Hadronic Matter Is Soft

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The stiffness of the hadronic equation of state has been extracted from the production rate of K^+ mesons in heavy-ion collisions around 1 AGeV incident energy. The data are best described with a compression modulus K around 200 MeV, a value which is usually called "soft." This is concluded from a detailed comparison of the results of transport theories with the experimental data using two different procedures: (i) the energy dependence of the ratio of K^+ from Au + Au and C + C collisions and (ii) the centrality dependence of the K^+ multiplicities. It is demonstrated that input quantities of these transport theories which are not precisely known, such as the kaon-nucleon potential, the $\Delta N \to NK^+\Lambda$ cross section, or the lifetime of the Δ in matter, do not modify this conclusion.

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For many years one of the most important challenges in nuclear physics has been to determine $E/A(\rho, T)$, the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density ρ and the temperature T. Only at equilibrium density, ρ_0, do we know the energy per nucleon $E/A(\rho = \rho_0, T = 0) = -16$ MeV by extrapolating the Weizsäcker mass formula to infinite matter. This quest has been dubbed "search for the nuclear equation of state (EoS)."

Modeling of neutron stars or supernovae has not yet constrained the nuclear equation of state [1]; therefore, the most promising approach in extracting $E/A(\rho, T)$ is to use heavy-ion reactions in which the density of the colliding nuclei changes significantly. Three principal experimental observables have been suggested in the course of this quest which carry—according to theoretical calculations—information on the nuclear EoS: (i) the strength distribution of the giant isoscalar monopole resonances [2,3], (ii) the in-plane sidewards flow of nucleons in semicentral heavy-ion reaction at energies between 100 and 400 AMeV [4], and (iii) the production of K^+ mesons in heavy-ion reactions at energies around 1 AGeV [5]. Although theory has predicted these effects qualitatively, a quantitative approach is confronted with two challenges: (a) The nucleus is finite and surface effects are not negligible, even for the largest nuclei and (b) in heavy-ion reactions the reacting system does not come into equilibrium. Therefore complicated nonequilibrium transport theories have to be employed and the conclusion on the nuclear equation of state can be only indirect.

(i) The study of monopole vibrations has been very successful, but the variation in density is minute; therefore, giant monopole resonances are sensitive to the energy which is necessary to change the density of a cold nucleus close to the equilibrium point ρ_0. According to theory the vibration frequency depends directly on the force that counteracts any deviation from the equilibrium and therefore the potential energy. The careful analysis of the isoscalar monopole strength in nonrelativistic [2] and relativistic mean field models has recently converged [3] due to a new parametrization of the relativistic potential. These calculations allow now for the determination of the compression modulus $K = \frac{1}{\kappa} = -V\frac{dp}{dV} = 9\rho^2 \frac{d^2E/A(\rho,T)}{(d\rho)^2}|_{\rho=\rho_0}$ which measures the curvature of $E/A(\rho, T)$ at the equilibrium point. κ is the compressibility. The values found are around K = 240 MeV and therefore close to what has been dubbed a "soft equation of state."

(ii) If the overlap zone of projectile and target becomes considerably compressed in semicentral heavy-ion collisions, an in-plane flow is created due to the transverse pressure on the baryons outside of the interaction region with this flow being proportional to the transverse pressure. In order to obtain a noticeable compression, the beam energy has to be large compared to the Fermi energy of the nucleons inside the nuclei and hence a beam energy of at least 100 AMeV is necessary. Compression goes along with excitation and therefore the compressional energy of excited nuclear matter is encoded in the in-plane flow. It has recently been demonstrated [6] that transport theories do not agree quantitatively yet and therefore conclusions [7] drawn previously from in-plane flow have to be taken with caution.

(iii) The third method is most promising for the study of nuclear matter at high densities and is the subject of this Letter. K^+ mesons produced far below the NN threshold cannot be created in first-chance collisions between projectile and target nucleons. They do not provide sufficient energy even if one includes the Fermi motion. The effective energy for the production of a K^+ meson in the NN center of mass system is 671 MeV as, in addition to the mass of the kaon, a nucleon has to be converted into a Λ to conserve strangeness. Before nucleons can create a K^+ at these subthreshold energies, they have to accumulate energy. The most effective way to do this is by conversion of a nucleon into a Δ and to produce a K^+ in subsequent collisions via K^+ meson via $\Delta N \to NK^+\Lambda$. Two effects link the yield of produced K^+ mesons with the density reached in the collision and the stiffness of the EoS. If less energy is needed to compress matter (i) more energy is available for K^+ production and (ii) the density which can be reached in these reactions will be higher. Higher density means a smaller mean free path and therefore the Δ will interact more often increasing the probability to produce a K^+ and hence, it has a lower chance to decay before it interacts. Consequently, the K^+ yield depends on the compressional energy. At beam energies around 1 *A*GeV matter becomes highly excited and mesons are formed. Therefore this process tests highly excited hadronic matter. At beam energies >2 *A*GeV first-chance collisions dominate and this sensitivity is lost. For details on how K^+ mesons behave in matter and on the transport theory approach to understand K^+ production in heavy-ion collisions we refer to the reviews [8].

In this Letter we report that the third approach allows us now to fix the stiffness of the equation of state. This is due to two new results. First, the calculations for two completely independent experimental observables, the ratio of the excitation functions of K^+ production in Au + Au and in C + C [9] and a new observable which measures the dependence of the K^+ yield on the number of participants, both agree with experiment if in these calculations it is assumed that nucleons interact with a potential which corresponds to a compression modulus of $\mathrm{K} \leq 200$ MeV in infinite matter in thermal equilibrium. This value extracted for hadronic matter at densities around 2.5 times the normal nuclear matter density is very similar to the one extracted at normal nuclear matter density. Second, the different implementation of all up to now unsolved physical questions, such as the $N\Delta \to K^+\Lambda N$ cross section, the KN interaction, as well as the lifetime of the nuclear resonances in the hadronic environment, do not affect this conclusion. Thus our results confirm the conclusions obtained by Fuchs *et al.* [10] which were based on the excitation function and the robustness of the results against a variation of the K^+ potential only.

In order to determine the energy necessary to compress nuclear matter in thermal equilibrium from heavy-ion reactions, one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by a Skyrme interaction. For this potential the potential energy in infinite nuclear matter is calculated. To determine the nuclear equation of state we average this (momentum-dependent) two-body potential over the momentum distribution of a given temperature T and add to it the kinetic energy. Expressed as a function of the density we obtain the desired nuclear equation of state $E/A(\rho, T)$. Our two-body potential has five parameters which are fixed by the binding energy of infinite nuclear matter at ρ_0, the compression modulus K, and the optical potential which has been measured in pA reactions.

Once the parameters are fixed we use the two-body potential with these parameters in the transport calculation. There is an infinite number of two-body potentials which give the same equation of state because the range of the potential does not play a role in infinite matter. The nuclear surface measured in electron scattering on nuclei fixes the range, however, quite well. The uncertainty which remains is of little relevance here (in contrast to the calculation of the in-plane flow which is very sensitive to the exact surface properties of the nuclei and hence to the range of the potential).

We employ the isospin quantum molecular dynamics (IQMD) [11] approach with Hamiltonian-type equations of motion where the expectation value of the total Hamiltonian reads as $\langle H\rangle = \langle T\rangle + \langle V\rangle$ with $\langle T\rangle = \sum_i \frac{p_i^2}{2m_i}$ and

$$\langle V\rangle = \sum_i \sum_{j>i} \int f_i(\vec{r}, \vec{p}, t) V^{ij} f_j(\vec{r}', \vec{p}', t) d\vec{r} d\vec{r}' d\vec{p} d\vec{p}', \quad (1)$$

where f_i is the Gaussian Wigner density of nucleon i. The baryon potential consists of a strong interaction supplemented by the Coulomb interaction between particles of charges Z_i and Z_j. The former can be further subdivided in a part containing the contact Skyrme-type interaction only, a contribution due to a finite range Yukawa potential, and a momentum-dependent part $V^{ij} = V^{ij}_{\mathrm{Skyrme}} + V^{ij}_{\mathrm{Yuk}} + V^{ij}_{\mathrm{mdi}} + V^{ij}_{\mathrm{Coul}}$ with

$$\begin{aligned} V^{ij} &= t_1 \delta(\vec{x}_i - \vec{x}_j) + t_2 \delta(\vec{x}_i - \vec{x}_j) \rho^{\gamma - 1}(\vec{x}_i) \\ &\quad + t_3 \frac{\exp\{-|\vec{x}_i - \vec{x}_j|/\mu\}}{|\vec{x}_i - \vec{x}_j|/\mu} + \frac{Z_i Z_j e^2}{|\vec{x}_i - \vec{x}_j|} \\ &\quad + t_4 \ln^2[1 + t_5(\vec{p}_i - \vec{p}_j)^2] \delta(\vec{x}_i - \vec{x}_j). \end{aligned} \quad (2)$$

For more details we refer to Refs. [11,12].

We include in this calculation all inelastic cross sections which are relevant to K^+ production. For details of these cross sections we refer to Ref. [13]. Unless specified differently, the change of the K^+ mass due to the kaon-nucleon (KN) interaction according to $m^K(\rho) = m_0^K(1 - 0.075\frac{\rho}{\rho_0})$ is taken into account, in agreement with recent self-consistent calculations of the spectral function of the K^+ [14]. The Λ potential is $2/3$ of the nucleon potential, assuming that the s quark does not interact with nonstrange nuclear matter.

In order to minimize the experimental systematic errors and the consequences of theoretical uncertainties, it is better to compare ratios of cross sections rather than absolute values [9]. We have made sure that the standard version of IQMD reproduces the excitation function for Au + Au as well as for C + C quite well [15]. These ratios are quite sensitive to the nuclear potentials because the compression obtained in Au + Au collisions is considerable (up to $3\rho_0$) and depends on the nuclear equation of state, whereas in C + C collisions the compression is small and almost independent on the stiffness of the EoS.

Figure 1 shows the comparison of the measured ratio of the K^+ multiplicities obtained in Au + Au and C + C reactions [9], together with transport model calculations, as a function of the beam energy.

We see clearly that the form of the yield ratio depends on the potential parameters (hard EoS: $K = 380$ MeV, thin lines and solid symbols; soft EoS: $K = 200$ MeV, thick lines and open symbols) in a quite sensible way and that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs *et al.* [10] given in the same graph agrees well with our findings.

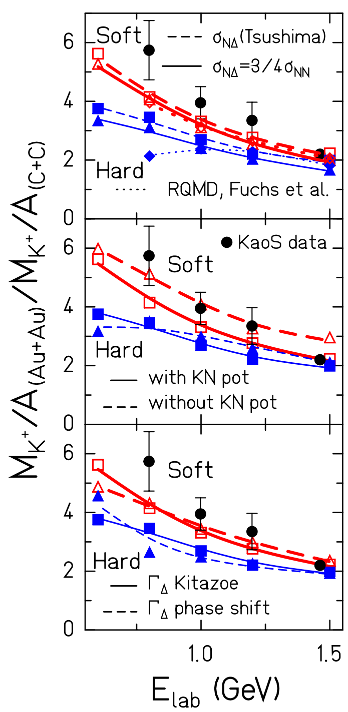

FIG. 1 (color online). Comparison of the measured excitation function of the ratio of the K^+ multiplicities per mass number A obtained in Au + Au and in C + C reactions (Ref. [9]) with various calculations. The use of a hard EoS is denoted by thin (blue) lines, a soft EoS by thick (red) lines. The calculated energies are given by the symbols; the lines are drawn to guide the eye. On top, two different versions of the $N\Delta \rightarrow K^+\Lambda N$ cross sections are used. One is based on isospin arguments [16]; the other is determined by a relativistic tree level calculation [17]. The calculations by Fuchs [10] are shown as dotted lines. Middle: IQMD calculations with and without KN potential are compared. Bottom: the influence of different options for the lifetime of Δ in matter is demonstrated.

This observation is, however, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. Therefore, it is necessary to verify that these uncertainties do not render this conclusion invalid. Figure 1, top, shows the influence of the unknown $N\Delta \rightarrow K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for ΔN interactions from Tsushima *et al.* [13]) with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ [16], which is based on isospin arguments and has been frequently used. Both cross sections differ up to a factor of 10 and change significantly the absolute yield of K^+ in heavy-ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known for the densities obtained in this reaction. This dependence has already been studied by Fuchs [10]. The uncertainties due to the Δ lifetime are discussed in the bottom part of the figure. Both calculations represent two extreme values for this lifetime [13] which is important because the disintegration of the Δ resonance competes with the production of K^+ from its interaction with nucleons.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential compared to a hard one and that the data of the KaoS Collaboration are compatible only with the soft EoS potential. The only possibility to change this conclusion is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. This would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

We would like to add that the smoothness of the excitation function also demonstrates that there are no density isomers in the regions which are obtained in these reactions because the K^+ excitation function would be very sensitive to such an isomeric state [18].

The conclusion that nuclear matter is best described by a soft EoS is supported by another observation, the dependence of the K^+ yield on the number of participating nucleons A_{part}. The prediction of the IQMD simulations in the standard version for this observable is shown in Fig. 2. The top of the figure shows the kaon yield M_{K^+}/A_{part} for Au + Au collisions at 1.5 AGeV as a function of the participant number A_{part} for a soft EoS using different options: standard version (soft, KN), calculations without kaon-nucleon interaction (soft, no KN), and with the isospin based $N\Delta \rightarrow N\Lambda K^+$ cross section (soft, KN, σ^*). These calculations are confronted with a standard calculation using the hard EoS potential. The scaling of the kaon yield with the participant number can be parameterized by $M_{K^+} = A_{part}^{\alpha}$.

All calculations with a soft EoS show a rather similar value of α—although the yields are very different—while

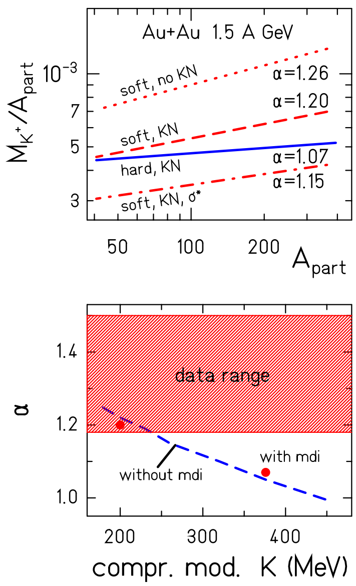

FIG. 2 (color online). Dependence of the K^+ scaling on the stiffness of the nuclear equation of state. We present this dependence in the form $M_{K^+} = A_{\text{part}}^{\alpha}$. On top, the dependence of M_{K^+}/A_{part} as a function of A_{part} is shown for different options: a "hard" EoS with KN potential (solid line), the other three lines show a "soft" EoS, without KN potential and $\sigma(N\Delta)$ from Tsushima [17] (dotted line), with KN potential and the same parametrization of the cross section (dashed line) and with KN potential and $\sigma(N\Delta) = 3/4\sigma(NN)$ (dash-dotted line). In the bottom figure, the fit exponent α is shown as a function of the compression modulus for calculations with and without mdi.

the calculation using a hard EoS shows a much smaller value. Therefore we can conclude that also the slope value α is a rather robust observable.

The bottom of Fig. 2 shows that α depends smoothly on the compression modulus K of the EoS. Whether we include the momentum-dependent interactions (mdi) of the nucleon-nucleon interaction or not [$t_4 = 0$ in Eq. (2)] does not change the value of α as long as the compression modulus is not changed—in stark contrast to the in-plane flow. Again, the measured centrality dependence for Au + Au at 1.5 AGeV from the KaoS Collaboration [19], $\alpha = 1.34 \pm 0.16$, is compatible only with a soft EoS potential.

This finding is also supported by a more recent analysis [20,21] of the in-plane flow which supersedes the former conclusion that the EoS is hard [22] (made before the momentum-dependent interaction has been included in the calculations). Because of the strong dependence of the in-plane flow on the potential range parameter and its dependence on the particles observed, these conclusions are much less firm at present. Comparisons of the out-of-plane squeeze of baryons also show a preference for a soft equation of state with momentum-dependent interactions [23].

In conclusion, we have shown that the two experimental observables which are most sensitive to the potential parameters of the nucleon-nucleon interaction are compatible only with those parameters which lead to a soft hadronic EoS. This conclusion is robust. Uncertainties of the input in these calculations, such as the KN potential at high densities, the lifetime of the Δ in matter, and the $\Delta N \to NK^+\Lambda$ cross section, do not influence this conclusion. The potential parameter K is even smaller than that extracted from the giant monopole vibrations. Thus the energy which is needed to compress hadronic matter is close to the lower bound of the value that has been discussed in the past.

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