Spin Wave Mode Excited by Spin-Polarized Current in a Magnetic Nanocontact is a Standing Self-Localized Wave Bullet

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We demonstrate that the lowest threshold of spin-wave excitation in an in-plane magnetized magnetic nanocontact driven by spin-polarized current is achieved for a nonlinear self-localized spin-wave mode—standing spin-wave bullet—stabilized by current-induced nonlinear dissipation. This nonlinear mode has a nonpropagating evanescent character, is localized in the region comparable with the contact radius, and has a frequency that is lower than the frequency of the linear ferromagnetic resonance. The threshold current and generated frequency at the threshold theoretically calculated for this mode are in quantitative agreement with experiment.

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Recently it has been theoretically predicted [1,2] and experimentally observed [3–7] that spin-polarized current passing through a thin magnetic layer (''free'' layer of a magnetic layered structure) can excite microwave magnetization oscillations in this layer. A spatially uniform nonlinear theory explaining many experimentally observed features of this phenomenon was developed in a series of theoretical papers [8–11]. At the same time, the exact nature of the dynamic mode excited in a magnetic nanocontact was not determined in [8–11].

The most complete theoretical analysis of the nature of the spin-wave eigenmode excited by spin-polarized current in a nanocontact geometry was performed by Slonczewski [8]. He developed a spatially nonuniform linear theory of spin-wave excitations in a nanocontact, where the "free" ferromagnetic layer is infinite in plane, while the spin-polarized current traversing this layer has a finite cross section $S = \pi R_c^2$, where R_c is the contact radius. Considering a perpendicularly magnetized nanocontact, Slonczewski showed that in the linear case the lowest threshold of excitation by spin-polarized current is achieved for an exchange-dominated cylindrical spin-wave mode having wave number $k_0 \simeq 1.2/R_c$, frequency

$$\omega(k_0) = \omega_0 + Dk_0^2, \quad (1)$$

and traveling out of the region of current localization [8]. Here ω_0 is the ferromagnetic resonance (FMR) frequency and D is the spin-wave dispersion coefficient determined by the exchange interaction.

It was also shown that the threshold current I_{th} in such a geometry consists of two additive terms: the first one arises from the radiative loss of energy carried by the propagating spin wave out of the region of current localization, while the second one is caused by the usual energy dissipation in the current-carrying region:

$$I_{th}^{lin} \simeq 1.86\frac{D}{\sigma R_c^2} + \frac{\Gamma(H)}{\sigma}. \quad (2)$$

Here $\sigma = \varepsilon g \mu_B / 2eM_0 dS$ [ε is the spin-polarization efficiency defined in [1,8], g is the spectroscopic Lande factor, μ_B is the Bohr magneton, e is the modulus of the electron charge, M_0 is the saturation magnetization, d is the thickness of the free magnetic layer, S is the cross-section area of the nanocontact], and $\Gamma(H)$ is the spin wave damping dependent on the bias magnetic field H.

It turns out that for a typical nanocontact of the radius $R_c \sim$ 20–30 nm the radiative losses are about 1 order of magnitude larger than the direct energy dissipation, and should give the main contribution into the threshold current. This result, however, contradicts experimental observations [see, e.g., [4]]: the experimentally measured magnitude of the threshold current in an in-plane magnetized nanocontact is much smaller than the value predicted by Eq. (2), although the dependence of this current on the magnetic field H is satisfactorily described by this equation.

In this Letter we develop a spatially nonuniform nonlinear theory of spin-wave excitation by spin-polarized current in a nanocontact geometry for the case of the in-plane magnetization. We show that in an in-plane magnetized magnetic film the competition between the nonlinearity and exchange-related dispersion leads to the formation of a stationary two-dimensional self-localized nonpropagating spin-wave mode. Such nonlinear self-localized wave modes in two- or three-dimensional cases are conventionally called wave "bullets" [12]. The frequency of this spin-wave "bullet" is shifted by the nonlinearity below the spectrum of linear spin waves and, therefore, this nonlinear mode has an evanescent character with vanishing radiative losses, which leads to a substantial decrease of its threshold current I_{th} in comparison to the linear propagating mode Eq. (2).

To describe the generation of a spin-wave bullet by the spin-polarized current we consider a free ferromagnetic layer, infinite in $y - z$ plane and having finite thickness d in the x direction (d is assumed to be sufficiently small for us to consider that the magnetization $\boldsymbol{M}$ is constant along the film thickness, and that the dipole-dipole interaction can be described by a simple demagnetization field). We assume that the internal magnetic field $H = H_{app} + H_{ex}$, consisting of the applied H_{app} and interlayer exchange H_{ex} fields, is applied in the z direction in the film plane. Using the standard Hamiltonian spin-wave formalism [13], which has been successfully used to develop a spatially uniform nonlinear model of spin-wave generation by spin-polarized current [9,10], one can derive an approximate equation for the dimensionless complex spin-wave amplitude $b \equiv b(t, \mathbf{r})$:

$$\frac{\partial b}{\partial t} = -i[\omega_0 b - D\Delta b + N|b|^2 b] - \Gamma b + f(r/R_c)\sigma I b - f(r/R_c)\sigma I |b|^2 b. \quad (3)$$

Here $\omega_0 \equiv \sqrt{\omega_H(\omega_H + \omega_M)}$ is the linear FMR frequency ($\omega_H \equiv \gamma H$, $\omega_M \equiv 4\pi\gamma M_0$, and γ is the gyromagnetic ratio), $D \equiv (2A/M_0)\partial\omega_0/\partial H = (2\gamma A/M_0)(\omega_H + \omega_M/2)/\omega_0$ is the dispersion coefficient for spin waves (A is the exchange stiffness), Δ is the two-dimensional Laplace operator in the film plane, $N = -\omega_H\omega_M(\omega_H + \omega_M/4)/\omega_0(\omega_H + \omega_M/2)$ is the coefficient describing nonlinear frequency shift, $\Gamma \equiv \alpha_G(\omega_H + \omega_M/2)$ is the spin-wave damping rate (α_G is the dimensionless Gilbert damping parameter). The dimensionless function $f(x)$ describes the spatial distribution of the spin-polarized current. The dimensionless spin-wave amplitude b is connected with the z component of the magnetization by the equation $|b|^2 = (M_0 - M_z)/2M_0$.

Equation (3) differs from the Eq. (9) in [8] [which resulted in the solution (2)] by the presence of two additional nonlinear terms: the term containing the coefficient N and describing a nonlinear frequency shift of the excited mode, and the last term describing the current-induced positive nonlinear damping that stops the increase of the amplitude of the excited mode at relatively large currents. Also, since the Eq. (3) was obtained as a Taylor expansion it is literally correct only for sufficiently small spin-wave amplitudes $|b| < 1$.

Without damping and current terms ($\Gamma = 0$, $I = 0$) Eq. (3) coincides with the well-known $(2 + 1)$-dimensional nonlinear Schrödinger equation (NSE) [14]. In the considered case of an in-plane magnetized film the nonlinear coefficient N is negative, and the nonlinearity and dispersion satisfy the well-known Lighthill criterion $ND < 0$ (i.e., they act in opposite directions), and the NSE has a nonlinear self-localized radially symmetric standing solitonic solution (or the solution in the form of a standing spin-wave bullet)

$$b(t, r) = B_0\psi(r/\ell)e^{-i\omega t}, \quad (4)$$

where dimensionless function $\psi(x)$, having maximum value of 2.2 at $x = 0$, describes the profile of the bullet. This function is the localized solution of the equation

$$\psi'' + \frac{1}{x}\psi' + \psi^3 - \psi = 0, \quad (5)$$

which has to be found numerically [see, e.g., [12,15]].

In Eq. (4) B_0, ℓ, and ω are the characteristic amplitude, characteristic size, and frequency of the bullet, respectively. Among these three parameters only one is independent. Taking the amplitude B_0 as an independent parameter, we can express the two other parameters as

$$\omega = \omega_0 + NB_0^2, \qquad \ell = \frac{\sqrt{|D/N|}}{B_0}. \quad (6)$$

We would like to stress that the frequency of the spin-wave bullet lies below the linear frequency ω_0 of the ferromagnetic resonance [see Eq. (6), and note that $N < 0$], i.e., outside the spectrum of linear spin waves. This is the main reason for the self-localization of the spin-wave bullet, as the effective wave number of the spin-wave mode with frequency (6) is purely imaginary. It also follows from Eq. (4) and the expansion condition $|b| < 1$ that the maximum magnitude of B_0 for which our perturbative approach is still correct is $B_0 = 0.46$.

It is well known [14] that the bulletlike solutions of $(2 + 1)$-dimensional NSE are unstable with respect to the small perturbations: the wave packets having the bullet shape (4), but amplitudes smaller than B_0, decay due to the dispersion spreading, while the wave packets having amplitudes higher than B_0 collapse due to the nonlinearity. At the same time, Eq. (3) with both Gilbert dissipation Γ and current I is a two-dimensional analog of a Ginzburg-Landau equation that is known to have stable localized solutions [see, e.g., review [16]].

One can assume that for a small damping rate Γ and current I the full nonconservative equation (3) will have a bulletlike solution, only slightly different from the exact solution Eq. (4) of the conservative NSE equation. It is clear, however, that not all of such solutions can be supported in our case. For example, small-amplitude bullets, for which $\ell \gg R_c$, practically do not interact with the spatially localized current and will decay due to the linear dissipation. The large-amplitude ($B_0 \geq 1$) bullets, on the other hand, will also decay because the effective damping $\Gamma - \sigma I(1 - |b|^2)$ for them changes sign and becomes positive.

To find if it is possible to balance the effects of the Gilbert damping and spin-polarized current for a certain amplitude of the excited spin-wave mode, we shall consider the time evolution of the mode energy

$$E \equiv \int_0^\infty |b(t, r)|^2 r dr, \quad (7)$$

which is conserved in the case of a conservative NSE (when $\Gamma = 0$, $I = 0$). From the full Eq. (3) one can find an exact equation for the time rate of the energy variation dE/dt:

$$\frac{dE}{dt} = -2\Gamma E + 2\sigma I \int_0^\infty f(r/R_c)|b|^2(1 - |b|^2) r dr. \quad (8)$$

Assuming that the mode profile is approximately the same as the profile of a bullet, one can substitute Eq. (4)

for b in Eq. (8). This gives the following equation

$$\frac{dE}{dt} = 2\left|\frac{D}{N}\right|\{\sigma I[\eta_2(qB_0) - B_0^2\eta_4(qB_0)] - \chi\Gamma\}, \quad (9)$$

where

$$\chi \equiv \int_0^\infty \psi^2(x)xdx \approx 1.86 \quad (10)$$

is the constant form factor of the bullet,

$$q \equiv \sqrt{\left|\frac{N}{D/R_c^2}\right|} \quad (11)$$

is the parameter describing the strength of nonlinearity relative to exchange-originated dispersion for the nanocontact of the radius R_c, and $\eta_n(qB_0)$ is defined as

$$\eta_n(qB_0) \equiv \int_0^\infty f(x/qB_0)\psi^n(x)xdx. \quad (12)$$

The stationary ($dE/dt = 0$) solution of Eq. (9) exists if

$$\frac{\sigma I}{\Gamma} = \frac{\chi}{\eta_2(qB_0) - B_0^2\eta_4(qB_0)}. \quad (13)$$

This equation implicitly defines the amplitude B_0 (and, therefore, frequency ω and size ℓ) of the stationary bullet as a function of the system parameters ($\sigma I/\Gamma$ and q).

On the other hand, Eq. (13) can be interpreted as an equation that defines a current magnitude I which is necessary to support a stationary spin-wave bullet of the amplitude B_0. The dependence of the normalized bias current $\zeta(B_0) = I(B_0)/I_{\rm min}$ (where $I_{\rm min} \equiv \Gamma/\sigma$) for two values of the nonlinearity factor q is shown in Fig. 1 for the case of a steplike current distribution [$f(x) = 1$ if $x < 1$ and $f(x) = 0$ otherwise]. This dependence has a clear minimum corresponding to the amplitude $B_0 = B_{\rm th}$ of a bullet formed at the threshold of microwave generation by spin-polarized current. Note that the normalized current $\zeta(B_0)$ shown in Fig. 1 was denoted as "supercriticality" ζ in our spatially uniform theory [10].

It is clear from Fig. 1 that in the case of a reasonably large nonlinearity factor $q \gtrsim 3$ the threshold current $I_{\rm th} = I(B_{\rm th})$, obtained by minimization of Eq. (13), only slightly exceeds the minimum possible value $I_{\rm min} \equiv \Gamma/\sigma$. Above the threshold, for any $I^* > I_{\rm th}$, there are two possible stationary amplitudes of the generated spin-wave bullet: $B^*_{\rm min}$ and $B^*_{\rm max}$. As usual, the low-amplitude branch $B^*_{\rm min}$ (for which $B_0 < B_{\rm th}$) is unstable, i.e., for any $B_0 < B^*_{\rm min}$ the mode amplitude will decay to the noise level ($B_0 \to 0$), while for any $B_0 > B^*_{\rm min}$ the mode amplitude will increase to the stable value $B^*_{\rm max}$, see Fig. 1.

Our analytical result Eq. (13) is heavily based on the assumption that the profile of the spin-wave mode generated at the threshold is approximately the same as the profile of a stationary bullet (4). To check the validity of this assumption we solved Eq. (3) numerically. The results of comparison of the spin-wave excitation profiles at the threshold obtained for a typical set of experimental parameters [4] from the analytical solution Eq. (4) (solid line) and numerical solution of Eq. (3) (black dots) are shown in Fig. 2. One clearly sees that the numerical profile of the nonlinear eigenmode is practically indistinguishable from

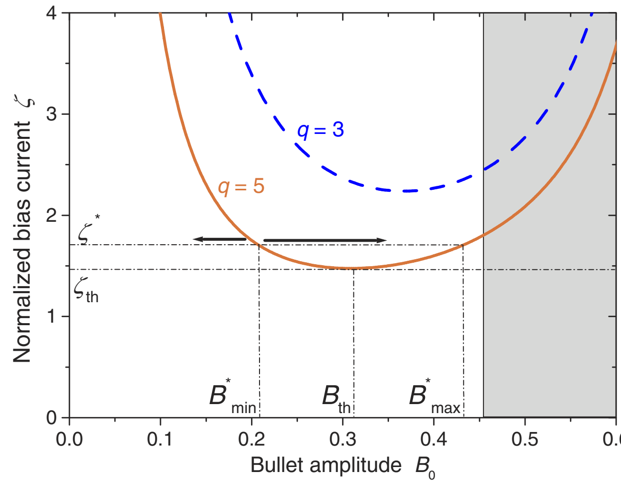

FIG. 1 (color online). Dependence (13) of the normalized current $\zeta = \sigma I/\Gamma$ on the bullet amplitude B_0 for two values of the nonlinearity factor q: solid line—$q = 5$, dashed line—$q = 3$. Dash-dotted lines indicate the threshold current $I_{\rm th}$, threshold bullet amplitude $B_{\rm th}$, and low ($B^*_{\rm min}$) and high ($B^*_{\rm max}$) bullet amplitudes for a certain supercritical current $I^* > I_{\rm th}$. The shaded area to the right of the vertical line $B_0 = 0.46$ indicates the region where our perturbative approach is not valid.

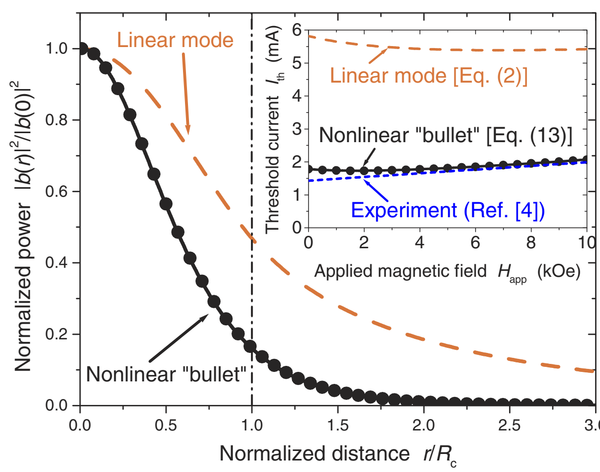

FIG. 2 (color online). Normalized profiles of the spin-wave mode generated by spin-polarized current at the threshold: solid line—bullet profile (4), circles—result of the numerical solution of Eq. (3), dashed line—profile of the linear eigenmode calculated from the linearized Eq. (3). Vertical dash-dotted line shows the region of current localization. The inset shows the dependence of the threshold current $I_{\rm th}$ on the applied magnetic field $H_{\rm app}$: solid line—nonlinear bullet Eq. (13), dashed line—Slonczewski-like linear mode (2). Dotted line—numerical fit $I({\rm mA}) = 1.43 + 0.056H_{\rm app}({\rm kOe})$ to the experimental data [4] [see Fig. 4 in [4]]. The parameters are: $4\pi M_0 = 16.6$ kG, $H_{\rm app} = H_{\rm ex} = 5$ kOe, $A = 2.85 \times 10^{-6}$ erg/cm, $\alpha_G = 0.015$, $d = 1.2$ nm, $R_c = 25$ nm, $\varepsilon = 0.3$.

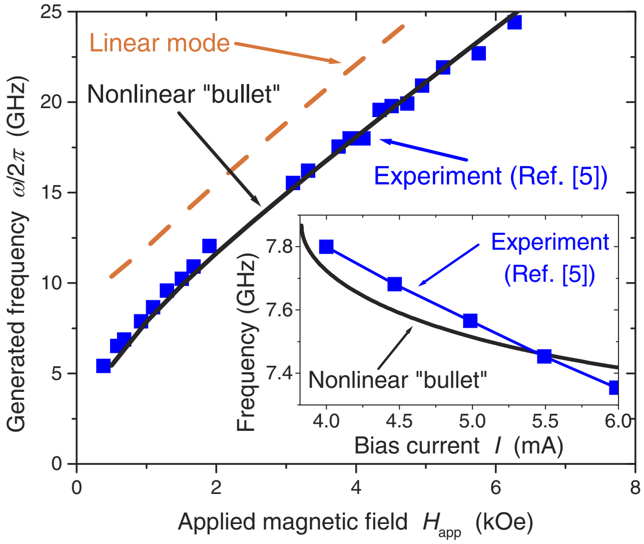

FIG. 3 (color online). Dependence of the frequency ω, generated at the threshold, on the applied magnetic field H_{app}. Solid line—frequency of the nonlinear bullet (6), dashed line—frequency of the linear mode (1), symbols—experiment from Fig. 2a in [5]. The inset shows the dependence of the generated frequency on the bias current for $H_{\text{app}} = 1$ kOe: solid line—nonlinear bullet, symbols—experiment from the inset of Fig. 1 in [5]. The parameters are: $4\pi M_0 = 8.0$ kG, $H_{\text{app}} = 1$ kOe, $H_{\text{ex}} = 0$, $A = 1.4 \times 10^{-6}$ erg/cm, $\alpha_G = 0.02$, $d = 5.0$ nm, $R_c = 20$ nm, $\varepsilon = 0.25$, spectroscopic Lande factor $g = 2$.

the approximate "bulletlike" profile, so the bullet model works exceptionally well in this case. For comparison, we also present in Fig. 2 the spatial profile of the Slonczhewski-like [8] linear mode that is obtained from the solution of Eq. (3) where the nonlinear terms (terms containing $|b|^2$) are omitted (dashed line). The amplitude of this linear mode at the threshold is vanishingly small, $|b(r)|^2 \to 0$.

In the inset of Fig. 2 we show the dependence of the threshold current on the applied magnetic field. One can see that our bullet model gives quantitative description of the threshold current experimentally measured in [4], and agrees with experiment much better than the linear threshold Eq. (2). Note, also, that the linear threshold in the in-plane magnetized nanocontact [in contrast with the case of perpendicular magnetization discussed in [8]] demonstrates a nonmonotonous dependence on the bias magnetic field H caused by the nonmonotonous behavior of the dispersion coefficient D in Eq. (2).

In the main panel of Fig. 3 we demonstrate the comparison of the predictions of our bullet model with the results of the experiment [5] [see Fig. 2a in [5]] for the magnitude of the spin-wave frequency generated at the threshold as a function of the applied magnetic field. It is again clear that the bullet model gives a quantitative description of the experiment.

In the inset of Fig. 3 we compare the theoretical dependence of the generated frequency on the bias current calculated from the bullet model in the above-threshold regime, with the experimental data taken from [5] [see the inset of Fig. 1b in [5]]. It can be seen that our theoretical curve is nonlinear and agrees with the experiment, demonstrating linear decrease of frequency with current, only qualitatively. We attribute this to the fact that our model Eq. (3) is correct only at a threshold and slightly above it, and a more sophisticated nonlinear model containing higher-order nonlinearities is needed to achieve a full quantitative agreement with experiment in the strongly nonlinear above-threshold regime.

In this Letter we have considered only the in-plane magnetized films. Similar results can be obtained for the magnetic films magnetized at small angles to their surface, for which the nonlinear frequency shift is still negative and Lighthill criterion $ND < 0$ is satisfied [see Fig. 8 in [10] for details]. The experimentally measured threshold currents for normally magnetized films [4], for which $ND > 0$ and no spin-wave bullets are possible, are also much lower than the values predicted by the linear model [8]. It was shown [17], however, that in one-dimensional case the nonconservative Eq. (3) can support localized solutions even in this case. The question of whether the nonlinear self-localized solutions of the two-dimensional Eq. (3) that have low excitation threshold really exist is still open and requires additional investigation.

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