Spin Polarization Dependence of Carrier Effective Mass in Semiconductor Structures: Spintronic Effective Mass

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We introduce the concept of a spintronic effective mass for spin-polarized carriers in semiconductor structures, which arises from the strong spin-polarization dependence of the renormalized effective mass in an interacting spin-polarized electron system. The majority-spin many-body effective mass renormalization differs by more than a factor of 2 at $r_s = 5$ between the unpolarized and the fully polarized two-dimensional system, whereas the polarization dependence ($\sim 15\%$) is more modest in three dimensions around metallic densities ($r_s \sim 5$). The spin-polarization dependence of the carrier effective mass is of significance in various spintronic applications.

Spintronics [1] involves extensive manipulation of spin-polarized carriers in semiconductors. We show in this Letter that such a spin-polarized semiconductor carrier system would have a new kind of carrier effective mass, the "spintronic effective mass" associated with it. This spintronic effective mass will depend crucially on both carrier density and carrier spin polarization.

Recent experimental measurements of [2–6] of various Fermi liquid parameters, such as the effective mass and the spin susceptibility, in two-dimensional (2D) carrier (both electron and hole) systems confined in semiconductor structures have vigorously renewed interest in one of the oldest problems [7,8] of quantum many-body theory, namely, the density dependence of quasiparticle many-body renormalization in interacting electron systems. The quasiparticle effective mass, $m^*(r_s)$, depends on the interaction parameter r_s, the so-called Wigner-Seitz radius, which is the dimensionless interparticle separation measured in the units of the effective Bohr radius: $r_s \propto n^{-1/2}(n^{-1/3})$ in 2D (3D), where n is the respective 2D (3D) density. (In 2D systems there is also a nonuniversal correction arising from the finite width of the quasi-2D layer in the quantization direction, which is a conceptually simple form-factor effect.) In general, $m^*(r_s)$ increases with increasing r_s (i.e., with decreasing density), except at very small r_s, and for 2D systems of current experimental interest, the many-body renormalization could be by as much as a factor of $2-3$ at experimentally relevant densities ($r_s \approx 5-10$). In this Letter we discuss another fundamental new aspect of the quasiparticle effective mass renormalization which, while being quite important quantitatively (particularly in 2D semiconductor systems of current interest), has received relatively minor attention. This is the dependence, $m^*(r_s, \zeta)$, of the quasiparticle effective mass on the spin-polarization parameter ζ ($\equiv |n_\uparrow - n_\downarrow|/n$, where $n \equiv n_\uparrow + n_\downarrow$ and $n_\uparrow$, $n_\downarrow$ is the spin-polarized carrier density). The fact that many-body effects must depend nontrivially on the spin-polarization parameter ζ (in addition to the density parameter r_s) is obvious—

for example, the completely spin-polarized $\zeta = 1$ system has a factor of 2 lower density of states at E_F (and a concomitantly larger, by 2 in 2D, Fermi energy), leading to substantially different many-body renormalization than the corresponding spin-unpolarized ($\zeta = 0$) paramagnetic system. We provide in this Letter the first complete calculation of the 2D and the 3D quasiparticle effective mass renormalization $m^*(r_s, \zeta)$ as a function of both r_s and ζ within the leading-order single-loop self-energy expansion (Fig. 1) in the dynamically screened Coulomb interaction (i.e., the infinite series of ring diagrams approximation). Our calculated effective mass renormalization manifests nontrivial dependence on the spin-polarization parameter ζ, which clearly needs to be incorporated in understanding the existing experimental data.

Our theory for the spin-polarization dependence of quasiparticle effective mass renormalization in interacting electron systems is motivated not only by fundamental many-body considerations, but also by practical and urgent experimental needs. In particular, the recent measurements [2–6] of 2D effective mass in semiconductor structures invariably involve the application of an external magnetic field (either parallel or perpendicular to the 2D layer and often both), which polarizes the carrier spin. This makes the interpretation of the measured effective mass as only a density-dependent Fermi liquid parameter $m^*(r_s)$, perhaps with the appropriate quasi-2D layer width corrections, highly conceptually suspect since the effective mass in
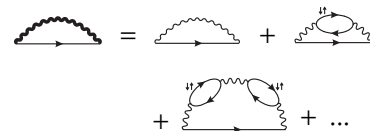

FIG. 1. Feynman diagram for self-energy: Solid lines denote the free electron Green's function and the wiggly lines the bare Coulomb interaction. At each vertex there is a conserved spin index that has to be explicitly accounted for in calculating the spin-polarization dependent self-energy.

such a spin-polarized system, $m^*(r_s, \zeta)$, should depend strongly on both density (i.e., r_s) and spin polarization (i.e., ζ). (The precise magnitude of the spin-polarization factor ζ is often not independently known, making an interpretation of the 2D effective mass measurement difficult.) In addition to the direct connection to the experimental 2D effective mass measurements, whose analyses must now be reexamined in light of our theoretical results, our calculated spin-polarization dependence of quasiparticle effective mass should also be of considerable significance to the fledgling subject of spintronics [1] where spin-polarized carriers are manipulated in semiconductor structures for logic and memory microelectronics applications. In spintronics applications, e.g., spin Hall effect [9,10] and spin transistors [11,12], the carriers are often spin polarized, and therefore the carrier effective mass would depend nontrivially on the spin-polarization parameter ζ as predicted in our work. Such a spin-polarization dependence of the carrier effective mass has so far not been taken into account in the spintronics literature to the best of our knowledge, but could turn out to be important in understanding experimental data and device modeling of spintronic systems.

The many-body self-energy Feynman diagrams we calculate for evaluating the quasiparticle effective mass are shown in Fig. 1. This is the single-loop dynamical screening approximation (i.e., the infinite series of ring diagrams) for the self-energy, generalized to the 2-component spin-polarized situation. The spin up (down) quasiparticle self-energy with momentum $\mathbf{k}$ and frequency ω in a polarized electron system can be written within our approximation as (see, e.g., [13])

$$\Sigma^{\uparrow(\downarrow)}(\mathbf{k}, \omega) = -\int \frac{d^d q\, d\nu}{i(2\pi)^{d+1}} \frac{v_q}{\epsilon(\mathbf{q}, \nu)} G_0^{\uparrow(\downarrow)}(\mathbf{q} + \mathbf{k}, \nu + \omega),$$

(1)

where $\hbar$ is always chosen to be 1, v_q is the bare Coulomb interaction between electrons with $v_q = 2\pi e^2/q$ in 2D and $v_q = 4\pi e^2/q^2$ in 3D,

$$G_0^{\uparrow(\downarrow)}(\mathbf{k}, \omega) = \frac{1 - n_F(\xi_{\mathbf{k}}^{\uparrow(\downarrow)})}{\omega - \xi_{\mathbf{k}}^{\uparrow(\downarrow)} + i\eta} + \frac{n_F(\xi_{\mathbf{k}}^{\uparrow(\downarrow)})}{\omega - \xi_{\mathbf{k}}^{\uparrow(\downarrow)} - i\eta} \quad (2)$$

is the Green's function for free spin up (down) electrons with the noninteracting energy dispersion $\xi_{\mathbf{k}}^{\uparrow(\downarrow)} = (k^2 - k_F^{2\uparrow(\downarrow)})/2m$ with m being the bare band mass, and $\epsilon(\mathbf{k}, \omega)$ is the dynamic dielectric function. Here $k_F^{\uparrow(\downarrow)}$ is the Fermi momentum for the spin up (down) electrons. $k_F^{\uparrow(\downarrow)} = k_F\sqrt{1 \pm \zeta}$ for 2D and $k_F^{\uparrow(\downarrow)} = k_F(1 \pm \zeta)^{1/3}$ for 3D, where k_F is the Fermi momentum for the unpolarized state. We use η to denote an infinitesimal positive number, and $n_F(x)$ for the Fermi function. At zero temperature, $n_F(x) = 1$ when $x \leq 0$ and 0 otherwise. Within our approximation, the dynamical screening is done by the infinite series of ring diagrams, and we have

$$\epsilon(\mathbf{k}, \omega) = 1 - v_q[\Pi^{\uparrow}(\mathbf{k}, \omega) + \Pi^{\downarrow}(\mathbf{k}, \omega)] \quad (3)$$

with $\Pi^{\uparrow(\downarrow)}(\mathbf{k}, \omega)$ the noninteracting electronic polarizability (i.e., the bare bubble in Fig. 1):

$$\Pi^{\uparrow(\downarrow)}(\mathbf{k}, \omega) = \int \frac{d^2 q}{(2\pi)^2} \frac{n_F(\xi_{\mathbf{q}}^{\uparrow(\downarrow)}) - n_F(\xi_{\mathbf{q}+\mathbf{k}}^{\uparrow(\downarrow)})}{\xi_{\mathbf{q}}^{\uparrow(\downarrow)} - \xi_{\mathbf{q}+\mathbf{k}}^{\uparrow(\downarrow)} + \omega}, \quad (4)$$

We note that this single-loop self-energy of Fig. 1 becomes asymptotically exact in the high-density $r_s \to 0$ limit, but is known to give reasonable results in the low-density $r_s > 1$ regime also [13,14]. The reason for this approximate validity of the single-loop self-energy well into the strong coupling regime is the fact that the dynamical screening expansion is not a series expansion in r_s, but is a self-consistent mean-field approximation where the effective expansion parameter is akin to $r_s/(C + r_s)$ with $C \gg 1$. Once the real part of the carrier self-energy, $\mathrm{Re}\Sigma^{\uparrow(\downarrow)}(k, \omega)$, is obtained, the effective mass is calculated by the on-shell quasiparticle approximation [13,14]:

$$\frac{m}{m_{\uparrow(\downarrow)}^*} = 1 + \frac{m}{k_F^{\uparrow(\downarrow)}} \frac{d}{dk} \mathrm{Re}\Sigma^{\uparrow(\downarrow)}(k, \omega = \xi_k^{\uparrow(\downarrow)})|_{k=k_F^{\uparrow(\downarrow)}}. \quad (5)$$

The on-shell effective mass approximation given in Eq. (5) is known to be a better approximation for the single-loop self-energy calculation (compared, for example, to solving the full Dyson's equation iteratively) as it is more consistent with the leading-order nature of the self-energy approximation itself—in fact, the on-shell approximation of Eq. (5) is the natural spin-polarized generalization of the quasiparticle effective mass renormalization in the usual spin-unpolarized case [13,14]. Using the Feynman diagrams of Fig. 1, we have calculated the on-shell effective mass renormalization m^*/m for both majority and minority-spin carriers in 2D and 3D electron systems as functions of the r_s and the ζ parameters, and below we present these results. For the sake of conceptual clarity we present both 2D and 3D results on the same footing without incorporating any finite quasi-2D width corrections, which are straightforward to incorporate and will reduce the 2D renormalization by factors of 1.2–2, depending on the carrier density and details of 2D confinement [14]. We take the bare electron-electron interaction to be the usual "$1/r$" Coulomb interaction in a 2D or 3D system, and the bare single-particle energy dispersion to be parabolic.

In Fig. 2 we present the calculated results for effective mass in an ideal 2D electron system with Coulomb interaction. For the majority electron mass, which is the likely experimentally measured quantity, effective mass decreases with increasing spin polarization. For small ζ values, this decrease is relatively small. However, as ζ approaches unity (i.e., near full spin polarization), the effect becomes much stronger. This is perhaps the reason for the misconception held in some of the earlier literature that the effective mass is spin-polarization independent, as it essentially is near $\zeta = 0$, but certainly not for $\zeta \sim 1$.
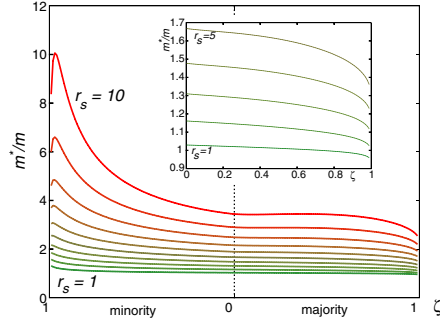
FIG. 2 (color online). Calculated 2D effective mass as a function of spin polarization ζ at different r_s values. Each curve corresponds to an r_s value of 1 to 10 with increment 1 from bottom to top. Inset: zoom-in figure for majority effective mass as a function of ζ for $r_s = 1$ to 5 from bottom to top with increment 1.

Figure 2 also shows that the minority effective mass increases with ζ first, and as ζ approaches one, it actually decreases sharply. The spin-polarization dependence of the effective mass is thus quite nontrivial and nonmonotonic.

In Fig. 3 we present the calculated results for the effective mass in an ideal 3D electron system with Coulomb interaction. The major difference between our 3D results shown in Fig. 3 and 2D results shown in Fig. 2 is that the 3D majority effective mass decreases more or less linearly with increasing ζ, but the nonmonotonic dependence of the minority effective mass on spin polarization is manifestly present in 3D also, except it is less sharp than in the 2D case.

In Fig. 4 we show the 2D majority effective mass as a function of r_s at $\zeta = 0$ or 1 for both on-shell and off-shell approximations. In the off-shell approximation the full Dyson's equation is solved for obtaining the quasiparticle effective mass leading to $m_{\uparrow\downarrow}^*/m = [1 - \partial_\omega \Sigma_{\uparrow\downarrow}(k,\omega)]/[1 + (m/k)\partial_k \Sigma_{\uparrow\downarrow}(k,\omega)]|_{k=k_{F\uparrow\downarrow}, \omega=0}$. The important difference between the definitions of the on-shell and the off-shell effective mass leads to a large quantitative difference between their calculated values, as apparent in Fig. 4. In general, the many-body renormalization corrections are substantially suppressed in the off-shell calculation (within

the single-loop self-energy approximation), and as has been argued extensively elsewhere [13,14], the on-shell approximation is the correct effective mass approximation for the single-loop self-energy used in our work, both for the sake of consistency and for the approximate inclusion of vertex correction. We note that the spin-polarization dependence of the off-shell effective mass (as well as the many-body renormalization itself) is weaker than the on-shell result. We point out that the only reason we are providing the off-shell effective mass results in this Letter (although within our approximation scheme of Fig. 1, the on-shell mass is the appropriate one) is to emphasize the fact that the off-shell result (which is often used in the literature) is quantitatively highly inaccurate for the one-loop self-energy approximation. For example, the on-shell 2D effective mass in our theory agrees far better [14] with experiment than the corresponding off-shell results.

In further discussing our theoretical results for the spin-polarization dependence of the quasiparticle effective mass $m^*(r_s, \zeta)$ we first note that the ζ dependence is rather weak in the high-density limit (small r_s). This trend can actually be analytically demonstrated by obtaining the ζ dependence of $m^*(r_s, \zeta)$ in the $r_s \to 0$ (and small ζ) limit, which for the single-loop self-energy gives through a straightforward calculation:

$$\frac{m_{\uparrow\downarrow}^*}{m} = 1 + \frac{r_s}{\sqrt{2}\pi}\ln r_s \mp \frac{r_s\zeta}{\sqrt{2}\pi}\ln r_s, \qquad (6)$$

where $r_s \equiv me^2/\sqrt{\pi n}$ in 2D, and

$$\frac{m_{\uparrow\downarrow}^*}{m} = 1 + \left(\frac{1}{2\pi}\right)\left(\frac{4}{9\pi}\right)^{1/3}\ln r_s \mp \left(\frac{1}{3\pi}\right)\left(\frac{4}{9\pi}\right)^{1/3}\zeta\ln r_s, \quad (7)$$

where $r_s \equiv me^2(4\pi n/3)^{-1/3}$ in 3D. Thus, in the small (r_s, ζ) limit, the spin-polarization dependence of m^* is small and linear in the spin polarization. For large spin polarization, however, the dependence of dynamical screening on the spin polarization is highly nontrivial, and the minority-spin effective mass (note that the minority-spin carrier density vanishes as the spin polarization approaches unity) shows a pronounced maximum (both in 2D and 3D) that is not captured in the leading-
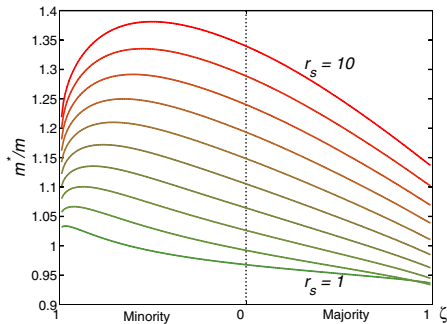

FIG. 3 (color online). Calculated 3D effective mass as a function of ζ for different r_s values. Each curve corresponds to an r_s value 1 to 10 with increment 1 from bottom to top.
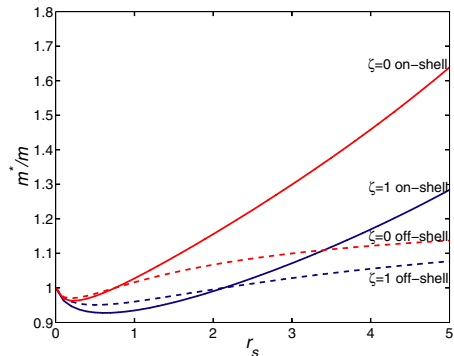

FIG. 4 (color online). Calculated 2D majority effective mass as a function of r_s for on-shell and off-shell approximations.
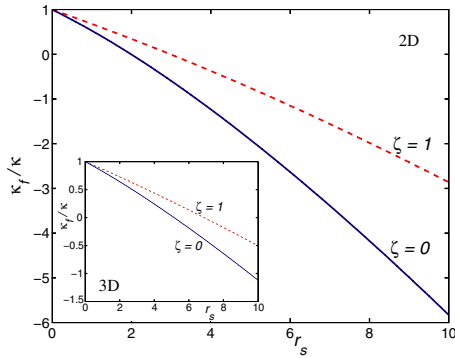
FIG. 5 (color online).　Calculated 2D and 3D compressibility as a function of r_s for fully spin-polarized and unpolarized states with κ_f as the noninteracting unpolarized Fermi gas compressibility and κ the renormalized compressibility.

order asymptotic expansion in the small r_s and ζ limit. A direct observation of this nonmonotonicity will indicate a nontrivial many-body aspect of the spin-polarization dependence of the spintronic effective mass renormalization.

We can now reexamine the previously mentioned experiments [2–6] using our spintronic effective mass results. These experiments all use external magnetic field induced Shubnikov–de Hass (SdH) oscillations to obtain the effective mass of two-dimensional electron systems. The SdH oscillation amplitude depends on the effective mass, temperature, and magnetic field according to the Dingle formula, through which effective mass can be obtained by data fitting [2–6]. However, it is important to notice that this is done in finite perpendicular magnetic field, which invariably spin polarizes the system. As we have already mentioned, it is conceptually wrong to assume that the derived effective mass corresponds to the zero-field mass because the effective mass depends on the spin polarization. From our results shown in Fig. 2, we see that when the polarization is less than $1/2$, the effective mass depends on the polarization weakly, and the above-mentioned method of experimentally determining the effective mass may serve as a good approximation. However, as spin polarization approaches one (such as the case in Ref. [5]), the effective mass depends strongly on the spin polarization, and fitting to the Dingle formula is no longer a suitable method to obtain the finite field effective mass, and certainly not the zero-field effective mass. One must incorporate the spin-polarization dependence of the quasiparticle effective mass in this situation in the experimental analysis.

Since the single-particle self-energy, the density of states, the dynamical screening, the Fermi momentum, and the Fermi energy are all affected by spin polarization, we expect all Fermi liquid parameters (not just the effective mass) to be strongly dependent on the spin-polarization parameter ζ (in addition to being dependent on r_s). An important thermodynamic quantity is the system compressibility, which is essentially the inverse of the volume derivative of the pressure of the system. As a

related application of the spin-polarization dependence of many-body effects we have calculated the effect of finite spin polarization on the compressibility of the interacting spin-polarized 2D and 3D electron systems within the same infinite ring-diagram approximation. We show these results (only for the unpolarized and the fully polarized cases) in Fig. 5, where again many-body spin-polarization corrections to the interacting compressibility are obvious. Since the interacting compressibility in 2D electron systems can be directly measured [15] with great accuracy, we suggest this as a possible way of estimating the spin-polarization effect on the many-body compressibility.

We conclude by emphasizing that the "spintronic" effective mass (and compressibility) in spin-polarized carrier systems could be strongly spin-polarization dependent and substantially different from the usual unpolarized paramagnetic values. Such a spin-polarization dependence should have serious implications in various spintronic applications. For example, both spin Hall effect [9,10] and spin transistors [11,12] involve the carrier effective mass, which would be explicitly spin-polarization dependent, complicating understanding of the experimental data. We suggest that experiments be carried out to directly test our predicted many-body spin-polarization dependence of the carrier effective mass in 2D semiconductor structure.

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