

## Kertész Line and Embedded Monopoles in QCD

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We propose a new class of defects in QCD which can be viewed as embedded monopoles made of quark and gluon fields. These objects are explicitly gauge invariant and they closely resemble the Nambu monopoles in the standard electroweak model. We argue that the “embedded QCD monopoles” are proliferating in the quark-gluon plasma phase while in the low-temperature hadronic phase the spatial proliferation of these objects is suppressed. At realistic quark masses and zero chemical potential the hadronic and quark-gluon phases are generally believed to be connected by a smooth crossover across which all thermodynamic quantities are nonsingular. We argue that these QCD phases are separated by a well-defined boundary—known as the Kertész line in condensed matter systems—associated with the onset of the proliferation of the embedded QCD monopoles in the quark-gluon plasma phase.

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The phase diagram of quantum chromodynamics has a rich structure in the “chemical potential” ( $\mu$ )—“temperature” ( $T$ ) plane [1]. In particular, at a small chemical potential QCD predicts the existence of a transition at  $T_c \approx 170$  MeV from the low-temperature hadronic (or, “confinement”) phase to the high-temperature quark-gluon (or, “deconfinement”) phase. It is generally believed that at realistic quark masses this transition is a smooth crossover across which all thermodynamic quantities and their derivatives are nonsingular [1,2]. At larger  $\mu$  the phase transition reemerges at a tricritical point and then continues as the first-order phase transition. At even higher chemical potential more exotic phases (such as the color superconductor phase and the color-flavor locking phase) appear [1]. Below we concentrate on the crossover region at moderately small chemical potential.

The  $\mu$ - $T$  phase diagram of QCD in a wide region around the tricritical point, Fig. 1(a), is qualitatively similar to the phase diagram of the standard model of electroweak (EW) interactions in the “Higgs boson mass” ( $M_H$ )—temperature ( $T$ ) plane, Fig. 1(b). As it is well known, the symmetric (high-temperature) and the Higgs (low-temperature) phases in the EW model are separated by a strong first-order EW phase transition at relatively small Higgs boson masses [3]. As the Higgs boson mass increases, the first-order transition weakens and stops at a tricritical end point ( $T^E, M_H^E$ )  $\approx$  (155 GeV, 72 GeV) at which the transition is of the second order [3,4]. At higher  $M_H$  the phase transition becomes a smooth crossover across which all thermodynamical quantities are smooth similarly to the case of QCD.

Another qualitative similarity between QCD and the EW model is that both field theories do *not* possess any topologically stable monopole or vortexlike defects. However, the absence of the stable topological defects does not make the topological structure of the EW model less interesting because it is well known [5] that this model contains the so-called “embedded” defects called the Nambu monopoles [6] and the  $Z$  vortices [7].

Analytical arguments [8] as well as dynamical simulations of hot EW model with the Higgs boson masses  $M_H \sim 30$  GeV and  $M_H \sim 70$  GeV show [9] that the first-order EW phase transition is accompanied by the percolation transition of the  $Z$  vortices and the Nambu monopoles. These embedded defects are suppressed in the Higgs phase and they are forming a dense percolating (condensed) medium in the symmetric phase. As the mass of the Higgs particle increases, the percolation transition does not stop at the tricritical point and it continues into the crossover region [10] still discriminating between the high- and low-temperature phases; Fig. 1(b).

In the condensed matter physics, the percolation transition realized in the absence of the thermodynamic phase transition is usually referred to as the Kertész line [11]. The simplest realization of the Kertész line appears in the Ising model in an external magnetic field. Each configurations of the Ising spins can be associated with a set of the Fortuin-Kasteleyn (FK) clusters [12] which are defined as a set of lattice links connecting nearest spins in the same spin states. The FK clusters are known to be proliferating (percolating) in the high-temperature phase. As the temperature gets lower the percolation of the FK clusters disappears (in the absence of the external magnetic field) at the phase transition (the Curie point). However, at non-zero external field the partition function is analytic in temperature and the phase transition is absent while the percolation transition (the Kertész line) still exists at any value of the external field.

The concept of the Kertész line appears naturally in QCD without reference to any (topological) defects. At high enough temperature or density of the quark matter—for example, in the heavy-ion collision experiments—the hadrons may overlap and form clusters within which the quarks are no more confined. The onset of the quark-gluon plasma phase may be associated with the percolation transition of the hadron clusters [13]. In the context of the field theory the Kertész line was also discussed for the monopole [14] and vortex [15] percolation in compact U(1)

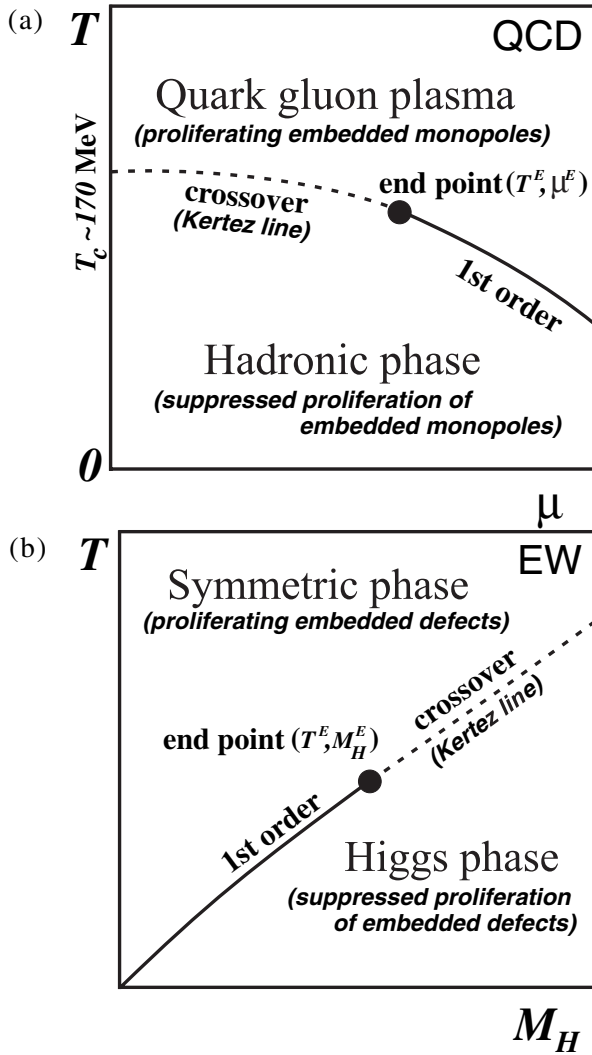


FIG. 1. (a) QCD and (b) EW phase diagrams around tricritical points. The properties of the embedded defects [suggested in QCD in this Letter and found in the EW model in Refs. [8–10]] are indicated in the brackets. The tricritical point, the first transition line, and the Kertész line are depicted as, respectively, the filled circle, the solid line, and the dashed line.

Higgs models, for the Nambu monopole [8], the Z-vortex [9,10], and the center vortex [16] percolation in the case of the SU(2) Higgs model.

Consider the embedded topological defects in the EW model with vanishing Weinberg angle,  $\theta_w = 0$ , for simplicity. The bosonic sector of this model is basically the SU(2) gauge model with the Higgs doublet  $\Phi(x) = (\phi_1(x), \phi_2(x))^T$ . The embedded defects in the EW model are defined with the help of the composite scalar field  $\chi^a(x)$ :

$$\chi^a(x) = -\Phi^\dagger(x)\tau^a\Phi(x), \quad (1)$$

where  $\tau^a$  are the Pauli matrices acting in the isospin space. The field  $\chi^a$  transforms in the adjoint representation of gauge group and can be treated similarly to the triplet Higgs field in the SO(3) Georgi-Glashow model.

In the unitary gauge of the EW model,  $\Phi(x) = (0, \phi(x))^T$ , the composite field  $\chi$  gets automatically fixed to the SO(3) unitary gauge,  $\chi^a = |\vec{\chi}|\delta^{a3}$ . The nonzero expectation value of the Higgs field  $\Phi$  in the Higgs (low-temperature) phase of the EW model guarantees a nonzero expectation value of the composite field  $\chi$  because of the identity  $\vec{\chi}^2 = (\Phi^\dagger\Phi)^2$ . Thus, if the Higgs field  $\Phi$  resides near the classical minimum of the Higgs potential,  $\langle\Phi\rangle = (0, \eta)^T$ , then the composite field  $\chi$  does so near the value [17]  $\chi_0^a \equiv \langle\chi^a\rangle = |\eta|^2\delta^{a3}$ , or

$$\langle\vec{\chi}^2\rangle = \langle\vec{\Phi}^2\rangle^2 \equiv |\eta|^4. \quad (2)$$

The nonzero vacuum expectation value of the composite field  $\chi^a$  in the Higgs phase makes it possible to construct a monopolelike configuration of the EW fields—called the electroweak Nambu or the electroweak monopole [6]—in a manner similar to the 't Hooft-Polyakov [18,19] construction of a monopole in the Georgi-Glashow model. The position of the monopole singularity can be identified with the help of the gauge-invariant 't Hooft tensor [18],

$$\mathcal{F}_{\mu\nu}(\chi, W) = F_{\mu\nu}^a\hat{\chi}^a + \frac{1}{g}\epsilon^{abc}\hat{\chi}^a(D_\mu^{\text{ad}}\hat{\chi})^b(D_\nu^{\text{ad}}\hat{\chi})^c, \quad (3)$$

$$\hat{\chi}^a = \frac{\chi^a}{|\vec{\chi}|},$$

where  $\hat{\chi}^a \equiv \hat{\chi}^a(\Phi)$  is the unit color vector, pointing into the direction of the composite  $\chi$  field (1),  $F_{\mu\nu}^a \equiv F_{\mu\nu}^a(W)$  is the field strength tensor for the SU(2) gauge field  $W_\mu^a$ , and  $(D_\mu^{\text{ad}})^{ab} = \delta^{ab}\partial_\mu + g\epsilon^{abc}W_\mu^c$  is the adjoint derivative. Equation (3) defines the gauge-invariant field strength tensor for the Z-component of the gauge field,  $Z_\mu = W_\mu^a\hat{\chi}^a$ .

The current of the Nambu monopole,

$$k_\nu^{\text{EW}} = \partial_\mu\tilde{\mathcal{F}}_{\mu\nu} \equiv \int_C \frac{\partial X_\nu^C(\tau)}{\partial\tau}\delta^{(4)}(x - X(\tau)), \quad (4)$$

$$\tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\mathcal{F}_{\alpha\beta}$$

has a  $\delta$ -like singularity at the monopole world line C parametrized by the vector  $x_\mu = X_\mu^C(\tau)$ . The location of the embedded monopoles are encoded in the gauge fields  $W_\mu$  and the Higgs fields  $\Phi$  via relations (1), (3), and (4).

The Z-magnetic charge of the Nambu monopole is quantized [5] in units of  $g_m = 4\pi/g$  and is conserved by virtue of its definition (4),  $\partial_\mu k_\mu^{\text{EW}} = 0$ , and therefore the Nambu monopoles can only disappear by annihilating with antimonopoles. In the unitary gauge,  $\hat{\chi}^a = \delta^{a3}$ , the electroweak monopole is just an Abelian monopole singularity in the diagonal gauge field  $Z_\mu \equiv W_\mu^3$ . Therefore, the electroweak monopole is an Abelian monopole embedded into the EW model.

Coming closer to QCD, let us consider for simplicity the SU(2) gauge theory with one species of a (generally, massive) fermion field  $\psi$  which transforms in the fundamental representation of the gauge group. Then one can construct two SU(2) QCD analogues of the electroweak  $\chi$

field (1):

$$\xi_{\Gamma}^a = \bar{\psi}(x)\Gamma\tau^a\psi(x), \quad \Gamma = 1, i\gamma_5, \quad (5)$$

where both the scalar  $\xi^a$  and the pseudoscalar  $\xi_5^a$  fields are the real-valued triplet vectors in the isospin space (we drop the index  $\Gamma$  in  $\xi_{\Gamma}$  if  $\Gamma = 1$ ).

The existence of the isospin vectors (5) allows us to define the currents of the gauge-invariant monopoles in the SU(2) QCD in a manner similar to the EW construction (4):

$$k_{\nu}^{\Gamma} = \partial_{\mu}\tilde{\mathcal{F}}_{\mu\nu}(\xi_{\Gamma}, A), \quad (6)$$

where  $\mathcal{F}_{\mu\nu}$  is the 't Hooft tensor (3) in which the EW gauge field  $W_{\mu}$  is replaced by the SU(2) gluon field  $A_{\mu}$ , and the EW composite field  $\chi$  is substituted by the fermionic composite fields (5). Equation (6) provides an explicitly gauge-independent way to identify monopolelike singularities in QCD using the fermionic degrees of freedom along the ideological line of Ref. [20]. The location of the embedded QCD monopoles is encoded in the gluon  $A_{\mu}^a$  and fermion  $\psi$  fields via relations (5), (3), and (6).

The  $k_{\mu}$  and  $k_{\mu}^5$  fermionic monopoles carry the magnetic charge with respect to, correspondingly,  $\mathcal{A}_{\mu} = A_{\mu}^a\hat{\xi}^a$  and  $\mathcal{A}_{\mu}^5 = A_{\mu}^a\hat{\xi}_5^a$  components of the gauge field. In the unitary  $\hat{\xi}^a = \delta^{a3}$  (or, in the ‘‘pseudo-unitary,’’  $\hat{\xi}_5^a = \delta^{a3}$ ) gauge the  $k_{\mu}$  (or, respectively,  $k_{\mu}^5$ ) fermionic monopoles correspond to monopoles embedded into the diagonal component,  $A_{\mu}^3$ , of the gluon field. One can also consider these monopoles as Abelian monopoles determined in an Abelian gauge [21] which is defined by a requirement of diagonalization of the corresponding composite fermionic field (5). Finally, in gauges, in which the gauge field  $A_{\mu}$  is smooth (i.e., in the Landau gauge), one can consider the embedded monopoles as the hedgehogs in the composite quark bilinears (5).

Thus, in the toy case of the  $N_f = 1$  SU(2) gauge theory one can define two types of the embedded QCD monopoles constructed from vector and pseudovector variables (6). The existence of the topologically nontrivial monopoles (6) is not a dynamically motivated fact. Instead, it is a simple (kinematical) consequence of the existence of the adjoint real-valued fields (5), which are not required to be condensed [22].

In the real case of QCD the zoo of the embedded monopoles is much richer. Indeed, in the SU(3) gauge theory with  $N_f$  massive fermions one can introduce two matrices in the flavor space instead of two composite scalar fields (5):

$$\Xi_{ff',\Gamma}^a(x) = \bar{\psi}_{f'}(x)\Gamma\lambda^a\psi_f(x), \quad (7)$$

where  $\lambda^a$ ,  $a = 1, \dots, 8$  are the SU(3) Gell-Mann color matrices and  $f, f' = 1, \dots, N_f$  are the flavor indices. Each element of these matrices transforms in the adjoint representation of the SU(3) gauge group.

To characterize the quark embedded defects in QCD we use the fact that the global flavor symmetry is explicitly broken by mass terms at the Lagrangian level (we consider the realistic case of nonequal quark masses). Using flavor transformations one can rotate the quark fields into a flavor basis where the mass matrix is diagonal. In this basis the diagonal elements of the matrices (7) should be considered as the real-valued color octet fields  $\xi_{f,\Gamma}^a \equiv \Xi_{ff,\Gamma}^a$  (no summation over the index  $f$ ). The diagonal elements  $\xi_{f,\Gamma}^a$  are then used to construct the gauge-invariant embedded monopoles as in the toy  $N_c = 2, N_f = 1$  case (6). Given the octet vectors (7) the monopole charges in the  $N_c = 3$  color case can be characterized by integer magnetic charges similarly to the monopoles in the SU( $N_c$ ) Higgs models [23].

Therefore, in QCD with  $N_f$  massive fermions there are two types of monopoles associated with each quark field (i.e., we have  $2N_f$  embedded monopoles in total). The trajectories and charges of these defects can be defined analogously Eq. (6). In principle, one can also define the ‘‘mixed’’ defects which involve quark-antiquark bilinears of different flavors. Note that in the massless case the chiral rotations mix the quark bilinears (5) with each other. However, in the real QCD case we expect that the monopoles constructed from scalar and pseudoscalar triplet fields must be different due to the breaking of the axial symmetry by the ground state.

The composite quark fields  $\xi_f^a$  and  $\xi_{f,5}^a$  play the role of the adjoint Higgs field in the SU(3) version of the Georgi-Glashow model. The existence of the stable monopoles in the Georgi-Glashow model is guaranteed by the spontaneous breaking of the SU(3) symmetry by the Higgs condensate. Contrary to the Georgi-Glashow model, the color symmetry in QCD is known to be unbroken [22]. Nevertheless we argue below that in QCD the role of the Higgs condensate is played by chiral condensates [24], which make the definition of the embedded QCD monopoles physically meaningful.

Properties of the defects can be guessed from the behavior of the condensates made of the octet,  $\xi^a = \sum_f \Xi_{ff}^a$ , and the axial octet,  $\xi_5^a = \sum_f \Xi_{ff,5}^a$ , fields. Obviously, due to the unbroken color invariance,  $\langle \xi_{\Gamma}^a \rangle = 0$ . Thus, the length of the composite Higgs octet field is characterized by the four-quark condensates  $\langle \tilde{\xi}_{\Gamma}^2 \rangle \equiv \langle \bar{\psi}\Gamma\lambda^a\psi\bar{\psi}\Gamma\lambda^a\psi \rangle$  with  $\Gamma = 1, \gamma_5$ . The factorization hypothesis [25] makes it possible to express the four-quark condensates in terms of the chiral condensate  $\langle \bar{\psi}\psi \rangle$ ,

$$\langle \tilde{\xi}_{\Gamma}^2 \rangle = C_{\Gamma} \langle \bar{\psi}\psi \rangle^2, \quad (8)$$

where  $C_{\Gamma}$  is a numerical factor, and  $\psi = u$  or  $d$ .

The QCD relation (8) is remarkably similar to the EW relation (2). At low temperatures the composite octet fields  $\xi_{\Gamma}^a$  are large similarly to the low-temperature behavior of the composite  $\chi$  field in the EW model. As temperature increases, both the four-quark condensate in QCD and the

Higgs expectation value in the EW model are diminishing, and they have small but nonvanishing values at the corresponding crossover temperatures.

In the EW model the large zero-temperature value of the  $\chi$ -field condensate gives rise to a large mass of the Nambu monopoles which suppresses the monopole formation. This fact may be understood intuitively since the field  $\chi$  must be vanishing inside the core of the Nambu monopole and this is unfavorable in the presence of the  $\chi$  condensate. Similarly, the embedded monopoles in QCD force the octet fields  $\xi_{\Gamma}^a$  to be vanishing in the center of the monopole in order to support their hedgehog structure. This is energetically unfavorable at low temperatures because of the presence of the four-quark condensates (8). However, as the temperature (and the chemical potential) increases, the condensates (8) continuously melt and the suppression of monopoles becomes less and less effective. At very high temperatures the value of the condensates is negligibly small and the embedded QCD monopoles must form a dense and percolating network—supported by thermal fluctuations—similarly to the behavior of the embedded EW defects [9]. Since at realistic quark masses the phase transition is absent [26], the onset of the percolation transition marks the Kertész line in QCD as shown in Fig. 1(a) by the dashed line. It seems very plausible that the percolation of the hadron clusters in the quark-gluon phase [13] is related to the proliferation of the embedded QCD monopoles.

Our considerations are based on the temperature behavior of the four-quark condensates which is qualitatively valid [27] beyond the simple factorization formula (8). Moreover, quantitative estimations of Ref. [27] show that as temperature increases the vacuum expectation value of the pseudoscalar octet fields is dropping faster compared to the scalar octet fields. Therefore, the QCD Kertész line, Fig. 1(a), may in fact be split into two lines since the onset of the percolation of the pseudoscalar embedded monopoles may happen at (much) lower temperature compared to the monopoles associated with the scalar  $\xi$  field. In the real QCD case the Kertész line should inevitably be split because the onset of percolation of the embedded monopoles associated with different quark fields should happen at different temperatures due to the difference in the quark masses.

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