Collisionless Reconnection in an Electron-Positron Plasma

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(Received 13 November 2004; revised manuscript received 8 April 2005; published 5 December 2005)

Electromagnetic particle-in-cell simulations of fast collisionless reconnection in a two-dimensional electron-positron plasma (without an equilibrium guide field) are presented. A generalized Ohm's law in which the Hall current cancels out exactly is given. It is suggested that the key to fast reconnection in this plasma is the localization caused by the off-diagonal components of the pressure tensors, which produce an effect analogous to a spatially localized resistivity.

DOI: 10.1103/PhysRevLett.95.245001 PACS numbers: 52.35.Vd, 52.27.Aj, 52.65.Rr

The conditions under which collisionless plasmas exhibit fast nonlinear reconnection have been a dominant subject of research over a few decades. Under standard approximations, collisionless hydrogen plasmas, consisting of electrons (of mass m_e and charge $-e$) and protons (of mass m_i and charge e), can be shown to obey the generalized Ohm's law

$$\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c} = \frac{m_e}{ne^2}\left[\frac{\partial \mathbf{J}}{\partial t} + \nabla\cdot(\mathbf{J}\mathbf{v} + \mathbf{v}\mathbf{J})\right] - \frac{\nabla\cdot\mathbf{P}_e}{ne} + \frac{\mathbf{J}\times\mathbf{B}}{nec} \tag{1}$$

discussed in textbooks [1], where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ is the plasma flow velocity, c is the speed of light, $\mathbf{J}$ is the current density, $\mathbf{P}_e$ is the electron pressure tensor, and n is the electron and ion densities. In recent years, it has been shown, primarily by means of simulations based on fluid, hybrid as well as particle-in-cell (PIC) methods, that the Hall current and electron pressure terms in the generalized Ohm's law (1) play an important role in realizing fast reconnection. It has been suggested that this is because of a separation of spatial scales between electron and ion flows, which, in turn, causes a separation of scales between the thin current sheet and the reconnection electric field produced in the reconnection layer. Whereas electrons contribute primarily to the current density in the thin current sheet, which has a characteristic width of the order of the electron skin depth, ions decouple from electrons over a broader spatial scale of the order of the ion skin depth (when the equilibrium guide field is zero), and control the reconnection rate. It has also been suggested that there is a strong link between fast reconnection and the excitation of whistler waves produced by the Hall current at small scales [2,3]. Unlike Alfvén waves, whistler waves have a dispersion relation that depends quadratically on the wave number, and hence their phase velocity increases linearly with increasing wave number (or decreasing spatial scale).

In this Letter, we study the problem of fast reconnection in electron-positron (or pair) plasmas. There has been growing interest in pair plasmas for their applications to astrophysical as well as laboratory plasma physics. Important astrophysical applications include extragalactic jets [4,5] and winds and jets from pulsars [6]. It has been suggested that such winds are "magnetically striped" [7], that is, they are composed of compartments of magnetic fields of alternate sign, where magnetic field reconnection and annihilation convert magnetic energy into particle energy. Recent laboratory experiments of electron-positron plasmas have offered significant new insights into beam-plasma instabilities [8,9], and new experiments on magnetically confined toroidal devices are under way [10]. Earlier analytic and simulation studies of pair plasmas have focused on issues of particle acceleration [11–13].

In addition to the various applications mentioned above, studies of magnetic reconnection in electron-positron plasmas present a new opportunity to examine critically the question of the ingredients that are essential in realizing regimes of fast magnetic reconnection. In a pair plasma, the electron and ion skin depth parameters are identical. We demonstrate, by means of PIC simulations, that fast reconnection occurs in a pair plasma without a separation of spatial scales between electron and positron flows, and without the intervention of the Hall current which cancels out exactly in the generalized Ohm's Law. Because of this cancellation, whistler waves do not exist in pair plasmas [14–16]. Despite the absence of the Hall current and whistler waves, our numerical results provide clear evidence of fast collisionless reconnection due to the localization caused by the off-diagonal components of the pressure tensors, which produce an effect analogous to a spatially localized resistivity.

We begin with a simple derivation of the generalized Ohm's law for collisionless pair plasmas. The momentum equation for the ion and electron fluids is given by

$$\frac{\partial}{\partial t}(m_{i,e}n_{i,e}\mathbf{v}_{i,e}) + \nabla\cdot(m_{i,e}n_{i,e}\mathbf{v}_{i,e}\mathbf{v}_{i,e}) = n_{i,e}q_{i,e}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_{i,e}\times\mathbf{B}\right) - \nabla\cdot\mathbf{P}_{i,e}, \tag{2}$$

where $q_i = e$, $q_e = -e$. Multiplying the positron equation by m_e, the electron equation by m_i, and subtracting the second from the first, we obtain

$$\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} = \frac{m_e}{2ne^2}\left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{J}\mathbf{v} + \mathbf{v}\mathbf{J})\right] - \frac{\nabla \cdot (\mathbf{P}_e - \mathbf{P}_i)}{2ne}, \tag{3}$$

where we have assumed quasineutrality, that is $n_e = n_i = n$, and used the exact relations $m_e = m_i$, $\mathbf{v}_i = \mathbf{v} + \mathbf{J}/2ne$, and $\mathbf{v}_e = \mathbf{v} - \mathbf{J}/2ne$. If we compare (1) with (3), we see readily that the Hall current term has cancelled out exactly in (3). However, the mechanisms that break field lines in a collisionless pair plasma are similar to those in a hydrogen plasma, namely, finite particle inertia and the pressure tensor. What is different in a pair plasma is the absence of scale separation between the electron and the positron skin depth.

To investigate whether a collisionless pair plasma can exhibit fast reconnection, we carry out electromagnetic PIC simulations using an initial condition similar to the geospace environmental modeling (GEM) reconnection challenge [17]. The plasma is assumed to be in a Harris equilibrium in the x-z plane with no equilibrium guide field ($B_y = 0$). The magnetic field B_x and the particle number density n are given, respectively, by $B_x = B_0 \tanh(z/a)$, and $n = n_0 \,\mathrm{sech}^2 z/a + n_b$, where B_0, n_0, and n_b are positive constants. The asymptotic magnetic field B_0 and the particle density n_0 satisfy the equilibrium condition $B_0^2/8\pi = n_0(T_e + T_i)$. In equilibrium, the positrons (electrons) have drift velocities $v_{di}(v_{de})$ in the y direction, and obey the relations $|v_{di} - v_{de}| = (2c/aeB_0)(T_e + T_i)$ and $v_{de}/v_{di} = -T_e/T_i$. Most of the parameters chosen are similar to those for the GEM challenge: the background density $n_b = 0.2n_0$, the ratio of the Alfvén velocity $v_A\{= B_0/([4\pi n_0(m_i + m_e)]^{1/2}\}$ to the speed of light c is $v_A/c = 1/20$, the thickness of the current sheet a is $0.5d_i$, where $d_i = c/\omega_{pi}$ is the positron skin depth, and $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$ is the positron plasma frequency. We consider two values of the temperature ratio, $T_i/T_e = 5$, which is the same as that of the GEM challenge, and $T_i/T_e = 1$. The total number of particles for each species is 3 725 056. Our time step is specified by the choice $\omega_{pi}\Delta t = 0.02$. The simulation domain has $L_x \times L_z = 512 \times 256$ grid points. In this system, $d_i = 20\Delta$, where Δ is the grid spacing. We use periodic boundary conditions along x, with the boundaries located at $x = -L_x/2$ and $x = L_x/2$. The z boundaries, $z = -L_z/2$ and $z = L_z/2$, are assumed to be conducting walls.

In this two-dimensional problem, the magnetic field can be represented as $\mathbf{B} = \hat{\mathbf{y}} \times \nabla\Psi(x, z, t) + B_y(x, z, t)\hat{\mathbf{y}}$, where $\Psi(x, z, t)$ is a flux function. We now disturb this equilibrium at $t = 0$ by imposing a large perturbation $\Psi_1 = 0.1B_0 d_i \cos(2\pi x/L_x)\cos(\pi z/L_z)$. The imposition of a large perturbation enables the realization of a nonlinear quasisteady state quickly. In what follows, we present results from three cases: (a) $m_i = m_e$, $T_i/T_e = 5$, (b) $m_i = m_e$, $T_i/T_e = 1$, and (c) $m_i = 25m_e$, $T_i/T_e = 5$.

We begin with case (b). Figure 1 shows contour plots for the magnetic flux function Ψ at the time $\Omega_i t = 10.7$ and $\Omega_i t = 17.7$, and the out-of-plane electric field E_y at the time $\Omega_i t = 17.7$, when the reconnection electric field E_y attains its maximum value $0.24B_0 v_A/c$. (Here Ω_i is the positron cyclotron frequency in the asymptotic equilibrium magnetic field B_0.) It turns out that this value is similar to that of the GEM challenge reconnection electric field, which lies in the range $0.2 \sim 0.3B_0 v_A/c$. (See [17] and companion papers that use PIC and Hall MHD simulations.) At $\Omega_i t = 17.7$, the system shows evidence of secondary tearing instabilities of the current sheet. Figure 2 (red curve) shows the time history of reconnection electric field E_y at the central X point. [For this plot, E_y was spatially averaged over 20Δ (x direction) $\times 10\Delta$ (z direction) around the X point, and temporarily averaged over $250\Delta t (= 0.67\Omega_i^{-1})$.] We note that already at $\Omega_i t = 10.7$, the reconnection electric field E_y has attained the large value of $0.1B_0 v_A/c$ (which is of the same order as the GEM value), without the intervention of secondary tearing instabilities. The field E_y at the X point eventually reaches its maximum value, $0.24B_0 v_A/c$ at $\Omega_i t = 17.7$, after which it gradually decays.

To delineate more clearly the role of secondary tearing instabilities, we perform two other simulations and plot the results for E_y in Fig. 2. The first (blue curve) uses a system of low aspect ratio, $L_x/L_z = 1(L_x \times L_z = 256 \times 256$ grid points), and uses the initial perturbation $\Psi_1 = 0.01B_0 d_i \cos(2\pi x/L_x)\cos(\pi z/L_z)$ to initiate reconnection, while the other (black curve) uses a system

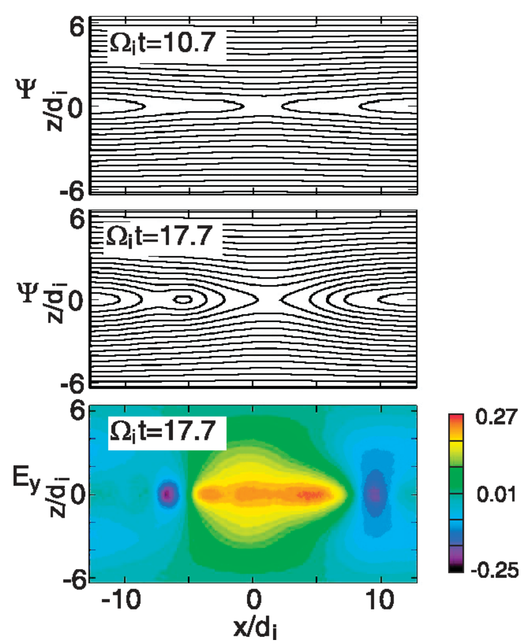

FIG. 1 (color). Contour plots of Ψ and E_y for $T_i/T_e = 1$.

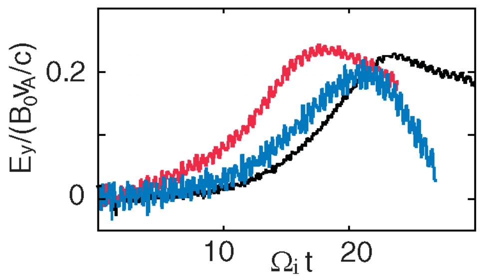

FIG. 2 (color). Time evolution of E_y at the X point for $T_i/T_e = 1$: (red) $\Psi_1 = 0.1 B_0 d_i$ and $L_x/L_z = 2$, (blue) $\Psi_1 = 0.01 B_0 d_i$ and $L_x/L_z = 1$, (black) $\Psi_1 = 0$ and $L_x/L_z = 2$.

of higher aspect ratio, $L_x/L_z = 2(L_x \times L_z = 512 \times 256$ grid points), and starts from noise, with $\Psi_1 = 0$. In the low-aspect-ratio case, the secondary tearing instability is absent, while in the case of higher aspect ratio evolving from noise, secondary tearing instabilities are seen to develop early. Despite these differences, the maximum reconnection rate attained in the two cases are not very different, providing further evidence for our claim that the realization of a large reconnection electric field in pair plasmas does not depend primarily on the excitation of secondary tearing instabilities.

We now investigate the structure of the B_y field in the quasisteady regime. In standard hydrogen plasmas, the self-generated B_y field is well known to have a quadrupolar structure, attributed to the scale separation between electrons and ions. Figure 3 shows the B_y field in the quasisteady nonlinear regime for three cases: (a), (b), and (c). Figure 3(c) shows clear quadrupolar structure. Whereas Fig. 3(a) for pair plasmas shows evidence of a self-generated B_y due to the different electron and positron flows in the x-z plane when $T_i \neq T_e$, it does not exhibit quadrupolar structure. On the other hand, Fig. 3(b) for equal-temperature pair plasmas shows little B_y-field generation. These findings on pair plasmas are qualitatively different from the results obtained for hydrogen plasmas [Fig. 3(c)].

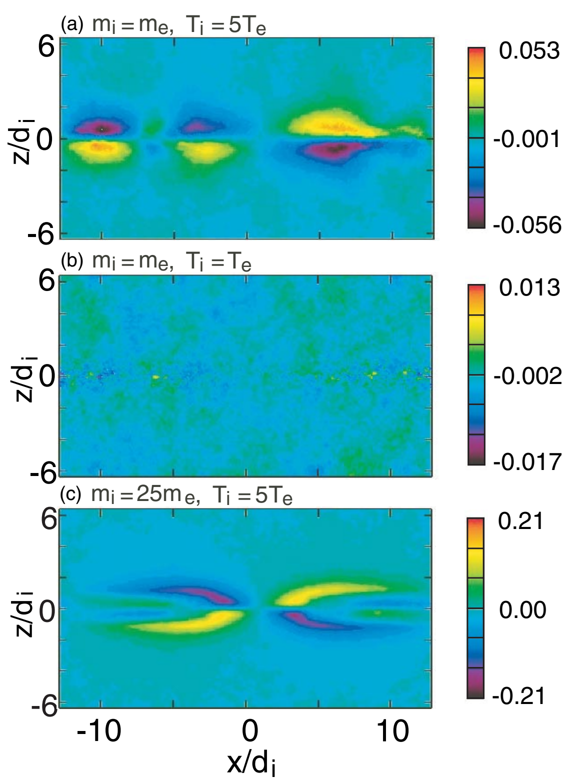

FIG. 3 (color). Contour plots of B_y/B_0: (a) $m_i = m_e$, $T_i/T_e = 5$ (b) $m_i = m_e$, $T_i/T_e = 1$ (c) $m_i = 25 m_e$, $T_i/T_e = 5$.

The top panel of Fig. 4 shows a plot of the reconnection electric field E_y along a straight line parallel to the z axis through the central X point. It is clear by inspection of Fig. 4 that the reconnection electric field is supported by contributions from the electron and positron pressure tensors and convective nonlinearities, and that contribution of the pair pressure tensor is localized in the vicinity of the X point.

We denote the electric field from the nonideal MHD terms in the generalized Ohm's law [the right-hand side of (3)] as E_y^*. In the middle panel of Fig. 4, we plot E_y^* and the current density J_y. The similarity in the spatial profiles of E_y^* and J_y suggests the definition of an effective enhanced resistivity $\eta^* = E_y^*/J_y$, localized in the diffusion region, with the spatial profile given in the bottom panel of Fig. 4. We propose that the notion of this effective resistivity, η^*, localized in the diffusion region, provides an explanation for the realization of the X-type geometry and fast reconnection of the Petschek type [18] in pair plasmas.

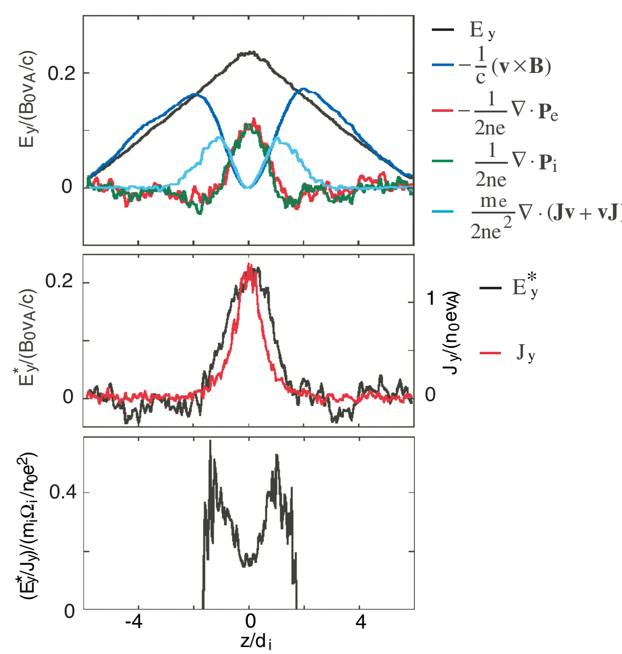

FIG. 4 (color). (top) E_y along a line through the X point for $T_i/T_e = 1$, (middle) E_y^* and J_y, (bottom) the ratio E_y^*/J_y in the diffusion region.

The physical origin of the effective resistivity can be understood at a microscopic level by a consideration of particle orbits. Because of the strong gradient of the magnetic field around the X point and the breakdown of adiabatic invariants, the particle motion becomes chaotic. As particles travel to the X point, they are accelerated in the y direction by the electric field E_y. After acceleration for a finite time during which the particles experience random kicks from the electric field, the particles are eventually expelled to the downstream region. This finite confinement time in the diffusion region produces an effective collisionless resistivity [19,20]. The resistivity at the X point can be estimated heuristically by the expression $\eta^* = m_i/(n_m e^2 \tau)$, where n_m is the density at $z = 0$ and τ is the acceleration time. Using the estimate $\tau = d/v_{zd}$ [20], where d is the half-width of the diffusion region and v_{zd} is the inflow speed at $z = d$, and substituting values observed in the simulation, that is, $v_{zd} \sim 0.22 v_A$, $d \sim 1.8 d_i$, and $n_m \sim 0.46 n_0$, we obtain $\eta^* \sim 0.19(m_i \Omega_i / n_0 e^2)$. If we multiply $J_y \sim 1.4 n_0 e v_A$ by η^*, we obtain $E_y \sim 0.27 B_0 v_A / c$, in agreement with the observed value.

In conclusion, we have simulated collisionless reconnection in an electron-positron plasma, and demonstrated fast reconnection rates, of the same order as those obtained in the GEM reconnection challenge. We demonstrate that the Hall current term, which is associated with scale separation between electrons and ions and whistler waves in a conventional hydrogen plasma and is widely believed to facilitate fast reconnection, is not necessary for fast reconnection in a pair plasma. The quadrupolar magnetic field pattern, which is often used as an observational proxy for collisionless reconnection in a conventional plasma, is absent in an electron-positron plasma. The nonlinear convective velocity and the electron and positron pressure tensors support the quasisteady reconnection electric field in such a plasma. Despite the absence of the Hall current and whistler waves, the reconnection geometry adjusts to form X points that can sustain a fast reconnection rate. This result appears to be a consequence of the localization of an effective collisionless resistivity produced by the pressure tensors of electrons and positrons.

We would like to thank Professor J. Goodman and Dr. M. Hesse for stimulating our interest in this subject, and Dr. C. S. Ng for useful discussions. We would also like to thank Dr. D. Newman for constructive criticism that helped improve the presentation of our results. This research is supported by the National Science Foundation, the Department of Energy, and the Paul Chair Endowment at the University of New Hampshire. Computational resources were provided by Center for Computational Sciences at Oak Ridge National Laboratory.

Note added.—After this work was submitted for publication, A. Ishizawa and R. Horiuchi informed us that their PIC simulations of quasisteady collisionless reconnection in an open system with mass ratio $m_e/m_i = 1/800$ exhibits suppression of the Hall current term due to gyroviscous cancellation at scales between the ion skin depth and the ion meandering orbit scale [21]. Thus fast collisionless reconnection appears to occur in their system despite this suppression. The results presented here are complementary to theirs because the Hall current term cancels out exactly everywhere in a pair plasma, yet we have fast reconnection.

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