Evidence of Holes in the Arnold Tongues of Flow Past Two Oscillating Cylinders

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The wake of two oscillating cylinders in a tandem arrangement is a nonlinear system that displays Arnold tongues. We show by numerical simulations that their geometry depends on the phase difference θ between the two oscillating cylinders. At $\theta = 0$ there may be holes inside these intraresonance regions unlike the solid Arnold tongues encountered in single-cylinder oscillations. This implies that, surprisingly, self-excitation of the system may be suppressed inside these holes, at conditions close to its natural frequency.

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Flow past two cylinders in a tandem arrangement represents an idealization of unsteady flow-structure interactions occurring in large arrays of cylinders encountered in diverse applications that include heat exchangers, tall buildings in proximity, and piles of offshore risers [1]. From the dynamical systems point of view, it represents two coupled periodic oscillators, with flow past each cylinder separately giving rise to a limit cycle corresponding to the classical von Karman vortex street. It is well established both experimentally and theoretically [1] that subjecting a *single* cylinder to oscillations transverse to the flow direction leads to the formation of "locked-in" regions within which the cylinder wake oscillates with a single frequency. For this to occur, the excitation frequency should be relatively close to the shedding frequency. Plotted in the amplitude-versus-frequency bifurcation diagram, these locked-in regions are termed Arnold tongues or resonance horns.

The question of the existence of Arnold tongues in the two-cylinder system was first raised in the work of Mahir and Rockwell [2]. They showed that, indeed, it is possible to attain a locked-in response over a range of excitation frequency wider than for a corresponding single cylinder. Furthermore, they showed that for closely spaced cylinders it is possible to attain a regime sensitive to the phase angle between the cylinders' oscillations. A fundamental question concerns the *shape* of the geometry of Arnold tongues in coupled oscillators subject to resonant excitation. In the current work we focus on this question of the "intraresonance region," and we employ numerical simulation to obtain detailed information on the experimental setup of Mahir and Rockwell.

Specifically, we consider two cylinders placed parallel to the flow with a center-to-center spacing equal to 2.5 diam ($P/D = 2.5$). The Reynolds number is $\mathrm{Re} = 160$ based on the free-stream velocity U_∞ and the cylinder diameter D. The selected spacing corresponds to flow in the *reattachment regime* for the stationary system at this Reynolds number and represents the strongest coupling between the two cylinders [3,4]. We have also performed simulations at $P/D = 3.5$, corresponding to a weaker coupling. The two cylinders oscillate with the same amplitude (A) and frequency (f_e); the amplitude is nondimensionalized by the cylinder diameter (D), and all frequencies by (U_∞/D). Also, the excitation frequency is normalized by the frequency of the corresponding unforced system. For the tandem cylinder system the latter is denoted f_0^{**}; we computed $f_0^{**} = 0.128$ for $P/D = 2.5$ and $f_0^{**} = 0.124$ for $P/D = 3.5$, both at $\mathrm{Re} = 160$. For the single cylinder we denote the natural frequency as f_0^{**}; we computed $f_0^{**} = 0.188$ at $\mathrm{Re} = 160$. We allow the two cylinders to oscillate with a phase difference θ between them, and we study in-phase ($\theta = 0$) and out-of-phase ($\theta = \pi$) oscillations of the two cylinders, following the experimental work of Mahir and Rockwell.

We solve the viscous incompressible Navier-Stokes equations using the spectral/hp element method [5]. Flexibility in the geometric discretization is achieved using an unstructured mesh in conjunction with an arbitrary Lagrangian Eulerian (ALE) scheme to accommodate motion of the moving grid. We ran a relatively large number (more than 175) of simulations in order to cover the parameter space of interest; parallel computing was employed to reduce simulation time. For all the cases studied we used the free-stream velocity U_∞ as the initial condition for the simulation. This ensures that there are no hysteresis effects due to continuation from another established flow state. The cylinder motion starts from zero deflection and velocity, and it smoothly reaches a stationary state within a few shedding periods. For every run, the time history of the velocity components and pressure was obtained at a number of downstream points. Each case is classified by examining the spectra of a point in the near wake of the downstream cylinder; in the gap region there is no vortex shedding at spacing $P/D = 2.5$ [2]. We calculate the spectra only after careful elimination of the transient part of the time signal; the total simulation time for each case was over 50 shedding cycles. A locked-in state of vortex formation exists when the same pattern of vortices consistently occurs at the same phase of oscillation of the cylinder(s). The velocity fluctuation in the near wake is dominated by the frequency of locked-in vortex formation, which coincides with the cylinder oscillation frequency (f_e). Additional peaks can occur at superharmonics of f_e. Phase portraits based on the streamwise (u) and crossflow (v) components of the velocity were also inspected to identify limit cycles.

We summarize our findings in the bifurcation diagrams of Figs. 1(a)–1(c), where the same point in the wake ($x/D = 5$; $y/D = 0.5$) was used to gather time-histories and analyze the results presented here [the $(0, 0)$ point is at the first cylinder center]. The different symbols in Figs. 1(a)–1(c) show all representative cases in the bifurcation diagram. In transition states (gray circles) the spectra are dominated by peaks at the excitation frequency f_e, but other small peaks may have *just* emerged resulting in a small modulation in the phase portrait, thus deviating from the tight limit cycle we obtain for the locked-in responses. The diagrams corresponding to each of the examined values of the phase angle θ are plotted first in Figs. 1(a) and 1(b) with the single cylinder on the bottom [Fig. 1(c)] for comparison. The diagram of Fig. 1(a) for $\theta = 0$ exhibits interesting structure and deviates substantially from the traditional Arnold tongue shape; the latter is solid and widens up with increasing values of the oscillation amplitude (A/D), as shown in the single-cylinder diagram of Fig. 1(c). A surprising result here is that there is a range of frequency ratios f_e/f_0^{**} between 1.25 and 1.5 for which the wake is locked-in for small amplitudes but loses synchronization at higher amplitudes—the latter region is enclosed by a dash line. Another rather unexpected result in the diagram of Fig. 1(a) is that as we increase the excitation frequency for a fixed oscillation amplitude (e.g., $A/D = 0.35$), locked-in states switch to quasiperiodic shedding and subsequently the wake returns to a locked-in state or an ‘‘almost’’ locked-in state again.

To investigate this transition path further, we plot phase portraits based on the two velocity components in Fig. 2(a)–2(e) for selected frequencies corresponding to $A/D = 0.35$ for which the aforementioned behavior is observed. The first case [Fig. 2(a)] corresponds to $f_e = 0.125$; ($f_e/f_0^{**} = 0.976$) and is clearly a locked-in case with peaks at the excitation frequency and its integer multiples (superharmonics). For $f_e = 0.157$; ($f_e/f_0^{**} = 1.226$) [Fig. 2(b)], on the other hand, there exists quasiperiodic shedding with several peaks present in the spectrum. The three most prominent of the peaks identified for this case correspond to f_e, $f_e/2$, and $3f_e/4$. Further increase of the excitation frequency to $f_e = 0.172$; ($f_e/f_0^{**} = 1.344$) [Fig. 2(c)] takes the wake back to a locked-in state with prominent peaks at $f_e, 2f_e, \ldots$. For

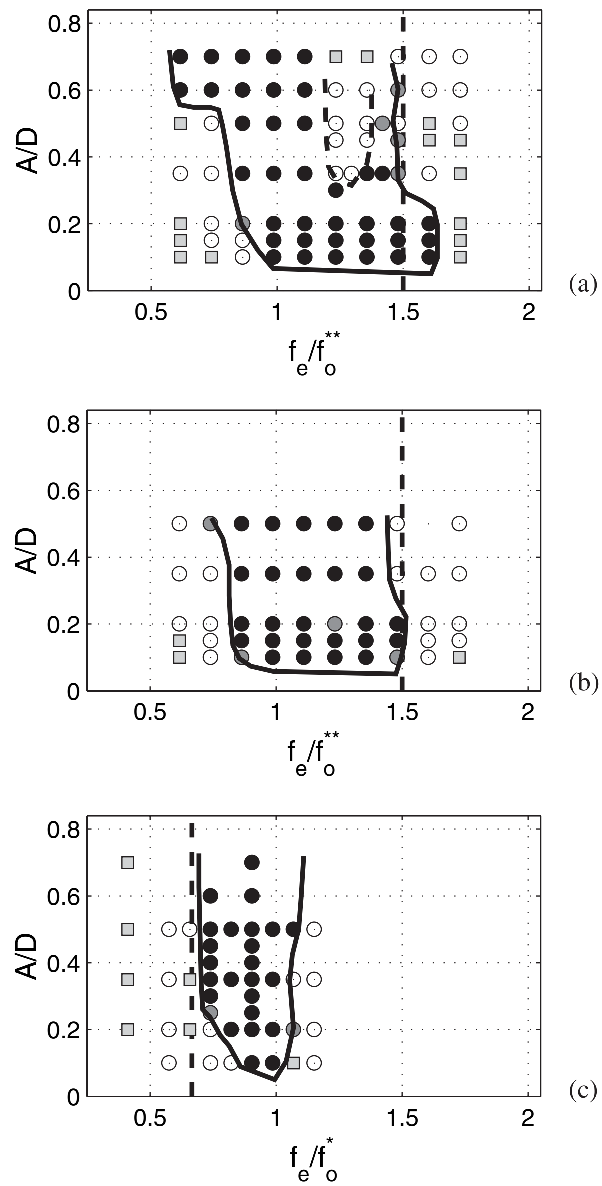

FIG. 1. Bifurcation diagrams plotted as normalized excitation amplitude versus normalized excitation frequency. Black circles indicate locked-in while white circles indicate quasiperiodic shedding. Squares correspond to period doubling and the gray circles correspond to transitional states. ($f_0^* = 0.188$; $f_0^{**} = 0.128$; their normalized values are denoted by the vertical dash line). (a) two cylinders, $\theta = 0$ degrees; (b) two cylinders, $\theta = 180°$; (c) single cylinder.

$f_e = 0.188$; ($f_e/f_0^{**} = 1.469$) [Fig. 2(d)], apart from the expected prominent peaks at f_e and $2f_e$, we identify peaks that are smaller by an order of magnitude but sufficiently large to cause some modulations. Finally, for $f_e = 0.220$; ($f_e/f_0^{**} = 1.719$) [Fig. 2(e)], the spectrum includes peaks at integer multiples of $f_e/2$, i.e., $\{f_e/2, f_e, 3f_e/2, 2f_e \ldots\}$.

To visualize the flow patterns at $\theta = 0$ corresponding to the aforementioned representative states, snapshots of the vorticity field for the corresponding cases are plotted in

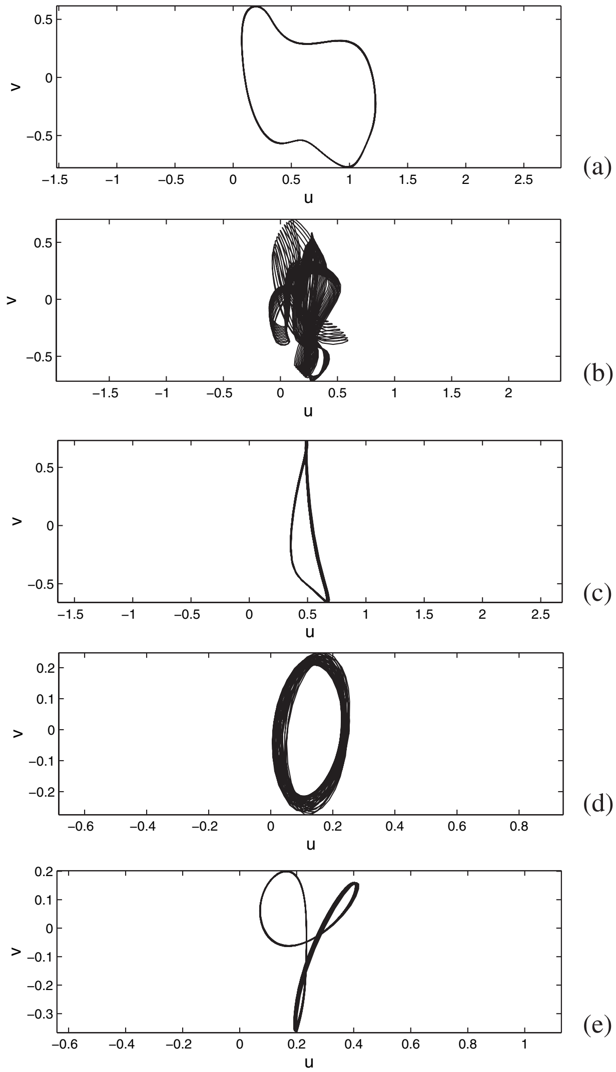

FIG. 2. Phase portraits showing the crossflow velocity versus the streamwise velocity in the near wake of the downstream cylinder for $A/D = 0.35$ and $\theta = 0.0$. (a) $f_e = 0.125$, locked-in; (b) $f_e = 0.157$, quasiperiodic; (c) $f_e = 0.172$, locked-in; (d) $f_e = 0.188$, transitional; (e) $f_e = 0.220$, period doubling.

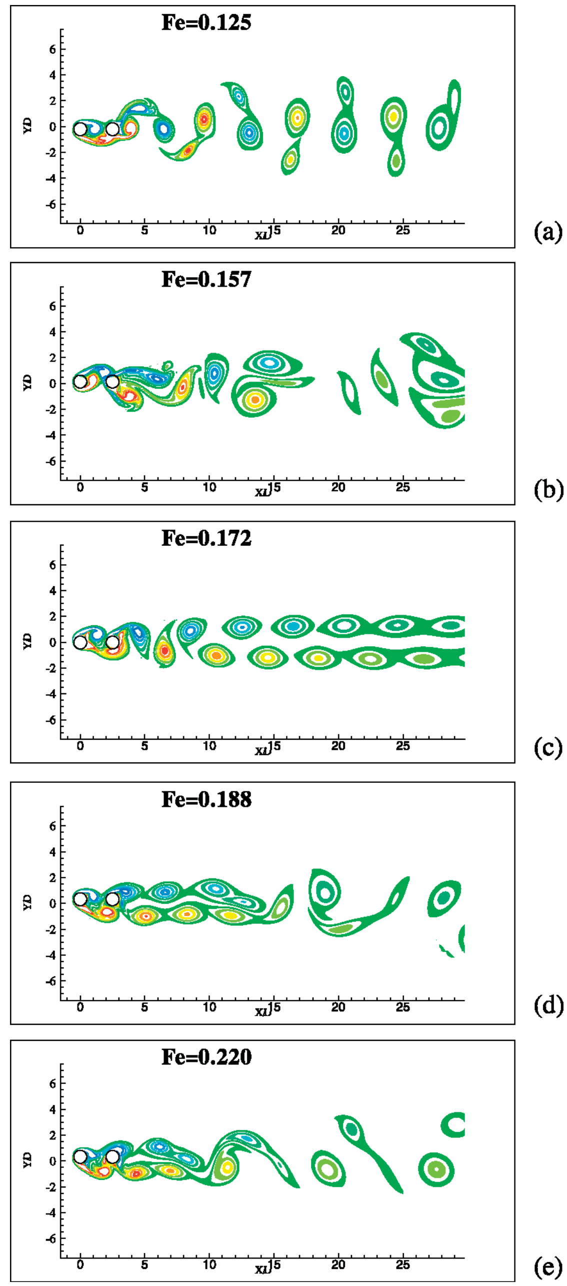

FIG. 3 (color). Vorticity fields corresponding to different frequencies for $A/D = 0.35$ and $\theta = 0.0$. Plots (a)–(e) correspond to the ones in Fig. 2.

Figs. 3(a)–3(e). For $f_e = 0.125$ [Fig. 3(a)] the wake exhibits a well organized symmetric ($S + P$) structure characterized by three shed vortices per cycle [6]. The vorticity field of the case $f_e = 0.157$ [Fig. 3(b)] shows a rather unorganized wake consistent with the findings of the spectral analysis. For excitation frequency $f_e = 0.172$ [Fig. 3(c)] the wake has a well organized $2S$ structure. The point for $f_e = 0.188$ [Fig. 3(d)] was marked "gray" in the bifurcation diagram, and its corresponding vorticity field shows an organized $2S$ vortex street, which collapses at a downstream distance relatively close to the cylinders. This is consistent with the peaks present in the corresponding spectrum. The organized near wake is typical of the locked-in case and leads to the single peak at f_e. The reorganization, however, of the vortex street downstream gives rise to the smaller peaks present in the spectra at lower frequencies. At $f_e = 0.220$ [Fig. 3(e)] the near-wake vortices are so strong and so (horizontally) close to each other that like-signed vortices merge relatively close to the cylinders, and an organized but wider wake emerges downstream. This merging leads to larger but slower vortices, thus generating frequency peaks at multiples of $f_e/2$. In contrast to these results obtained for $\theta = 0$, in the antiphase ($\theta = \pi$) case there is only mild narrowing from the high frequency border of the synchronization region, see Fig. 1(b). We also note the widening of the synchronization region compared to the single-cylinder case, especially toward higher frequencies. These findings are in accord with the experimental results of Mahir and Rockwell, who reported that at $\theta = 0$ ''modulated or locked-in response can exist intermittently'' [2], unlike the cases for large values of θ.

In order to investigate the stability of the flow states associated with the apparent holes in the Arnold tongue we performed simulations with perturbations introduced in the prescribed cylinders' motion given by $y(t) = A\cos[\omega_e t + \epsilon\omega_e \sin(q\omega_e t)]$, where $\omega_e = 2\pi f_e$. In particular, we considered the case $f_e = 0.157$ and $A/D = 0.35$ [see Figs. 2(b) and 3(b)] and set $\epsilon = 0.1$ and $q = 20$. The perturbation caused a strong modulation in the velocity and acceleration of the cylinders and we observed larger values in the enstrophy and fluctuating lift compared to the unperturbed case. However, there were only small differences in the vortex street which maintained the same structure as the unperturbed vortex street of Fig. 3(b). Spectral analysis showed that the same peaks (as in the unperturbed case) are present, suggesting that the examined state is stable under the perturbations we considered. We also introduced similar perturbations to locked-in states and for the single-cylinder system and we found stability of those states as well. In another set of simulations we examined the effect of cylinder coupling for the in-phase case and set the spacing at $P/D = 3.5$ for which we have observed weaker coupling compared to the case with $P/D = 2.5$; we only simulated cases for $A/D = 0.35$ in the frequency range from $f_e = 0.0783$ to $f_e = 0.220$ (a total of 11 cases). Here too we observed a ''hole'' around the frequency $f_e = 0.157$ although of a smaller size compared to the one in the strongly coupled case at $P/D = 2.5$.

The Arnold tongues formed in flow past a single cylinder have been modeled by Olinger and Sreenivasan [7] using a circle sine map, which is typically a good model for small A/D values. For the single cylinder case no such holes representing non-locked-in response appear inside the tongues. McGehee and Peckham [8] were able to create a multitude of bifurcation features interior to resonance regions by manipulating the coefficients in the Fourier series for the forcing function in the circle map but no specific physical systems were identified that are susceptible to such bifurcations. Perhaps the dynamical model that is closer to the flow that we consider in this work is the doubly forced periodic oscillators of Peckham and Kevrekidis [9]. This is a system in which each oscillator separately has its own limit cycle, and increasing values in the forcing amplitude lead to bifurcations inside the Arnold tongue, just like in our two-cylinder system. These bifurcation regions are relatively thin vertical zones that widen as the forcing amplitude increases and are located close to the natural frequency; because of their shape they are termed ''Arnold flames.'' However, there is a big difference of the holes that we discovered compared to the Arnold flames of Peckham and Kevrekidis [9]: Arnold flames are regions within which multiple periodic orbits exist, whereas our holes indicate an absence of any periodic orbits, rather than the presence of extra orbits. To the best of our knowledge, such states inside the Arnold tongue have not been reported before. In unpublished work, Peckham [10] has observed holes of the type we report here for periodically forced oscillators by constructing corresponding resonant surfaces with a disk or cylinder topology and a ''handle'' attached to them. The projection of the resonant surfaces to the parameter space gives the resonance tongues while the projection of the handle gives a hole in the resonance tongue.

In summary, we have presented numerical evidence revealing the existence of *new bifurcations* that lead to nonperiodic states inside the intraresonance regions in flow past two cylinders in tandem arrangement. From the practical standpoint, our findings suggest that, surprisingly, self-excitation of the two cylinders system may be suppressed at conditions close to its natural frequency and large amplitude, where we typically expect large oscillatory response. We suspect that other periodically forced nonlinear systems of general type exhibit similar behavior.

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