Reversible Destruction of Dynamical Localization

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Dynamical localization is a localization phenomenon taking place, for example, in the quantum periodically driven kicked rotor. It is due to subtle quantum destructive interferences and is thus of intrinsic quantum origin. It has been shown that deviation from strict periodicity in the driving rapidly destroys dynamical localization. We report experimental results showing that this destruction is partially reversible when the deterministic perturbation that destroyed it is slowly reversed. We also provide an explanation for the partial character of the reversibility.

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Dynamical localization (DL) is one of the most dramatic manifestations of how the quantum behavior of a complex system may differ from that of its classical counterpart [1]. It takes place in one-dimensional time-periodic Hamiltonian systems where the deterministic motion is classically chaotic and, on the average, equivalent to a diffusive expansion in momentum space (the so-called chaotic diffusion behavior). Because of subtle destructive interference effects, the quantum dynamics is substantially different: while this dynamics is similar to the classical one for short times, the diffusive behavior stops after some break time and the quantum momentum distribution gets frozen to a steady state at long times. Interest in the DL also comes from the fact that it can be easily observed experimentally, e.g., by placing laser-cooled atoms in a periodically kicked laser standing wave, the so-called "kicked rotor" [2]. The quantum inhibition of classical transport is a rather generic behavior in one-dimensional time-periodic Hamiltonian systems. It relies on the existence of a class of states which are stationary under the one-cycle evolution operator, the so-called Floquet states, forming a basis of the Hilbert space. DL is thus a rather robust feature, which can be observed for a large class of initial states, either pure states or statistical mixtures.

Another fascinating feature of DL is its sensitivity to external nonperiodic perturbations or deviations from the temporal periodicity of the system [3]. Various ways of breaking DL have been studied experimentally and theoretically. One way is to add amplitude noise to the kicks [4]. In such cases, it has been observed that DL is destroyed, i.e., that the quantum motion remains diffusive at long times, as the classical motion. This destruction has also been observed by adding a second series of kicks at an incommensurate frequency [5], an experiment that has also evidenced a very high sensitivity to frequency differences, allowing observation of sub-Fourier resonances [6]. Another qualitatively different way of destroying DL is to introduce a small amount of spontaneous emission in the system, thus breaking its quantum coherences [4,7]. While the first two examples correspond to a purely Hamiltonian evolution, the latter one introduces an irreversible dissipative evolution.

In the case of a purely Hamiltonian dynamics, a fundamental question remains, concerning the nature of the DL destruction: is this destruction complete and irreversible or is it possible to stop the diffusive behavior? Even better, is it possible to reverse the evolution and reconstruct a more localized state? The purpose of this Letter is to report experimental results showing that such a relocalization is possible (at least partially) when the nonperiodic perturbation that destroys DL is slowly reversed in time.

Let us first consider the standard kicked rotor Hamiltonian of a single atom in a pulsed standing wave (SW):

$$H_0 = \frac{P^2}{2} + K \sin\theta \sum_{n=0}^{N-1} \delta_\tau(t-n), \tag{1}$$

where P is the reduced momentum along the SW axis in units of $M/(2k_L T_1)$ (k_L is the laser wave number and M the mass of the atom), $\theta = 2k_L z$ the reduced position of the atom along the SW axis, and $K = \Omega^2 T_1 \tau \hbar k_L^2/(2M\Delta_L)$ the kick strength (Ω is the resonant Rabi frequency of the SW, Δ_L its detuning from the atomic resonance). The time t is measured in units of the period T_1 of the kicks. N is the number of kicks, and δ_τ is a Dirac-like function; τ is the finite duration of the kicks. In the limit $\tau \to 0$, the dynamics of this Hamiltonian system is well known and relies upon only two parameters: K and the effective Planck constant $\hbar\!\!\!\!/ = 4\hbar k_L^2 T_1/M$. For $K \gg 1$, the classical dynamics is a chaotic diffusion; a localized set of initial conditions will spread in momentum space like a Gaussian of width $\propto t^{1/2}$. Below the break time, the classical and the quantum dynamics of an initially localized state are identical. After the localization time, the quantum dynamics is frozen, the average kinetic energy ceases to grow; at the same time, the momentum distribution evolves from a characteristic Gaussian shape in the diffusive regime to an exponential shape $\sim \exp(-|P|/L)$ (with L being the localization length) characteristic of the localized regime [8,9].

Consider now an experiment in which a slowly increasing and then decreasing perturbation is added. This perturbation is added to Hamiltonian (1) as a second series of kicks with the same frequency but with a time dependent amplitude (see upper frame in Fig. 2):

$$H = H_0 + \frac{K}{2}\sin\theta\left[1 - \varepsilon\cos\left(\frac{2\pi t}{\Theta}\right)\right]\sum_{n=0}^{N-1}\delta_\tau\left(t - n - \frac{\phi}{2\pi}\right) \tag{2}$$

where $\Theta \gg 1$ is the period of the perturbation, ϕ the relative phase between the two kick series, and ε the modulation amplitude, with $\varepsilon \sim 1$. Experimental values are $\Theta = 35$, $\phi/2\pi = 1/6$, and $\varepsilon = 0.94$. At time $t = N$, the system has been exposed to N kicks of the primary sequence (with fixed strength K) and N kicks of the secondary sequence (with time-varying strength), i.e., to a total of $2N$ pulses.

In order to experimentally realize the Hamiltonian equation (2), a sample of cold cesium atoms is produced in a standard magneto-optical trap and released in the $F_g = 4$ hyperfine ground-state sublevel. A double sequence of N pulses built according to Eq. (2) is applied. The SW is detuned by $\Delta_L/2\pi = 20$ GHz ($\sim$3800 Γ, where Γ is the natural width of the atomic transition) with respect to the $6S_{1/2}$, $F_g = 4 \rightarrow 6P_{3/2}$, $F_e = 5$ hyperfine transition of the Cesium $D2$ line ($\lambda_L = 852$ nm). Such largely detuned radiation essentially induces *stimulated* transitions responsible for conservative momentum exchanges with the atoms, so that the dynamics is Hamiltonian. However, the SW laser line presents a very broad low-level background (several hundreds of GHz) responsible for a significant rate of dissipative spontaneous transitions. To get rid of this problem, the SW passes through a 10 cm cesium cell before interacting with the cold atoms. This filtering reduces the background by more than 1 order of magnitude in a bandwidth of about 500 MHz around the cesium transitions. Finally, after being transported by a polarization-maintaining fiber, 92 mW of laser light, collimated to a 1.5 mm waist, is available for the experiment, and retro-reflected to build the SW. The frequency of each kick series is set to 30 kHz, and the duration of each kick to $\tau = 0.6$ μs. The parameter K is $\sim$9, $\hbar \sim 3.46$, and the localization time $\sim$10 periods. The localization time measured in units of the pulse period and the localization length L measured in units of $2\hbar k_L$ coincide, and both scale roughly as $(K/\hbar)^2 \approx 7$. These parameters have been chosen in order to optimize the experimentally detected signal. (The setup allows variations of K from $\sim$0 to 15, and $\hbar$ from $\sim$1.5 to 6). The spontaneous emission rate is estimated to 0.06 per atom for the maximum duration of the experiment. Once the SW sequence is over, the atomic momentum distribution is probed with a *velocity selective* Raman pulse. Thanks to the Doppler effect, and a well-chosen detuning, this pulse transfers the atoms in a well-defined velocity class from the hyperfine sublevel $F_g = 4$ to the $F_g = 3$ sublevel [10,11]. The atoms remaining in the $F_g = 4$ sublevel are pushed away by a resonant laser beam. A resonant pulse brings the selected atoms back in the $F_g = 4$ level where their number is measured by a resonant probe. The whole cycle starts again to measure the population in another velocity class, allowing us to reconstruct the full atomic momentum distribution. Such a measurement is then performed for increasing pulse numbers N.

A last precaution must, however, be taken. As discussed above, the SW is intense enough to induce—for a few atoms—a real transition from the level $F_g = 4$ to the excited state, followed by spontaneous emission leading possibly to the hyperfine level $F_g = 3$, whatever their momentum. Those atoms would be optically repumped to the $F_g = 4$ sublevel and detected, generating an incoherent, N dependant, background. For each experiment, the Raman detuning is set very far away (at 10 MHz, more than 1000 recoil velocities), where the probability of finding a Raman resonant atom is very low. Except for this modification, the experiment is launched in exactly the same conditions. The stray background is corrected by subtracting the resulting signal from the resonant one.

Figure 1 shows the velocity distribution as a function of N. As expected, the early dynamics is diffusive. DL is expected around $t = 10$. Since the perturbation starts increasing from $t = 0$, DL is not visible and one could assume it is destroyed before it could be seen. However,

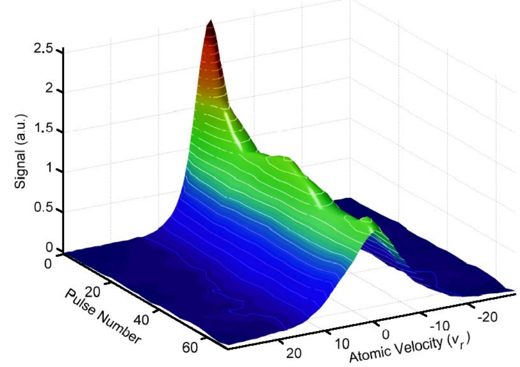

FIG. 1 (color online). Experimentally measured velocity distributions as a function of time. The atomic velocity is measured in units of the recoil velocity $v_r = \hbar k_L/M \simeq 3.5$ mm/s. At short times, the diffusive broadening (or the reduction of the zero-velocity population) of the velocity distribution is observed. When the slowly changing perturbation is reversed (around $t = 17$), the velocity distribution *starts to shrink.* This is highly nontrivial behavior, showing that the destruction of dynamical localization by a slowly time-varying kick sequence is reversible. After a second cycle of the perturbation a second relocalization of the velocity distribution is observed (around $t = 70$).

when the perturbation is reversed, the distribution *shrinks* and takes an exponential shape (see Fig. 2), signing a partial ‘‘revival’’ of the localization. This is clearly visible in Fig. 2 which displays the zero-velocity population Π_0 as a function of N (red crossed solid line), which is inversely proportional to the width of the distribution and therefore directly proportional to the degree of localization. Figures 2(a) and 2(b) display, respectively, the velocity distribution at $t = 17 \sim \Theta/2$, where the perturbation reaches its maximum amplitude, and at $t = \Theta = 35$, where it is back to its minimum initial value. In 2(a), the distribution is very well fitted by a Gaussian, whereas the distribution in 2(b) is better fitted by an exponential. The exponential shape at $t = \Theta$ is not the only remarkable fact. The fact that the momentum distribution gets narrower (Π_0 increases) is highly significative. Indeed, the classical dynamics is diffusive and irreversible, forbidding, in the general case [12], a return to a narrower distribution. Furthermore, DL leads to the suppression of the classical diffusion, i.e., a freezing of the velocity distribution; but it cannot lead to a narrowing of the distribution, which is precisely what is experimentally observed in Fig. 2. This is a key point of the present experiment: it proves that the exponential shape observed at $t = \Theta$, where the perturbation is zero, does not simply results from the DL that would be observed in the periodic case, with no perturbation.

It is even possible to go further by proving the coherent nature of the reversibility process. We have performed an

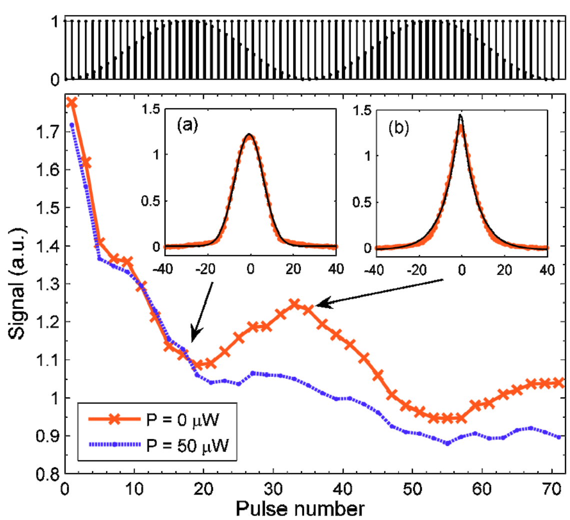

FIG. 2 (color online). Upper frame: kick sequence. Main frame: Population in the zero-velocity class as a function of the duration N of the pulse sequence with no resonant light (crossed solid line) and with a 50 μW pulse of resonant light applied at $t = 17$ (dashed line). The absence of revival in the presence of resonant light (decoherence) is a clear-cut proof of the importance of quantum interference for the reversibility of the DL destruction. The inset (a) shows the Gaussian velocity distribution at $t = 17$, near the maximum of the perturbation, in the absence of resonant light; inset (b) shows an exponential shape near the minimum of the perturbation, $t = 35$.

additional experiment where a resonant laser pulse irradiates the atomic cloud at $t = 17$. The intensity and detuning of this pulse are set such that, on average, only one or two spontaneous photons are emitted per atom. Its purpose is to destroy the quantum coherences, with minimal heating and mechanical effects. The two curves in Fig. 2 (with and without the resonant pulse) are practically identical before $t = 17$, which indicates that heating resulting from the resonant pulse is negligible [13]. In the presence of resonant light, the revival at $t = 35$ disappears almost completely (dashed line curve in Fig. 2). Moreover, the velocity distribution (not shown in Fig. 2) is Gaussian around $t = 35$. This clearly proves that, in the absence of spontaneous emission, although DL is not observed before $t = 17$, there is a memory in the system which is destroyed by spontaneous emission. This reinforces the idea that, when the perturbation is reversed, a dynamically localized state is recovered, at least partially. In fact, the revival of DL is only partial, and a part of it is irremediably destroyed. As shown in Figs. 1 and 2, a second perturbation cycle from $t = \Theta$ to 2Θ has been performed, and a second revival is observed. However, its amplitude is smaller than the first one, and the shape of the velocity distribution is also damaged. This is due to fundamental reasons, although spontaneous emission or experimental imperfections could also contribute.

A detailed discussion of the physical processes at work in our experiment is beyond the scope of this Letter, and will be published elsewhere. We give here a few guidelines to the theoretical interpretation. The robust structure behind DL is the existence of Floquet states for a time-periodic Hamiltonian system, which are eigenstates of the evolution operator over one period. By their definition, such states repeat identically (except for a phase factor) at each kick and thus do not spread in momentum space. Any initial state can be expanded on the complete set of Floquet states. Chaotic diffusion is—in this picture—due to a gradual dephasing of the various Floquet states (of different eigenenergies) that contribute to the initial state. However, a nontrivial property of the periodically kicked rotor is that all Floquet states are *localized* in momentum space [14]: this is the temporal analogy to Anderson localization in disordered one-dimensional systems, as put on firm ground in Ref. [15]. Only Floquet states localized close to the initial (zero) momentum significantly overlap with the initial state and contribute to the long term dynamics. At sufficiently long times, the various Floquet states significantly contributing to the dynamics are completely dephased (in a characteristic time which is but the break time), the momentum distribution covers all significantly populated Floquet states, but cannot extend further in momentum space, leading to the freezing of the diffusive growth. When the dynamics is no longer exactly periodic, population is transferred among the various Floquet states, and Floquet states localized farther from $P = 0$

can be populated. In this situation, DL is thus expected to be destroyed. There is, however, a situation where such an evolution can be controlled: if, at any time, the Hamiltonian is almost periodic with, for example, a kick strength $K(t)$ slowly changing with time t, an adiabatic approximation can be used. The atomic state at time t can be expanded in terms of the ''instantaneous'' Floquet eigenbasis corresponding to the local value of $K(t)$. If the variation of $K(t)$ is slow enough, the evolution is adiabatic in the Floquet basis, meaning that the populations of the Floquet eigenstates do not change with time, while the eigenstates themselves evolve [16,17]. This leads to an apparent diffusive broadening of the momentum distribution [18], but the robust Floquet structure is still underlying. To recover the localization, it is sufficient to reverse the evolution of $K(t)$ back to its initial value. One then recovers the initial well localized Floquet eigenstates with unchanged populations, i.e., a dynamically localized momentum distribution. This is the deep origin of the revival of the localization experimentally observed above. Any phenomenon breaking phase coherence (such as a spontaneous emission) will redistribute the atomic wave function over other Floquet states, eliminating all possibility of revival. However, the revival is only partial, because the evolution cannot be made 100% adiabatic. Indeed, even for very slow changes of $K(t)$, there are some avoided crossings between various Floquet states of such size that they will be crossed neither diabatically, nor adiabatically, and will consequently redistribute the population over the Floquet states, partially destroying the reversibility.

To summarize, we have observed that the destruction of dynamical localization in the kicked rotor, induced by a nonperiodic driving can be partially reversed. If the external driving evolves sufficiently slowly, some information is carefully stored in the populations of the various Floquet states. Although it is not visible in the momentum distribution—which seems to follow an irreversible diffusive broadening—it can be easily restored by reverting the driving back to its initial value, producing a relocalization of the wave function. We have also shown that this intrinsically quantum behavior is destroyed by decoherence, i.e., by adding spontaneous emission to the experiment.

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[12] One can indeed conceive carefully prepared initial states that would evolve to a narrower shape, but this is clearly not the case here.
[13] Each point in Fig. 2 represents a different experiment with a different N. The resonant pulse of light is always applied at $t = 17$ and the Raman detection then performed. Any significant heating effect would enlarge the momentum distribution and would be detected.
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