (b); $k = 0.25$ (c); $k = 0.5$ (d); $k = 0.75$ (e); $k = 1$ (f).

double the energy of the single light bullet. The increased intensity breaks the balance between the Kerr self-focusing and plasma defocusing processes and results in a larger energy loss through the multiphoton ionization. Therefore, the formed channel by a single light bullet with more energy is less stable than a channel formed by the fusion of two light bullets. When the two light bullets are out of phase, the destructive interference leads to the reduction of the intensity of each light bullet. The intensity in the overlapping region is always lower. This results in the decrease of refractive index in the center. Thus, more light energy disperses outside. Two out-of-phase light bullets appear to repulse each other and disperse quickly, as shown in Figs. 2(e) and 2(f). Under such a circumstance, the channel stability becomes much lower than that of a single light bullet. If the phase shift is $0 < \phi < 0.5\pi$, two light bullets also attract and fuse, but the stability of the formed channel becomes lower with the increase of phase shift, as shown in Figs. 2(c) and 2(d). The interaction mechanisms are more complicated. The interference between two light bullets causes the asymmetric intensity distribution. This leads to the asymmetric distribution of refractive index, which causes the energy transfer from one light bullet to the other. The attraction of two laser beams in underdense plasmas has been observed before [16,17]. The attraction there is not the Kerr effect but the combined effect of the relativistic effect and the ponderomotive force. As a result, we can conclude that in order to form a long stable channel, the phase shift between two light bullets should be as small as possible. The situation may be different in the case of multiple light bullets in the transverse plane, but the mechanism is the same. From this point of view, the MF pattern can be controlled by use of a tilting lens [13], i.e., by changing the phase shift between the light bullets.

If the power of a femtosecond laser beam is high enough, the filamentation can start before the geometric focus, and the light bullets interact on the geometric focus with a crossing angle. To investigate the effect of the crossing angle on the light bullets' interaction, we model the interaction of two light bullets with different incident angles for both cases of in-phase and out-of-phase. Here, the input pulse envelope can be written as $A(x, y, z, t)|_{z=0} = A_b(x - \frac{a}{2}, y, t) \exp(i(x - \frac{a}{2}) \tan\theta) \times \exp(i\phi) + A_b(x + \frac{a}{2}, y, t) \exp(-i(x + \frac{a}{2}) \tan\theta)$, where $a = 0.5$ mm represents the separating distance, ϕ refers to the phase shift between two light bullets and θ represents the incident angle. Simulations are performed, for two light bullets with a small ($\theta = 0.01°$) and a large ($\theta = 0.1°$) incident angles. For the small incident angle $\theta = 0.01°$, the two light bullets fuse into a long stable channel in Fig. 3(a), similar to the parallel light bullets, as shown in Fig. 2(b). Figure 3(b) shows the channel evolution with the large incident angle of $\theta = 0.1°$. In this case, after converging, two in-phase light bullets go through each other and disperse quickly. Figs. 3(c) and 3(d) shows the interaction of two out-of-phase light bullets with the small ($\theta = 0.01°$) and the large ($\theta = 0.1°$) incident angles, respectively. For the small incident angle, the interaction is similar to the case of parallel light bullets. Although the large incident angle makes the two out-of-phase light bullets move close, they repulse each other and then diverge. It seems that the two out-of-phase light bullets collide with each other, as shown in Fig. 3(d). The light bullets will disappear quickly after the collision. This situation is caused by the quick change of electron density in the interaction zone, due to the interference of the laser field. The balance between the Kerr self-focusing and plasma defocusing processes is then broken. Comparing the case of small incident angle with that of large one, we suggest that a longer channel can be formed by using a lens of long focal length which can form the light bullets with a small crossing angle (2θ), as proved by the experiments [18].

If the initial propagating direction of two light bullets is not in one plane, their interaction shows very different features. In this case, the input beam profile can be written as $A(x,y,z,t)|_{z=0} = A_b(x-\frac{a}{2}, y-\frac{b}{2}, t)\exp(i(x-\frac{a}{2})\tan\theta) \times \exp(i\phi) + A_b(x+\frac{a}{2}, y+\frac{b}{2}, t)\exp((-i(x+\frac{a}{2})\tan\theta)$. Here, a

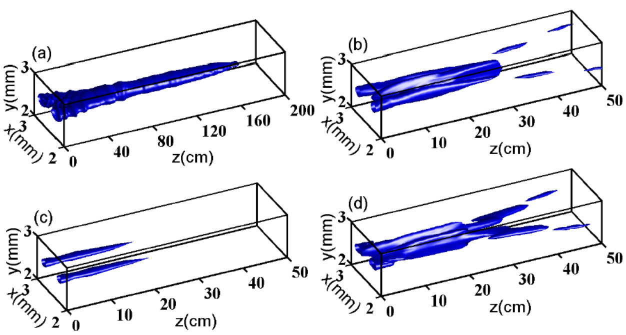


FIG. 3 (color online). The energy fluence distribution of two interacting bullets with a separating distance $a = 0.5$ mm, individual incident angle θ, and relative phase shift ϕ; (a) $\theta = 0.01°$ and $\phi = 0$, (b) $\theta = 0.1°$ and $\phi = 0$, (c) $\theta = 0.01°$ and $\phi = \pi$, (d) $\theta = 0.1°$ and $\phi = \pi$.

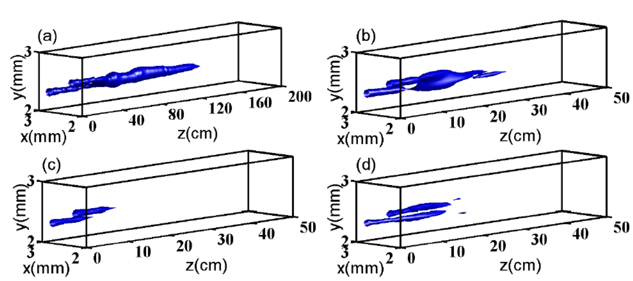


FIG. 4 (color online). The energy fluence distribution ($\text{fluence}_{\text{iso}} = 2.99$) of two bullets with a separating distance $a = 0.5$ mm in the x direction and $b = 0.2$ mm in the y direction, and incident angle θ and relative phase shift ϕ; (a) $\theta = 0.01°$ and $\phi = 0$, (b) $\theta = 0.1°$ and $\phi = 0$, (c) $\theta = 0.01°$ and $\phi = \pi$, (d) $\theta = 0.1°$ and $\phi = \pi$.

and b refer to the separating distance in the x and y directions, respectively. Figure 4 shows the light bullets' interaction when $a = 0.5$ mm, $b = 0.2$ mm, $\theta = 0.01°$, $\phi = 0$ [Fig. 4(a)], $\theta = 0.1°$, $\phi = 0$ [Fig. 4(b)], $\theta = 0.01°$, $\phi = \pi$ [Fig. 4(c)], $\theta = 0.1°$, $\phi = \pi$ [Fig. 4(d)]. When the incident angle is small, and two light bullets are in phase [Fig. 4(a)], the dominating process is still fusion. A long stable channel is formed. When the incident angle is large, we can see the evident spiral propagation of two light bullets [Fig. 4(b)]. This is an interesting phenomenon which was observed in soliton interaction before [15]; nevertheless, the stability of the channel is much lower than the case of a small crossing angle. Figures 4(c) and 4(d) show that the repulsion of two out-of-phase light bullets, with the small (0.01°) and the large (0.1°) incident angles.

It should be noted that in our simulations we have neglected the Raman nonlinearity, i.e., delayed Kerr response of air ($\frac{1}{\tau_k}\int_{-\infty}^{t}\exp(-(t-t')/\tau_k)|E(t')|^2 dt'$) [5,7,9]. Because the duration of a single light bullet is much shorter than the characteristic time of Raman nonlinearity, the interaction process of the light bullets cannot be significantly influenced by including the Raman nonlinearity.

In conclusion, the interaction of two light bullets in air is investigated numerically by solving (3D + 1) NLS. The simulations are performed for different phase shifts and crossing angles between the two light bullets. For different phase shifts, the interaction causes attraction, fusion, or repulsion between the two light bullets. The simulation results can be used to explain the mechanism hidden in earlier experiments on controlling multiple filamentation of femtosecond laser pulses in air. It is shown that two crossing light bullets cannot form a stable channel when the crossing angle is large. Two in-phase light bullets in different planes are found to be spiral during the propagation.

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