Realization of a Minimal Disturbance Quantum Measurement

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We report the first experimental realization of an "optimal" quantum device able to perform a minimal disturbance measurement on polarization encoded qubits saturating the theoretical boundary established between the classical knowledge acquired of any input state, i.e., a "classical guess," and the fidelity of the same state after disturbance due to measurement. The device has been physically realized by means of a linear optical qubit manipulation, postselection measurement, and a classical feed-forward process.

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The measurement process represents the most innovative and distinctive aspect of quantum mechanics with respect to classical physics. The main result of the quantum measurement theory is the unavoidable disturbance of the quantum state induced by the measuring process itself, as epitomized by the early Heisenberg x-ray microscope thought experiment [1]. The balance between the information available on an unknown quantum system and the perturbation induced by the measurement process is of utmost relevance when investigating the quantum world [2–5]. In spite of this relevance, only in the last years and in the context of quantum information (QI) for finite dimensional systems, an exact quantum theoretical formulation of this problem has been developed [6]. When measuring an unknown quantum system $|\phi\rangle$ two main questions arise: (A) How good is the estimation of the state obtained by the measuring process? (B) How close is the final state to the input one? Adopting the tools developed within QI, the previous questions can be answered by introducing suitable quantitative figures of merit to assess the classical information acquired on the state and the resemblance of the final quantum system to the initial one [7]. The classical guess G attained by applying a state estimation strategy is defined as the mean overlap between the unknown state $|\phi\rangle$ and the state inferred from the measurement ρ_G: $G = \langle\phi|\rho_G|\phi\rangle$ while the closeness of the output quantum state ρ_F to the input one is expressed by the quantum fidelity $F = \langle\phi|\rho_F|\phi\rangle$ [Fig. 1(a)]. The final problem is then to establish which kind of relation connects these two quantities. The higher the information achieved, the higher the disturbance applied, and vice versa. For instance, to carry out an optimal state estimation strategy on $|\phi\rangle$ we should perform a von Neuman measurement [8], i.e., a "strong disturbance" one, thus leading to the maximal modification of the initial state. In this case the output quantum fidelity is identical to the classical one. This represents an extreme point of the $F - G$ boundary. On the contrary, if we want to maintain unchanged the quantum state, i.e., $F = 1$, we cannot obtain any information about it. This point defines the other extreme of the $F - G$ plot.

In the present work we consider the basic element of quantum information, the qubit, which is encoded in a two-dimensional quantum system and represents the quantum analogue of the classical bit. Let us start from the situation in which no *a priori* information is available on the qubit; i.e., this one belongs to the "*universal*" set of input states $|\phi_{\rm univ}\rangle = \alpha|0\rangle + \beta|1\rangle$ with any $\alpha, \beta \in C^2$, $|\alpha|^2 + |\beta|^2 = 1$. The optimal trade-off condition between $G_{\rm univ}$ and $F_{\rm univ}$ was found by Banaszek [6] and reads

$$F_{\rm univ} \le \frac{2}{3} + \frac{\sqrt{1-(6G_{\rm univ}-3)^2}}{3}. \tag{1}$$

The two extreme situations outlined above correspond to the points ($F_{\rm univ} = 2/3$, $G_{\rm univ} = 2/3$) and ($F_{\rm univ} = 1$, $G_{\rm univ} = 1/2$). When partial *a priori* information on the qubit to be measured is available, a better guess of the state can be attained introducing at the same time less disturbance on the system. Within this framework, a particular simple case is represented by the set of states called *phase qubits,* for which the information is encoded in the phase φ_i of the input qubit represented by any point on any equatorial plane i of the corresponding Bloch sphere, i.e., $|\phi_{\rm cov}\rangle = 2^{-1/2}(|\Psi_+\rangle + e^{i\varphi_i}|\Psi_-\rangle)$ for a convenient orthonormal basis $\{|\Psi_+\rangle, |\Psi_-\rangle\}$. For $|\Psi_\pm\rangle = 2^{-1/2}(|0\rangle \pm i|1\rangle)$ we have $|\phi_{\rm cov}\rangle = \cos\gamma|0\rangle + \sin\gamma|1\rangle$. The *phase qubits* are adopted in most of the quantum key distribution cryptographic protocols [9], and the trade-off between phase estimation and disturbance limited fidelity lies at the basis of the security assessment problem. In this simpler case the quantum bound reads [10]

$$F_{\rm cov} \le \frac{3}{4} + \frac{\sqrt{1-(4G_{\rm cov}-2)^2}}{4}, \tag{2}$$

while the two extreme situations correspond to the points ($F_{\rm cov} = 3/4, G_{\rm cov} = 3/4$) and ($F_{\rm cov} = 1, G_{\rm cov} = 1/2$).

Let us now describe the procedure which saturates the quantum mechanical bounds, that is, performs the minimal disturbance measurement (MDM) protocol in our work: Fig. 1(b). The main idea underlying the physical apparatus is to exploit suitable interaction of the input qubit with an *ancilla qubit,* i.e., a *probe,* and subsequently measure the ancilla to extract information about the system that we want to guess. By varying the ancilla readout, we are able to tune the strength of the measurement on the input qubit ranging from the maximum extraction of achievable classical information, i.e., leading to maximum state disturbance, to no collection of classical information leaving the input qubit completely unchanged. Let us use the ancilla P to be prepared in the state $2^{-1/2}(|0\rangle_P + |1\rangle_P)$ and the input qubit S in the generic: $|\phi_{\rm univ}\rangle_S = \alpha|0\rangle_S + \beta|1\rangle_S$. The interaction between input qubit and probe is achieved by verifying the ''parity'' of the two qubits when they are expressed in the computational basis $|0\rangle, |1\rangle$. Such *parity check operation* entangles the two qubits when they are in a superposition state of the basis vectors. To perform this inspection, we apply the mutual orthogonal projectors $E_0 = [|0\rangle|0\rangle\langle 0|\langle 0| + |1\rangle|1\rangle\langle 1|\langle 1|]$, commonly referred as *parity check* operator [11], and $E_1 = I - E_0$ where I is the identity operator. After successful implementation of the E_i projection with probability equal to $1/2$ independently from the input state $|\phi\rangle$, the overall output state reads $|\Phi_i^{\rm out}\rangle_{SP} = 2^{-1/2}(\alpha|0\rangle_S|i\rangle_P + \beta|1\rangle_S|i \oplus 1\rangle_P)$ where the symbol $\oplus$ denotes the sum operation modulo 2. Let us consider the case in which E_0 is realized. The ancilla is measured in the rotated basis $\{|G_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, |G_1\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle\}$. The parameter θ determines the strength of the measurement. The value $\theta = 0$ corresponds to optimal state estimation process, i.e., maximum G, while for $\theta = \frac{\pi}{4}$ the input qubit is left unchanged, $F = 1$. If the measurement is successful for the ancillary state $|G_0\rangle$ (or $|G_1\rangle$) then the input qubit is guessed to be in the state $|0\rangle$ (or $|1\rangle$). To complete the protocol, a unitary operator is applied on the qubit S depending from the measurement outcome on the probe. In particular, if the state $|G_1\rangle$ is detected the operation σ_Z is applied, that is, $|0\rangle \to |0\rangle$ and $|1\rangle \to -|1\rangle$, while no operation is applied when the state $|G_0\rangle$ has been measured. A similar procedure is applied when E_1 is successful. In this case, however, the role of the states $|G_0\rangle, |G_1\rangle$ must be inverted; that is, $|G_0\rangle$ ($|G_1\rangle$) corresponds to $|1\rangle$ ($|0\rangle$) and the σ_Z is triggered by a click of the $|G_0\rangle$ detector. In summary, after the projection, the measurement of the probe and the feed-forward, the output qubit density matrix ρ_F is achieved by tracing over the probe Hilbert space and is found in the state $\rho_F = |\phi_{G_i}\rangle\langle\phi_{G_i}| + |\phi_{G_{i\oplus 1}}\rangle\langle\phi_{G_{i\oplus 1}}|$ where $|\phi_{G_i}\rangle = \alpha\cos\theta|0\rangle + \beta\sin\theta|1\rangle$ and $|\phi_{G_{i\oplus 1}}\rangle = \alpha\sin\theta|0\rangle + \beta\cos\theta|1\rangle$. At the same time the input state is guessed to be in the state $\rho_G = p_{G_i}|0\rangle\langle 0| + p_{G_{i\oplus 1}}|1\rangle\langle 1|$ where $p_{G_i} = |\alpha|^2\cos^2\theta + |\beta|^2\sin^2\theta$ and $p_{G_{i\oplus 1}} = |\alpha|^2\sin^2\theta + |\beta|^2\cos^2\theta$. From the previous results we obtain the *state-dependent* quantum fidelity and the classical guess as a function of the parameters α, β: $F_\phi = 1 - 2|\alpha|^2|\beta|^2(1 - \sin 2\theta)$ and $G_\phi = \langle\phi|\rho_G|\phi\rangle = \frac{1}{2} + \frac{\cos 2\theta}{2}(1 - 4|\alpha|^2|\beta|^2)$. By averaging the classical guess and the output fidelity over the ensemble of possible input qubit states, we obtain $G_{\rm univ} = [3 + \cos(2\theta)]/6$ and $F_{\rm univ} = [2 + \sin(2\theta)]/3$, which saturate the inequality given by Eq. (1). Interestingly, the previous scheme can also be applied to input *phase qubit* states belonging to the equatorial plane of the Bloch sphere, characterized by real value of the parameters $\{\alpha, \beta\}$. In this case the average classical guess and output fidelity are, respectively, $F_{\rm cov} = [3 + \sin(2\theta)]/4$ and $G_{\rm cov} = [2 + \cos(2\theta)]/4$ that satisfy the inequality given by Eq. (2).

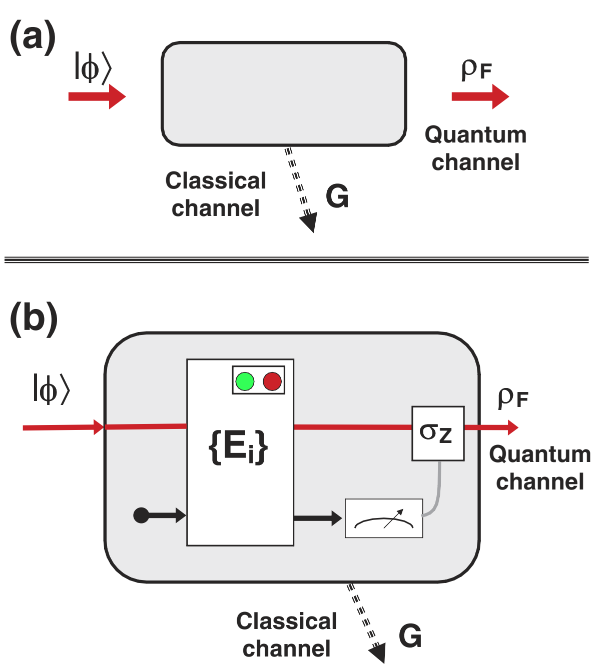

FIG. 1 (color online). (a) Diagram of a minimal disturbance measurement (MDM) performed on a single qubit in the state $|\phi\rangle$. The device provides an output state ρ_F with fidelity F and a ''classical guess'' G. (b) Realization of a MDM by the projector $\{E_i\}$, the measurement of the probe qubit, and the classical feed-forward σ_Z.

Let us now turn our attention to the actual implementation of the protocol for qubits encoded in the polarization state of a single photon by adopting the isomorphism $|0\rangle \equiv |H\rangle, |1\rangle \equiv |V\rangle$ where $|H\rangle, |V\rangle$ denote the horizontal and vertical polarizations, respectively. In order to carry out the projective operations we have exploited the interference of the two photons, the input qubit to be measured and the ancilla, at the layer of a polarizing beam splitter (PBS), PBS_M in Fig. 2. PBS transmits the horizontal polarization and reflects the vertical one; thus when injecting the PBS with a single photon for each input mode, the successful implementation of the ''parity check'' E_0 operator corresponds to the emission of one photon for each output mode. Indeed, this event implies that photons are simultaneously both transmitted or reflected when exhibiting the same

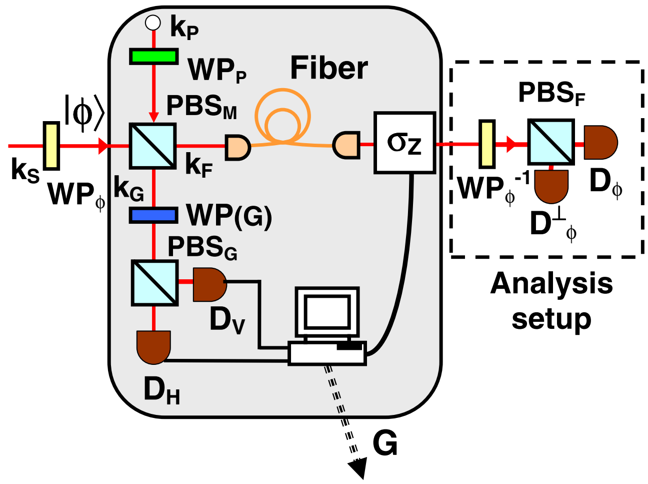

FIG. 2 (color online). Optical setup implementing the MDM. The output is characterized adopting the analysis setup illustrated in the dashed box.

parity. The signature of a success event is the detection of a single photon in the probe output. Actually the occurrence of the E_0 operator was experimentally identified by detecting a photon on each output mode. Such requirement is not a significant limitation since most of the linear optics quantum information protocols with polarization encoded qubits ends up in photon number measurements of all involved modes [12]. On the contrary, the implementation of the projector E_1 is associated with the emission of a two-photon state in one of the two output modes. In this case, since the two photons are indistinguishable, the signal and the probe qubits cannot be correctly addressed, and these events are then discarded. The experimental MDM device works, therefore, with probability $p = 1/2$. Nevertheless, this probabilistic feature does not spoil the main physical result of the present procedure since the trade-off conditions are not altered by any probabilistic procedure [13,14].

In the present experiment two photons with equal wavelength $\lambda = 795$ nm and with a coherence time $\tau_{\text{coh}} = 600$ fs were generated in a nonentangled state on the modes k_S and k_P (Fig. 2) by spontaneous parametric down conversion in a type I β-barium-borate crystal in the initial polarization product state $|H\rangle_S|H\rangle_P$ [15]. The input qubit was codified on the mode k_S into the *polarization* state $|\phi\rangle_S = \alpha|H\rangle_S + \beta|V\rangle_S$ by means of a half and a quarter wave plate (WP_ϕ), whereas the ancilla qubit was polarization encoded in the state $2^{-1/2}(|H\rangle_P + |V\rangle_P)$ adopting the half wave plate WP_P. The photons S and P were then injected on the two input modes of the polarizing beam splitter PBS_M with an adjustable mutual temporal delay Δt. The condition $\Delta t = 0$ has been identified observing the bunching of two input photons in the states $|H\rangle_S$ and $|V\rangle_P$ over the same spatial and temporal output mode. By this method we ensured the optimal temporal overlap of the two-photon wave packets at the PBS layer and hence maximized their mutual interference.

The mode k_F corresponds to the output quantum channel of the MDM device, while the photon belonging on mode k_G enters the classical measurement apparatus adopted to infer the classical guess G. This estimation task is realized by means of a tunable half wave plate WP(G), a polarizing beam splitter PBS_G, and two detectors D_H, D_V. The angular position of WP(G), $\vartheta_G = \theta/2$, determines the strength of the measurement. The complete protocol implies a classical feed-forward on the polarization state of the photon belonging to the mode k_S depending on which detector (D_H or D_V) is fired: precisely if the detector D_V clicks a σ_Z Pauli operation is applied, in the other case no transformation is implemented on the quantum channel. To carry out the σ_Z transformation, we adopted a fast $LiNbO_3$ Pockels cell (PC) electronically driven by a transistor array activated by a click of detector D_V. The σ_Z transformation was implemented by applying to the PC a $\lambda/2$ voltage, i.e., leading to a $\lambda/2$ induced phase shift of the $|V\rangle$ polarization component. Details on the electronic circuit piloting the electro-optic Pockels cell can be found in Ref. [16]. In order to synchronize the active window of the Pockel cell with the output qubit, the photon over the mode k_F was delayed through propagation over a 30 m long single mode optical fiber. The polarization state on the mode k_F after the propagation through the system fiber + PC was analyzed by the combination of the wave plate WP_ϕ^{-1} and of the polarization beam splitter PBS_F. For each input polarization state $|\phi\rangle_S$, WP_ϕ^{-1} was set in order to make PBS_F transmit $|\phi\rangle$ and reflect $|\phi^\perp\rangle$.

Two different experiments have been carried out. In the first one the device has been characterized for a universal set of input qubits, in the second one for a covariant set. To demonstrate the realization of the MDM apparatus, it is sufficient to use a finite set of nonorthogonal quantum states from mutually maximally complementary bases. For the universal MDM, we have adopted the three maximally complementary bases $|H\rangle$, $|V\rangle$, $|L_\pm\rangle = 2^{-1/2}(|H\rangle \pm |V\rangle)$ and $|C_\pm\rangle = 2^{-1/2}(|H\rangle \pm i|V\rangle)$, whereas for phase covariant MDM we employed the $|H\rangle$, $|V\rangle$, and $|L_\pm\rangle$ bases only. Such sets of states are adopted in the conventional quantum cryptographic protocols [9]. For each state $|\phi\rangle_S$, the corresponding values of F_ϕ and G_ϕ were measured for different ϑ_G settings. This task was achieved by collecting the twofold coincidences between the two sets of detectors $\{D_H, D_V\}$ and $\{D_\phi, D_\phi^\perp\}$ and then extracting the joint probabilities of the two-photon states $p_{H\phi}$, $p_{V\phi}$, $p_{H\phi\perp}$, $p_{V\phi\perp}$ where p_{ij} is the relative frequency of the coincidence count $D_i - D_j$. The fidelity of the output state ρ_{out} can be evaluated as $F_\phi = \langle\phi|\rho_{\text{out}}|\phi\rangle = p_{H\phi} + p_{V\phi}$. To extract the value G_ϕ, we first calculate the occurrence probability P_i $\{i = H, V\}$ of the measurement $|i\rangle\langle i|$, as $P_i = p_{i\phi} + p_{i\phi\perp}$. In this case the input state is guessed to be in the quantum state $|i\rangle$ leading to a fidelity $|\langle\phi|i\rangle|^2$. Hence for each state $|\phi\rangle$ the resulting estimation fidelity is obtained as

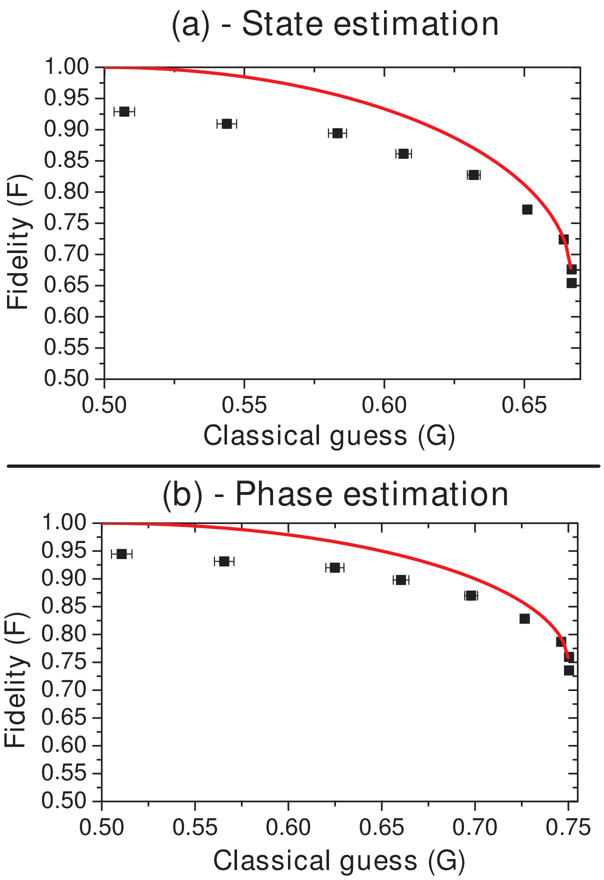

FIG. 3 (color online). (a) Experimental data of the quantum fidelity F versus the classical guess G for an arbitrary input qubit. The fidelities have been averaged over the six states $\{|H\rangle, |V\rangle, |L_\pm\rangle, |C_\pm\rangle\}$; solid line: optimal trade-off between F_{univ} and G_{univ} [Eq. (1)]. (b) Experimental data of the quantum fidelity F versus the classical guess G for an equatorial input qubit. The fidelities have been averaged over the four states $\{|H\rangle, |V\rangle, |L_\pm\rangle\}$; solid line: optimal trade-off between F_{cov} and G_{cov} [Eq. (2)].

$G_\phi = \sum_i P_i |\langle\phi|i\rangle|^2$. The mean quantum fidelities and classical guesses were averaged over all the input states. The experimental data are reported in Fig. 3. For the universal MDM the extreme experimental points are ($G^{\text{exp}}_{\text{univ}} = 0.666 \pm 0.001$; $F^{\text{exp}}_{\text{univ}} = 0.654 \pm 0.004$) and ($0.507 \pm 0.004$; 0.929 ± 0.002), corresponding to the settings $\vartheta_G = 0°$ and $\vartheta_G = 22.5°$. These figures are to be compared with the theoretical limits: ($G^{\text{th}}_{\text{univ}} = 0.666$; $F^{\text{th}}_{\text{univ}} = 0.666$) and (0.5; 1). Likewise, for the phase covariant MDM the extremal experimental points are ($G^{\text{exp}}_{\text{cov}} = 0.750 \pm 0.001$; $F^{\text{exp}}_{\text{cov}} = 0.735 \pm 0.004$) and ($0.511 \pm 0.006$; 0.945 ± 0.003) to be compared with the theoretical ones: ($G^{\text{th}}_{\text{univ}} = 0.75$; $F^{\text{th}}_{\text{univ}} = 0.75$) and ($G^{\text{th}}_{\text{univ}} = 0.5$; $F^{\text{th}}_{\text{univ}} = 1$). The discrepancies between the theoretical and experimental curves are mainly due to not perfect interference visibility at the PBS, which partially spoil the "parity check" operation. The deviation equal to ~7% between the theoretical value $F^{\text{th}}_{\text{univ}} = 1$ and the experimental one 0.93 can be attributed to the PBS (3% due to a nonvanishing reflectivity for the H polarization), decoherence in optical fiber propagation and classical feed-forward (2%), spatial matching of the overlapping modes (2%). A simple analysis leads to the consideration that the value of ($F^{\text{th}} - F^{\text{exp}}$) decreases for a lower value of F^{th}.

In summary, we realized conditional implementation of minimal disturbance measurement saturating the quantum mechanical $F - G$ trade-off, both for universal and for phase covariant sets of states. The present procedure can be adopted for different qubit hardware and can have interesting applications in the framework of quantum communication to improve the transmission fidelity of a lossy quantum channel [17].

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