QCD Corrections to Static Heavy-Quark Form Factors

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Interactions of heavy quarks, in particular, of top quarks, with electroweak gauge bosons are expected to be very sensitive to new physics effects related to electroweak symmetry breaking. These interactions are described by the so-called static form factors, which include anomalous magnetic moments and the effective weak charges. We compute the second-order QCD corrections to these static form factors, which turn out to be sizable and need to be taken into account in searches for new anomalous coupling effects.

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The top quark, so far the heaviest known fundamental particle, will serve as an excellent probe of the fundamental interactions at energies of a few hundred GeV, once more and more top-quark events will be accumulated at the Tevatron and, especially, after the CERN Large Hadron Collider (LHC) will have started operating. Even more detailed investigations into top-quark properties will be possible at a future linear electron-positron collider (ILC), where top-quark pairs can be studied in a very clean and well-defined environment.

In view of its large mass the top quark is, in particular, a unique probe of the dynamics that breaks the electroweak gauge symmetry. If this mechanism differs from the Higgs mechanism of the standard model (SM) of particle physics, observable effects could be found first in top-quark production and decay. They may manifest themselves as deviations of the top-quark gauge-boson couplings from the values predicted by the SM (c.f. [1,2] for overviews).

There has been an enormous effort in recent years to investigate the potential of top quarks, and also of bottom quarks, for new physics effects. Specifically, the couplings to photons and Z bosons, which are the subject of this Letter, have been studied in detail—both for heavy-quark production at hadron colliders [3,4] and at a future high-luminosity high-energetic linear electron-positron collider [5] (for reviews, see [6,7]). Indirect constraints, which are rather tight for b quarks, on anomalous contributions to these couplings can be obtained also from electroweak precision observables measured at LEP [3,8–10]. Many of these studies attempt to use a "model-independent" approach in that the effect of new interactions is described in terms of effective Lagrangians or, equivalently, in terms of anomalous couplings of heavy quarks, in particular, to the SM gauge bosons. In this context the obvious question arises about the size of these effective couplings within the SM, in order to assess the margin of detectability of truly anomalous new physics effects, given the experimental sensitivity of some observable.

In this Letter we investigate the couplings of heavy quarks, notably of the t and b quark, to the photon and Z boson in higher-order QCD. In view of the asymptotic freedom property of the strong interactions and of the large energy scale set by the heavy-quark mass, $m_Q \gg \Lambda_{\rm QCD}$, these effective couplings can be computed perturbatively. These couplings do in general depend on the precise kinematics at the vertex, which is expressed in terms of the so-called vertex form factors, depending on the momentum transfer at the vertex. In many applications, it is justified to approximate these form factors by their limits at zero momentum transfer, the so-called static form factors. The most prominent static form factor is the electromagnetic spin-flipping form factor: the anomalous magnetic moment. At present, radiative corrections to the static form factors of quarks are known to one loop in the SM [11,12], while pure QED corrections to the anomalous magnetic moment are known analytically to three loops [13]. As far as these quantities are concerned, the largest radiative corrections are often those due to QCD. Here we present analytic results for the static γQQ and ZQQ form factors to second order in the QCD coupling α_s and we predict the size of these moments for t and b quarks within the SM. Moreover, we briefly discuss the implications of our results in view of an existing upper bound on the anomalous magnetic moment of the b quark and on future precision measurements of the γtt and Ztt couplings.

To start, we define the static form factors of a heavy quark Q by considering for definiteness the kinematical situation $V^* \to Q(p_1) + \bar{Q}(p_2)$ (V^* = off shell photon or Z boson). We study the $V^* Q\bar{Q}$ vertex functions $\bar{u}(p_1)\Gamma_Q^{\mu,V}(q)v(p_2)$ for on-shell external quarks in the limit of zero four-momentum transfer $q = p_1 + p_2$. In general, this vertex function can be decomposed, using Lorentz covariance, into six form factors, two of which are CP violating. CP violation in a flavor-conserving vertex function is, in the SM, a tiny, higher-loop induced effect, and we do not consider it here any further. Assuming CP invariance $\Gamma_Q^{\mu,V}$ then depends on four form factors:

$$i\Gamma_Q^{\mu,V} = e\bigg(F_{1,Q}^V\gamma^\mu + \frac{i}{2m_Q}F_{2,Q}^V\sigma^{\mu\nu}q_\nu + G_{1,Q}^V\gamma^\mu\gamma_5 + \frac{1}{2m_Q}G_{2,Q}^V\gamma_5 q^\mu\bigg), \tag{1}$$

where the form factors are functions of $s = q^2$. Further $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and $e > 0$ denotes the positron charge. If one considers the matrix element of the electromagnetic current, the parity-violating part of Eq. (1) is to be replaced by $G_Q(\gamma_\mu\gamma_5 s - 2m_Q\gamma_5 q_\mu)$, where $G_Q(0)$ yields the anapole moment of Q.

We recall that for a reaction involving the VQQ vertices the physical object is the S matrix element, but not, in general, these form factors. The static quantities $F_{i,Q}^\gamma(0)$, $G_{i,Q}^\gamma(0)$ do have a physical meaning: they are definable as the residues of the photon pole in scattering amplitudes in the soft photon limit, and they are gauge invariant (with respect to the full SM gauge group) and infrared finite. Likewise, the $F_{i,Q}^Z(m_Z^2)$, $G_{i,Q}^Z(m_Z^2)$, which determine the S matrix element of the decay of an on-shell Z boson into a $Q\bar{Q}$ ($Q = b, c$) quark pair, are gauge invariant—but they are not infrared finite, as the $Q\bar{Q}$ state above threshold is degenerate with states containing in addition soft photons and gluons and/or collinear massless partons. On the other hand, the static form factors $F_{i,Q}^Z(0)$, $G_{i,Q}^Z(0)$ are infrared finite, but gauge invariant only with respect to QCD or pure QED.

Here, our primary aim is to compute the anomalous magnetic moments of heavy quarks in QCD, which are physical quantities. In addition, we determine the heavy-quark anomalous weak magnetic moment and its axial charge to second order in α_s. Although these are well-defined objects in QCD only, these SM predictions should serve as useful reference values, in particular, for new physics models whose effect on, for instance, the $e^+e^- \to Q\bar{Q}$ amplitude is essentially confined to the $VQ\bar{Q}$ vertices.

To lowest order in the SM couplings, only $F_{1,Q}^{\gamma,Z}$ and $G_{1,Q}^Z$ are nonzero. The other form factors are generated by one-loop radiative corrections: $G_{1,2,Q}^\gamma$ by parity-violating weak corrections, and $F_{2,Q}^{\gamma,Z}$ and $G_{2,Q}^Z$ by strong and electroweak corrections. In this Letter we consider the QCD corrections to these form factors. To lowest order in the electroweak couplings we have $G_{1,2,Q}^\gamma = 0$ and $F_{i,Q}^V = v_Q^V F_{i,Q}$ ($i = 1$, 2), where $v_Q^\gamma = Q_Q$ and $v_Q^Z = (T_3^Q/2 - s_w^2 Q_Q)/(s_w c_w)$, $a_Q = -T_3^Q/(2s_w c_w)$ are the SM vector and axial vector couplings of Q to the Z boson and the photon in units of e, where $s_w(c_w)$ is the sine (cosine) of the weak mixing angle, T_3^Q the third component of the weak isospin, and Q_Q is the charge of the heavy quark.

Here we determine these form factors in the static limit to second order in α_s, using the results for nonzero momentum transfer s given in [14]. We work in QCD with N_l massless quarks and one quark Q with mass m_Q (defined in the on-shell scheme). This includes the case of six quarks with all quarks but the top quark taken to be massless. The QCD coupling $\alpha_s = \alpha_s(\mu)$ is defined in the standard $\overline{\text{MS}}$ scheme in $N_f = N_l + 1$ flavor QCD with μ being the renormalization scale.

Because of conservation of the electromagnetic and the neutral vector current, the QCD corrections to $F_{1,Q}$ vanish for $s \to 0$; i.e., $F_{1,Q}(s = 0) = 1$. The form factors $F_{2,Q}$, $G_{1,Q}$, and $G_{2,Q}$ become infrared finite in the static limit. We obtain

$$F_{2,Q}(s = 0) = \frac{\alpha_s}{2\pi}C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 F_{2,Q}^{(2l)}, \tag{2}$$

with

$$\begin{aligned} F_{2,Q}^{(2l)} = {} & C_F^2\left(-\frac{31}{4} + 2\zeta_2(5 - 6\ln(2)) + 3\zeta_3\right) \\ & + C_F C_A\left(\frac{317}{36} + 3\zeta_2(-1 + 2\ln(2)) - \frac{3}{2}\zeta_3\right) \\ & + C_F T_F\left(\frac{119}{9} - 8\zeta_2\right) - \frac{25}{9}C_F T_F N_l \\ & + C_F\beta_0\ln(r_Q), \end{aligned} \tag{3}$$

where $r_Q = \mu^2/m_Q^2$, ζ_n is the Riemann zeta function, and $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$, $T_F = 1/2$ with $N_c = 3$ being the number of colors. Further $\beta_0 = (11C_A - 4T_F(N_l + 1))/6$. For $G_{1,Q}$ we have, to order α_s^2

$$G_{1,Q}^Z = a_Q(G_{1,Q}^{(A)} + G_{1,Q}^{(B)}), \tag{4}$$

where the superscripts A, B denote universal and nonuniversal corrections, respectively. We get for the type A term

$$G_{1,Q}^{(A)}(s = 0) = 1 - \frac{\alpha_s}{2\pi}C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 G_{1,Q}^{(2l)}, \tag{5}$$

with

$$\begin{aligned} G_{1,Q}^{(2l)} = {} & C_F^2\left(-\frac{29}{12} + 8\zeta_2(1 - \ln(2)) + 2\zeta_3\right) \\ & + C_F C_A\left(-\frac{143}{36} + 2\zeta_2(-1 + 2\ln(2)) - \zeta_3\right) \\ & + C_F T_F\left(\frac{115}{9} - 8\zeta_2\right) + \frac{7}{9}C_F T_F N_l - C_F\beta_0\ln(r_Q). \end{aligned} \tag{6}$$

The term $G_{1,Q}^{(B)}$ is obtained by summing over all weak isospin quark doublets in the second-order triangle diagram contributions to the ZQQ vertex [14]. For definiteness we consider here only the static t and b quark form factors. In determining $G_{1,Q}^{(B)}$ we neglect in both cases the mass of the b quark with respect to that of the t quark in the respective triangle diagram. Then we get

TABLE I. Input values used in computing the static top and bottom form factors. The mass of the Z boson and the weak mixing angle are taken to be $m_Z = 91.187$ GeV and $\sin^2\theta_W = 0.231$ [15]. $\alpha_s(m_Z)$ is the $\overline{\text{MS}}$ QCD coupling for $N_f = 5$ flavors, taken from [15], while $\alpha_s(m_t)$ is the coupling for $N_f = 6$ flavors, obtained from $\alpha_s(m_Z)$ by renormalization-group evolution.

	t quark		b quark
m_Q	175 GeV		5 GeV
Q_Q	2/3		−1/3
T_Q^3	1/2		−1/2
μ	m_t	m_b	m_Z
$\alpha_s(\mu)$	0.1080	0.2145	0.1187

TABLE II. One- and two-loop QCD contributions, and their sums, to the anomalous magnetic and weak magnetic moments of the top and bottom quark, for different values of the renormalization scale μ.

	t ($\mu = m_t$)	b ($\mu = m_b$)	b ($\mu = m_Z$)
$(g-2)_Q^{\gamma,(1l)}/2$	1.53×10^{-2}	-1.52×10^{-2}	-8.4×10^{-3}
$(g-2)_Q^{\gamma,(2l)}/2$	4.7×10^{-3}	-1.00×10^{-2}	-6.6×10^{-3}
$(g-2)_Q^{\gamma}/2$	2.00×10^{-2}	-2.52×10^{-2}	-1.50×10^{-2}
$(g-2)_Q^{Z,(1l)}/2$	5.2×10^{-3}	-1.87×10^{-2}	-1.03×10^{-2}
$(g-2)_Q^{Z,(2l)}/2$	1.6×10^{-3}	-1.24×10^{-2}	-8.1×10^{-3}
$(g-2)_Q^{Z}/2$	6.8×10^{-3}	-3.11×10^{-2}	-1.85×10^{-2}

$$
\begin{aligned}
G_{1,t}^{(B)}(0) &= \left(\frac{\alpha_s}{2\pi}\right)^2 C_F T_F [\mathcal{G}_{1,m_t,m_t}(0) - \mathcal{G}_{1,0,m_t}(0)],\\
G_{1,b}^{(B)}(0) &= \left(\frac{\alpha_s}{2\pi}\right)^2 C_F T_F [\mathcal{G}_{1,m_b,m_b}(0) - \mathcal{G}_{1,m_t,0}(0)],
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
\mathcal{G}_{1,m_Q,m_Q}(0) &= -\frac{19}{3} + \frac{16}{3}\zeta_2 - 3\ln(r_Q),\\
\mathcal{G}_{1,0,m_Q}(0) &= -7 - 3\ln(r_Q),\\
\mathcal{G}_{1,m_Q,0}(0) &= \frac{3}{2} - 3\ln(r_Q).
\end{aligned} \tag{8}
$$

The induced pseudoscalar form factor $G_{2,Q}(0)$ can also be computed to order α_s^2 from the results of [14]. However, it is absent in Eq. (1) for on-shell photons or Z bosons, and irrelevant for the couplings of heavy quarks to leptons and light quarks via V boson exchange. Thus it seems not to be experimentally accessible in the foreseeable future. Therefore we shall not present it here.

Let us now consider the static magnetic and weak magnetic form factor of a quark Q. We consider

$$
\left(\frac{g-2}{2}\right)_Q^{\gamma,Z} \equiv F_{2,Q}^{\gamma,Z}(0) = v_Q^{\gamma,Z} F_{2,Q}(0), \tag{9}
$$

which corresponds to the anomalous magnetic (MDM) and weak magnetic (WMDM) moments of Q. [Notice that in the literature the WMDM is often associated with $F_{2,Q}^Z(s = m_Z^2)$.] We determine the numerical values of these moments for t and b quarks with the above formulas and the input values given in Table I. In the case of $(g-2)_t^{\gamma,Z}/2$ we work in $N_f = 6$ flavor QCD with all quarks but the top quark taken to be massless, while $(g-2)_b^{\gamma,Z}/2$ is computed in the effective $N_f = 5$ flavor theory with $m_i = 0$ ($i = u, d, s, c$) and $m_b \neq 0$. The results are given in Table II. The two-loop QCD contributions to $(g-2)_t^{\gamma,Z}/2$ and $(g-2)_b^{\gamma,Z}/2$ are sizable: they are about 30% and 70%, respectively, of the order α_s moments. In the case of the b quark we have evaluated the static magnetic form factors for two different values of the renormalization scale μ, which would apply to different physical situations, for instance, the leptoproduction of $b\bar{b}$ quark pairs above threshold and at the Z resonance. The dependence on μ, which is mainly due to the dependence of α_s on this scale, indicates also the size of the unknown higher-order QCD corrections. As to the SM electroweak one-loop contributions to the (W)MDMs of b and t quarks [12]: for the b quark they are significantly smaller than the QCD-induced moments given in Table II. The weak contributions to the WMDM and, if the SM Higgs boson is light, to the MDM of the t quark are of the order of a few times 10^{-3}.

Next we consider the weak axial vector charge $\mathbf{a}_Q^Z$ of a heavy quark Q defined by

$$
\mathbf{a}_Q^Z \equiv G_{1,Q}^Z(0), \tag{10}
$$

where, to second order in the QCD coupling, $G_{1,Q}^Z$ receives the type A and B contributions given above. Numerical evaluation of these formulas for t and b quarks gives the values of Table III. In the case of the t quark the two-loop type A and B contributions almost cancel, while for the b quark the second-order corrections are again sizable. Because of the parity invariance of QCD, the axial vector form factor of the photon, $G_{1,Q}^\gamma$, remains zero also at higher orders in QCD.

What do these numbers tell us? For the b quark an upper bound on its magnetic moment was derived in [16] from an analysis of LEP1 data, which, in our convention, reads

TABLE III. One- and two-loop QCD contributions to the static form factor $G_{1,Q}^Z$ as defined in Eq. (10) for top and bottom quarks.

	t ($\mu = m_t$)	b ($\mu = m_b$)	b ($\mu = m_Z$)
$G_{1,Q}^{(A,1l)}$	-2.29×10^{-2}	-4.55×10^{-2}	-2.52×10^{-2}
$G_{1,Q}^{(A,2l)}$	-1.81×10^{-3}	-7.74×10^{-3}	-1.30×10^{-2}
$G_{1,Q}^{(B)}$	1.86×10^{-3}	-1.58×10^{-2}	-4.85×10^{-3}
$G_{1,Q}^Z/a_Q$	$1 - 2.29 \times 10^{-2}$	$1 - 6.90 \times 10^{-2}$	$1 - 4.31 \times 10^{-2}$

$\delta(g-2)^{\gamma}_{b}/2 < 1.5 \times 10^{-2}$ (68% C.L.). Comparing it with Table II we see that the QCD-induced contributions to the b quark magnetic moment saturate this bound, which implies that there is limited room for new physics contributions to this quantity. At a future linear collider [6,7], when operated at the Z resonance, the sensitivity to this variable could be improved substantially, either by global fits or by analyzing appropriate angular distributions in $b\bar{b}$ and $b\bar{b}\gamma$ events.

As to the static form factors of the top quark, no such tight constraints exist so far on possible contributions from new interactions (see [3,9,10] for a review). As emphasized above, these quantities are particularly sensitive to the dynamics of electroweak symmetry breaking. For instance, in various models with a strongly coupled symmetry breaking sector one may expect contributions from this sector to the static t quark form factors at the 5%–10% level [1,2,17]. The QCD-induced anomalous magnetic moment and the QCD corrections to the axial charge of the top quark are of the same order of magnitude. Future colliders have the potential to reach this level of sensitivity. At the LHC the $tt\gamma$ and ttZ couplings can be separately measured in associated $tt\gamma$ and ttZ production, respectively. A detailed study of these reactions showed [3,4] that at the LHC a statistical sensitivity of about 5% to the photonic couplings $F^{\gamma}_{1,2,t}$, $G^{\gamma}_{1,t}$ may eventually be reached, while the sensitivity to the analogous couplings to the Z boson is significantly lower. At a future high-luminosity linear e^+e^- collider with polarized beams [18], both the Z and the photonic couplings of the top quark will be measurable with a precision of a few percent [6,7]. Thus it is mandatory to determine these quantities within the SM as precisely as possible.

In conclusion, we have determined the static form factors of b and t quarks, notably their anomalous magnetic moments and axial charges, to second order in the QCD coupling α_s. Precise knowledge of these effective couplings to photons and Z bosons within the standard model is mandatory in order to assess the margin of detectablity of new physics contributions and to interpret correctly (future) measurements. For the b quark we have found that the QCD contributions to its anomalous magnetic moment saturate the existing experimental upper bound, which implies that there is not much room for new physics effects on this quantity. The QCD corrections to the static form factors of the top quark are of the same order of magnitude as the precision with which these couplings may eventually be measured at future colliders and must therefore be taken into account in searches for anomalous coupling effects.

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