Spatiotemporal Synchronization of Coupled Oscillators in a Laboratory Plasma

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The spatiotemporal synchronization between two plasma instabilities of autonomous glow discharge tubes is observed experimentally. For this purpose, two tubes are placed separately and two chaotic waves interact with each other through a coupler. When the coupling strength is changed, the coupled oscillators exhibit synchronization in time and space. This is the first experimental evidence of spatiotemporal synchronization by mutual chaotic wave interaction in plasma.

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The synchronization of two chaotic oscillators [1,2] has attracted considerable attention in many branches of science [3–7], influenced by the possibility of the widespread applications of coupled nonlinear oscillators. It is well known that two chaotic oscillators can synchronize through a coupling interaction. The behavior of coupled nonlinear oscillators is a phenomenon of interest in plasma physics and some branches of science. Thus far, although only the temporal appearance has been analyzed, spatiotemporal structure has recently attracted considerable attention [8–11]. However, spatiotemporal synchronization is as of yet rarely observed in nature. It has recently been reported that spatiotemporal synchronization is observed in laser experiments. The possibility of observing the synchronization of spatiotemporal chaos in other nonlinear systems has also been mentioned [11]. Plasma, which often appears in nature, is a typical nonlinear dynamical system with large degrees of spatiotemporal freedom; it is of interest as a medium for testing the universal characteristics of chaos. In spatially extended systems such as plasma and fluids, an understanding in space and time is currently required. We believe that the investigation of spatiotemporal synchronization in plasma will have a novel influence on nonlinear physics.

A schematic of the experimental setup of two coupled oscillators is shown in Fig. 1. Experiments are performed using a glass tube with diameter and length of 2 and 75 cm, respectively. After evacuating the tube to high vacuum, neon gas is introduced into each tube at a pressure of 4.78 mb. When a high dc voltage is applied to the electrodes, plasma is produced by a glow discharge between the electrodes, and ionization waves [12] propagate from the cathode to the anode. These waves are characterized by being backward waves because the direction of phase velocity is opposite to that of group velocity. The typical ion and electron temperatures are approximately 0.025 and 10 eV, respectively. A current source is used in order to adjust the discharge current. Here, discharge currents in tubes 1 and 2 are fixed at 24.3 and 24.1 mA, respectively. The two spatiotemporal oscillators, i.e., plasma tubes, are operated under almost identical environments. The two tubes are operated independently, and ionization waves are self-excited due to plasma instability. The two unstable waves interact with each other through the coupling of a variable resistor. Here, the value of the resistor is a control parameter for deciding the coupling strength in coupled oscillators. Using photodiodes, the time series signals for the analysis are obtained as fluctuations in the light intensity, and they are sampled using a digital oscilloscope. In this study, the system of ionization waves generated in a glow discharge is appropriate for investigating the spatiotemporal structure; this is because it is easy to measure the structure of this system by using a CCD camera. The spatiotemporal data are obtained as fluctuations in the light intensity by using the camera, and these data are sampled using a computer.

When the values described above are selected as discharge current and gas pressure, the system exhibits chaotic oscillation in time and space. Dynamical behaviors of

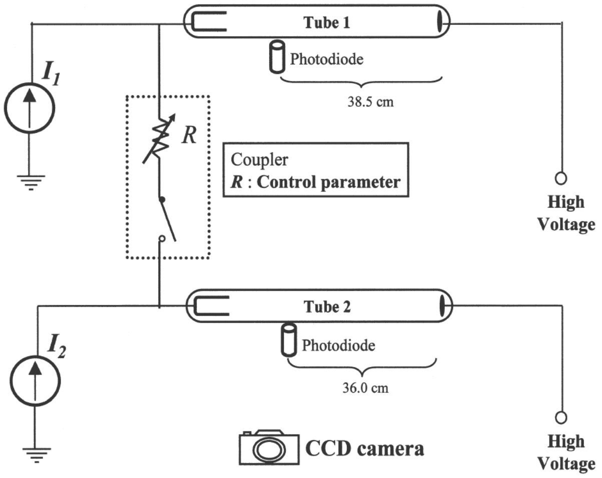

FIG. 1. Schematic of the experimental setup of two coupled oscillators. The pressure in each tube is 4.78 mb, and the discharge currents in tubes 1 and 2 are 24.3 and 24.1 mA, respectively.

ionization waves in a glow discharge have been investigated extensively and chaotic behaviors have been reported [13]. Figure 2 shows the time series signals (wave 1 sampled from tube 1; wave 2 sampled from tube 2) and the Wave 1 Wave 2 plot, with respect to two cases: (a) before coupling and (b) in the process of coupling. Here, the values of the resistor, which is the control parameter of the coupler, is 51.8 kΩ. Figure 2(a) shows that two chaotic waves oscillate independently and they are uncorrelated. Figure 2(b) shows that the phases of two oscillators are locked. Figure 3 shows the phase difference $|\Phi_1 - \Phi_2|$ of two oscillators (a) before coupling, (b) in the process of coupling with the values at 100 and 82.3 kΩ resistors, and (c) in the process of coupling with the value of the resistor at 51.8 kΩ. Figure 3(a) shows that the phase difference $|\Phi_1 - \Phi_2|$ increases with time. Thus, two waves do not synchronize before coupling. Figure 3(b) also shows that the phase difference $|\Phi_1 - \Phi_2|$ increases with time. The two waves do not synchronize for these values of the resistor. However, the slope of the line becomes smooth when the value of the resistor is decreased; the strength of coupling, in particular, increases. Figure 3(c) shows that the phase difference $|\Phi_1 - \Phi_2|$ of two coupled oscillators practically remains at zero with time. Thus, the phases of the coupled oscillators synchronize in the process of coupling; in other words, "phase synchronization" [14,15] occurs. Phase synchronization between a plasma discharge and external circuit has been reported [16]. However, to the best of our knowledge in nonlinear and plasma physics, phase synchronization in mutual coupling of two chaotic waves in plasma has not been observed thus far. Therefore, in this study, we claim that phase synchronization of coupled chaotic oscillators in plasma is observed for the first time. The correlation between the amplitudes of wave 1 and wave 2 also increases in the process of coupling. However, it does not reach a completely correlated state although the phases are completely locked. The coupled systems exhibit highly coherent oscillations when phase synchronization occurs.

Figure 4 shows the time series signals as the fluctuating components of the currents in the coupler as a function of the control parameter R, which is the value of the resistor in the coupler. The power spectra corresponding to before and after the threshold are shown simultaneously; these are constructed on the basis of the fluctuating components of the currents in the coupler. When the value of the resistor decreases, the coupling strength increases and phase synchronization occurs in a certain range. This figure shows the time series signals around the phase synchronization threshold. The fluctuating components of the currents in the coupler oscillate chaotically, and the power spectrum exhibits a very broad peak before the threshold. On the other hand, under synchronization, the fluctuating components of the currents in the coupler oscillate in a somewhat stable manner; after the threshold, sharp peaks in spectra are observed. Moreover, the fluctuating components of the

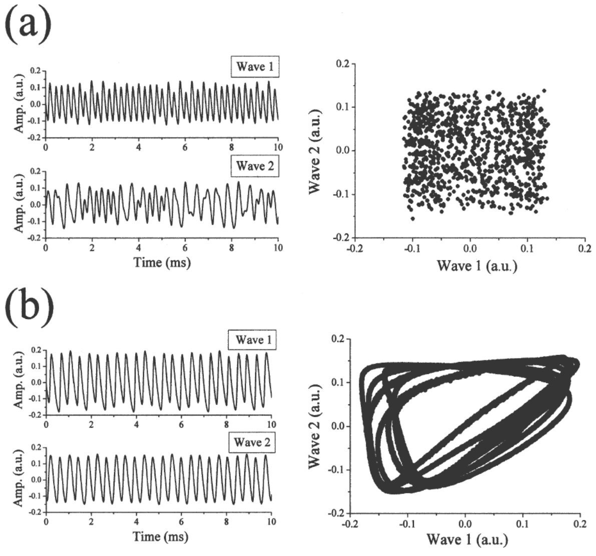

FIG. 2. Time series signals (wave 1 sampled from tube 1; wave 2 sampled from tube 2) and the Wave 1 Wave 2 plot: (a) before coupling and (b) in the process of coupling. The value of the resistor, which is the control parameter of the coupler, is 51.8 kΩ. Here, the time series signals for analysis are obtained as fluctuations in the light intensity using photodiodes.

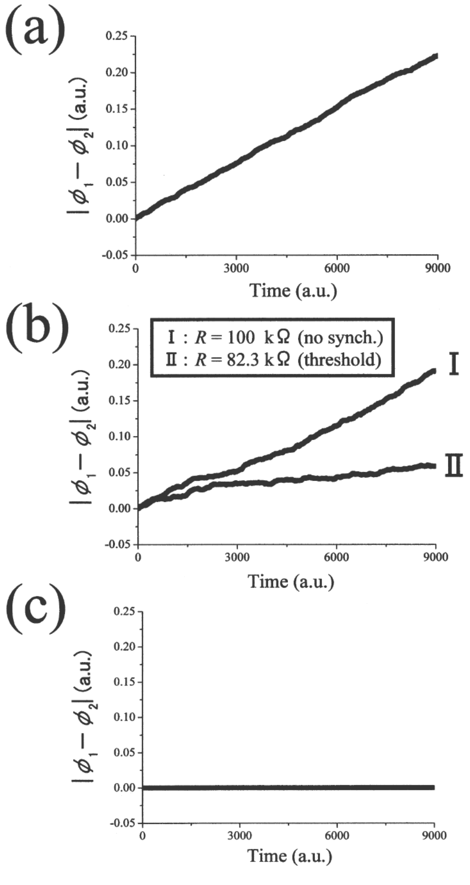

FIG. 3. Phase difference $|\Phi_1 - \Phi_2|$ of two oscillators: (a) before coupling, (b) in the process of coupling with the values of the resistors at 100 and 82.3 kΩ, and (c) in the process of coupling with the value of the resistor at 51.8 kΩ.

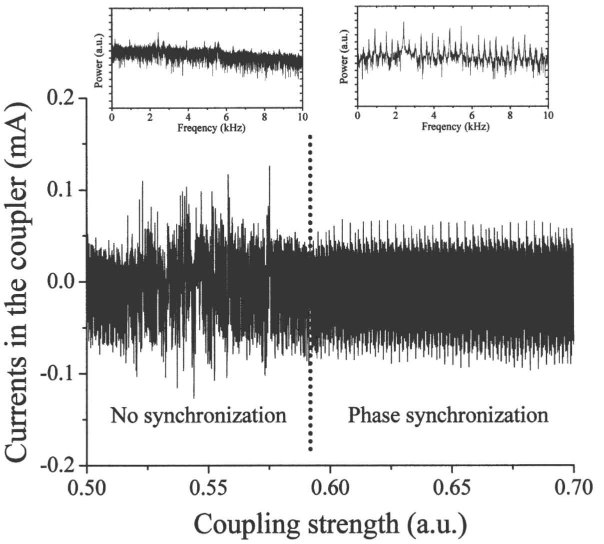

FIG. 4. Time series signals as the fluctuating components of currents in the coupler as a function of the control parameter R, which is the value of the resistor in the coupler, the power spectra corresponding to before and after the threshold are shown simultaneously; these are constructed by the fluctuating components of currents in the coupler. The time series signals around the phase synchronization threshold are shown.

currents in the coupler are less than 0.5% (± 0.1 mA) of the discharge current ($\sim$24 mA) of each plasma. It is concluded that two waves can couple and interact as a result of the exchange of fairly low current, thus indicating the nonlinearly of coupled oscillators.

Figures 5(a) and 5(b) show the spatiotemporal appearance of two oscillators in space and time (a) before coupling and (b) in the process of coupling, respectively. This figure corresponds to the spatially extended states shown in Figs. 2(a) (before coupling) and 2(b) (in the process of coupling), respectively. The spatial spectra are simultaneously shown on the basis of the spatiotemporal appearance. The left and right traces correspond to wave 1 and wave 2, respectively. Here, the time series signals are obtained every 38.68 μs and the spatial data are obtained as the fluctuations in the light intensity of 100 positions spaced equally in each tube, using a CCD camera. Thus, the spatiotemporal appearances are constructed. Before coupling, the spatiotemporal appearances of each oscillator exhibit spatiotemporal chaos and their oscillations are chaotic and uncorrelated over space. They are not synchronized over time or space. When phase synchronization occurs with time through the coupling of two chaotic oscillators, coupled oscillators are also correlated over space according to the spatial spectra. The peaks of each system at 5.96 and 9.78 cm increase and become very sharp. Coupled systems exhibit coherent oscillations. They are synchronized over time and space; i.e., spatiotemporal synchronization is achieved in coupled oscillators. The spatiotemporal oscillations of the coupled systems change from a chaotic state to a rather periodic one. The possibility that spatiotemporal synchronization in coupled oscillators can lead to the spatiotemporal control of chaos [17–21] will be investigated in future studies.

Spatial reconnection [22] is observed in spatiotemporal appearances, as shown in Fig. 5. Chaotic reconnection is observed before coupling, as shown in Fig. 5(a). Spatiotemporal chaos can be caused by this reconnection. On the other hand, chaotic reconnection observed without synchronization transforms to a somewhat regular one when spatiotemporal synchronization occurs. As shown

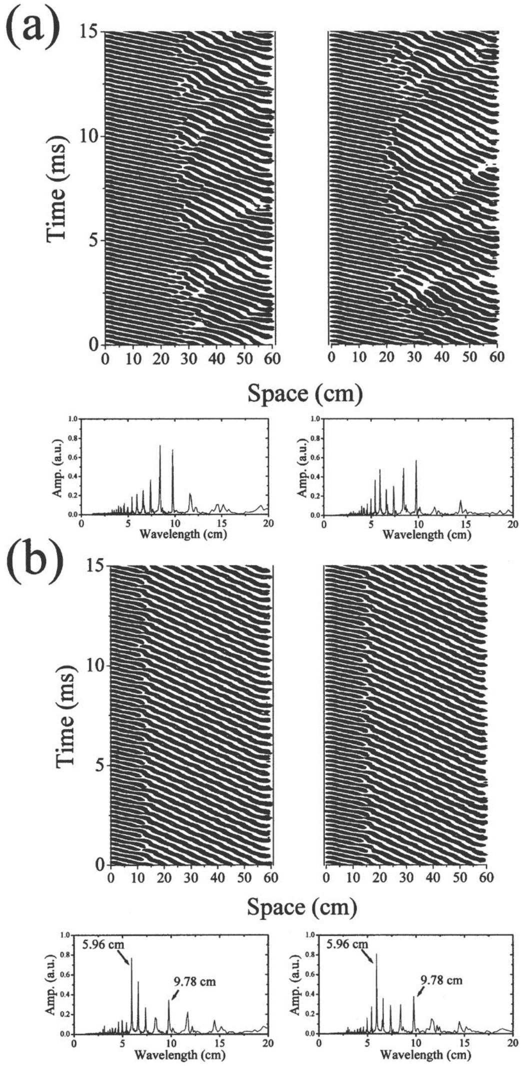

FIG. 5. The spatiotemporal appearances of two oscillators in space and time (a) before coupling and (b) in the process of coupling. They correspond to spatially extended states shown in Figs. 2(a) (before coupling) and 2(b) (in the process of coupling), respectively. The spatial spectra are simultaneously shown. The left and right traces correspond to wave 1 and wave 2, respectively. Here, the time series signals are obtained every 38.68 μs and the spatial data are obtained as the fluctuations in the light intensity of 100 positions spaced equally in each tube, using CCD camera. Thus, the spatiotemporal appearances are constructed.

in Fig. 5(b), the frequencies before and after the reconnection are 2.42 and 3.94 kHz, respectively. Their approximate ratio is 3:5. Furthermore, it is observed that the phases between the two spatial structures slip around the position of the reconnection. Thus, it is natural that spatial reconnection affects spatiotemporal pattern formation in plasma. It may be considered that such a wave reconnection in plasma has a good analogy with the dynamics in liquid crystals, chemical waves, and so on [23,24].

In summary, the dynamical behavior of coupled oscillators between the two instabilities of autonomous discharge tubes in a glow discharge is studied. When the value of the resistor is changed varying the coupling strength, the coupled oscillators exhibit phase synchronization. The fluctuating components of the currents in the coupler oscillate chaotically, and their power spectrum exhibits broad peaks before reaching the phase synchronization threshold. On the other hand, under synchronization, the fluctuating components of the currents in the coupler oscillate in a somewhat stable manner; after the threshold, sharp peaks are observed. Before coupling, the spatiotemporal appearances of each oscillator reveal spatiotemporal chaos and their oscillations are chaotic and uncorrelated over space as well. When phase synchronization occurs with time, the coupled oscillators are synchronized over time and space; i.e., spatiotemporal synchronization is achieved in coupled oscillators. The spatiotemporal oscillations of the coupled systems change from a chaotic state to a rather periodic one. Moreover, spatial reconnection, which exhibits a frequency ratio of 3:5, is observed during spatiotemporal synchronization.

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