de Sitter Vacua via Consistent D Terms

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We introduce a new mechanism for producing locally stable de Sitter or Minkowski vacua, with spontaneously broken $N = 1$ supersymmetry and no massless scalars, applicable to superstring and M-theory compactifications with fluxes. We illustrate the mechanism with a simple $N = 1$ supergravity model that provides parametric control on the sign and the size of the vacuum energy. The crucial ingredient is a gauged $U(1)$ that involves both an axionic shift and an R symmetry, and severely constrains the F- and D-term contributions to the potential.

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Introduction.—Superstring and M-theory vacua with exact or spontaneously broken $N = 1$ supersymmetry deserve special attention. Their effective $D = 4$ supergravities admit chiral fermions and inherit some strong symmetry properties from the underlying higher-dimensional theory. So far, perturbative compactifications with fluxes and branes could produce, at best, either Minkowski vacua of the no-scale type, with broken supersymmetry and at least one complex flat direction, or anti–de Sitter (AdS) vacua with all geometrical moduli stabilized.

It is important to go further, exploring the possible existence of locally stable de Sitter (dS) or Minkowski vacua, with no residual flat directions. Some interesting attempts along these lines do indeed exist [1–4]. Most of them rely on the positive contributions to the potential associated with the gauge symmetry of the theory, the so-called D terms. However, the subtle consistency requirements dictated by the coexistence of the two local symmetries, supersymmetry and the gauge symmetry, are known on general grounds [5,6], but were never thoroughly examined in this context.

Reference [1] used a superpotential motivated by nonperturbative effects to produce a supersymmetric AdS vacuum, then uplifted the vacuum energy by a positive contribution to the potential ascribed to $\overline{D3}$ branes. So far, however, no consistent supergravity description of such a mechanism was found. Reference [2] proposed to replace the $\overline{D3}$-brane contribution with a D-term contribution originated by magnetic fluxes but, as will be clear in the following, such a proposal does not fulfill the abovementioned consistency requirements. Reference [3] introduced nonperturbative superpotentials and D terms that satisfy all known consistency conditions, but did not perform a full minimization of the supergravity potential with respect to all fields.

In the present Letter, we first review the general consistency conditions associated with D terms in $N = 1$ supergravity. We then construct a simple explicit model fulfilling all such conditions. We show that, for a wide range of parameters, the model admits locally stable vacua with spontaneously broken supersymmetry and positive or negative vacuum energy. The vacuum energy can be zero, or very small and positive, for special values of the parameters. The key features of the model can be present in flux compactifications of superstring theories. The crucial one is a gauged $U(1)$ symmetry that combines an R symmetry with an axionic shift, and severely constrains the form of the F- and D-term contributions to the potential. The rigid version of such a symmetry, and some of its consequences if unbroken, were previously studied in [7]. Supergravity models with gauged R symmetry [8] were considered in [6,9]. The models of [3] have instead a gauged axionic symmetry but no gauged R symmetry. We finally comment on the extension of our mechanism to more general models and on its string/M-theory embedding, leaving a detailed exploration for future work.

D terms in $N = 1$ supergravity.—The gauge-invariant two-derivative action for $N = 1$, $D = 4$ supergravity with chiral multiplets $\phi^i \sim (z^i, \psi^i)$ and vector multiplets $V^a \sim (\lambda^a, A^a_\mu)$ is completely fixed by three ingredients [5]. The first is the real and gauge-invariant Kähler function G, which can be written in terms of a real Kähler potential K and a holomorphic superpotential W as

$$G = K + \log|W|^2. \qquad (1)$$

The second is the holomorphic gauge kinetic function f_{ab}, which transforms as a symmetric product of adjoint representations, plus a possible imaginary shift associated with anomaly cancellation. Generalized Chern-Simons terms may also be needed [10], but they will not play any role in the simple case discussed in this Letter. The third are the holomorphic Killing vectors $X_a = X^i_a(z)(\partial/\partial z^i)$, which generate the analytic isometries of the Kähler manifold for the scalar fields that are gauged by the vector fields. In the following it will suffice to think of G, f_{ab}, and X_a as functions of the complex scalars z^i rather than the superfields ϕ^i.

The gauge transformation laws and covariant derivatives for the scalars in the chiral multiplets read

$$\delta z^i = X^i_a \epsilon^a, \qquad D_\mu z^i = \partial_\mu z^i - A^a_\mu X^i_a, \tag{2}$$

where ϵ^a are real parameters. The scalar potential is

$$V = V_F + V_D = e^G(G^i G_i - 3) + \frac{1}{2} D_a D^a, \tag{3}$$

where $G_i = \partial G/\partial z^i$, scalar field indices are raised with the inverse Kähler metric $G^{i\bar{k}}$, gauge indices are raised with $[(\mathrm{Re} f)^{-1}]^{ab}$, and D_a are the Killing potentials, real solutions of the complex Killing equations:

$$X^i_a = -iG^{i\bar{k}} \frac{\partial D_a}{\partial \bar{z}^{\bar{k}}}. \tag{4}$$

The general solution to the Killing equation for D_a, compatible with gauge invariance, is then

$$D_a = iG_i X^i_a = iK_i X^i_a + i\frac{W_i}{W} X^i_a. \tag{5}$$

Gauge invariance of G requires that K and W be invariant up to a Kähler transformation

$$K' = K + H + \bar{H}, \qquad W' = We^{-H}, \tag{6}$$

where H is a holomorphic function, thus it will not be restrictive to assume that K is gauge invariant. If W is also gauge invariant, Eq. (5) reduces to the standard form

$$D_a = iK_i X^i_a. \tag{7}$$

Otherwise, it must be

$$i\frac{W_i}{W} X^i_a = \xi_a, \qquad (\xi_a \in \mathbb{R}), \tag{8}$$

so that the gauge noninvariance of W can be at most an overall phase with real parameter ξ_a, for the Abelian factors $U(1)_a$. The constants ξ_a correspond to gaugings of the R symmetry, and give rise to the supergravity expression for the D terms [5,6]:

$$D_a = iK_i X^i_a + \xi_a. \tag{9}$$

The ξ_a are then the genuine Fayet-Iliopoulos (FI) terms of supergravity. For a linearly realized gauge symmetry, $iK_i X^i_a = -K_i (T_a)^i_k z^k$, and we recover the standard expression of [11] for the D terms. For an axionic realization, $X^i_a = iq^i_a$, where q^i_a is a real constant, and we obtain the so-called field-dependent FI terms.

A few comments are now in order. Equation (5) shows that D terms are actually proportional to F terms, $F_i = e^{G/2} G_i$. Two facts, frequently forgotten in the recent literature, then become obvious. First, and in contrast with the rigid case, there cannot be pure D breaking of supergravity, unless the gravitino mass vanishes and the D-term contribution to the vacuum energy is uncanceled, as in the limit of global supersymmetry. Second, if V_F admits a supersymmetric AdS vacuum configuration, $\langle G_i \rangle = 0$ $(\forall i)$ and $\langle e^G \rangle \neq 0$, such configuration automatically minimizes V_D at zero. For this kind of vacua, as already stressed in [12], D terms cannot be used to raise the vacuum energy from negative to positive or zero. Moreover, for the theory to be consistent, W must be gauge invariant, up to an overall phase for $U(1)$ factors. This severely restricts the possibility of constructing superstring-inspired supergravity models with both nonperturbative superpotentials and D terms. D terms associated with a gauged $U(1)$ symmetry cannot coexist with "racetrack" (sums of exponentials) or other (e.g., constant plus exponential) superpotentials, when the latter break such a symmetry. This is in agreement with some recent results in superstring compactifications, where it was shown that the gauged isometries are protected from being broken, both by instanton-induced [13] and by flux-induced [14] superpotentials. On the other hand, a rigid axionic and/or R symmetry, possibly the remnant of a gauged symmetry spontaneously broken at the string scale, can be explicitly broken to a discrete subgroup by nonperturbative effects.

A model with stable de Sitter vacua.—Consider a model with a single chiral multiplet $S \sim (S, \chi)$ and Kähler potential

$$K = -p \log(S + \bar{S}) + K_0, \qquad (0 < p \in \mathbb{R}), \tag{10}$$

with K_0 real and S independent. This form of K is familiar from superstring compactifications. Decomposing the complex scalar as $S = s + i\sigma$, s can stand here for the string dilaton, a geometrical (Kähler or complex structure) modulus of the compactification manifold, or a combination thereof; σ could instead carry the degree of freedom of some component of the NS-NS or R-R potentials, or even of the internal metric. The $U(1)$ isometry acting as a shift on the "axion" σ can be gauged by a vector multiplet. The corresponding holomorphic Killing vector is just an imaginary constant,

$$X^S = iq, \qquad (q \in \mathbb{R}). \tag{11}$$

The most general form of the superpotential compatible with the gauged $U(1)$ symmetry is then

$$W = W_0 e^{-kS}, \qquad (k \in \mathbb{R}), \tag{12}$$

where W_0 is S independent. Equation (12) has the typical form of the nonperturbative superpotentials induced by instantons or gaugino condensation [15]. Notice that the gauged $U(1)$ is a combination of the axionic $U(1)$, acting as a shift on σ and leaving all the other fields invariant, and the $U(1)$ R symmetry, with charge $+(\xi/2)$ for χ, $-(\xi/2)$ for the gravitino ψ_μ and the gaugino λ, and zero for all the bosonic fields. $\xi = kq$ is the constant FI term. For the gauge kinetic function, we take

$$f = S, \tag{13}$$

as typical of superstring compactifications, but the model would still work for $f = aS + b$, the most general form compatible with the gauged symmetry. Notice, finally, that the gauge kinetic function f of the possible gauge group factor associated with instantons or gaugino condensation does not need to coincide with the $U(1)$ gauge kinetic function of Eq. (13): this may be of help in obtaining vacua at weak coupling.

The scalar potential of Eq. (3) then reads

$$V_F = \frac{e^{G_0} e^{-2ks}}{(2s)^p} \left[\frac{(2s)^2}{p} \left(k + \frac{p}{2s} \right)^2 - 3 \right], \tag{14}$$

$$V_D = \frac{q^2}{2s} \left(k + \frac{p}{2s} \right)^2, \tag{15}$$

where $e^{G_0} \equiv |W_0|^2 e^{K_0}$. As required by gauge invariance, V does not depend on σ: the axion is absorbed by the massive $U(1)$ vector boson via the Higgs effect [16]. However, both V_F and V_D depend nontrivially on s. For $k < 0$, there is always a stable supersymmetric AdS vacuum at $\langle s \rangle = -p/(2k)$, but there can be also metastable dS vacua for suitable values of the parameters. For $k > 0$ and $p \geq 3$, V is positive definite and monotonically decreasing. For $k > 0$ and $p < 3$, V_F is unbounded from below for $s \to 0$, but V_D is positive definite and diverges for $s \to 0$. As a result, for a wide range of parameters there is a locally stable dS (or stable AdS) minimum of V at a finite value $\langle s \rangle$, with spontaneously broken supersymmetry. At this level, having approximate Minkowski vacua requires a tuning of the parameters so that $\langle V_D \rangle \simeq -\langle V_F \rangle \gg \langle V \rangle$. In principle, this may find an explanation in the correlations of the underlying string theory. Notice that $\langle s \rangle$ can be continuously varied by rescaling the parameters as:

$$k \to \alpha^{-1} k, \qquad e^{G_0} \to \alpha^p e^{G_0}, \qquad q \to \alpha^{3/2} q, \tag{16}$$

($0 < \alpha \in \mathbb{R}$), which leads to $\langle s \rangle \to \alpha \langle s \rangle$. A representative example is shown in Fig. 1.

Discussion.—The simple model discussed above can be easily generalized. The inclusion of additional gauge multiplets is straightforward (apart from anomaly cancellation, see below), thus we consider the inclusion of additional chiral multiplets ϕ^i, transforming linearly under the axionic $U(1)$, with charges q^i. Since the R charge is fixed to be vanishing for the z^i and $+(\xi/2)$ for the ψ^i, the corresponding charges under the gauged $U(1)$ will be q^i and $(q^i + \xi/2)$, respectively.

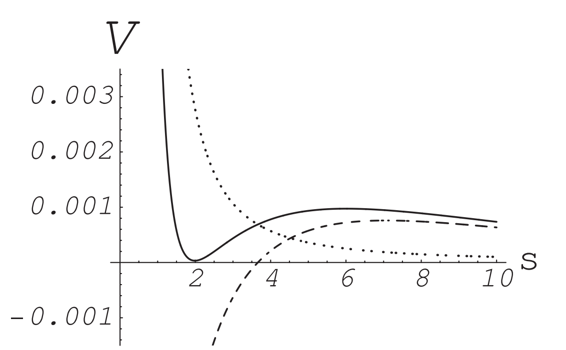

FIG. 1. V (solid line), V_F (dashed line), and V_D (dotted line) for $p = 1$, $q = 0.3$, $e^{G_0} = 1/64$, and $k = 0.1$.

Consider first the simple case where the full K can be written as in (10), with K_0 real gauge-invariant function of the z^i. Assume also a factorized W as in (12), with W_0 the analytic gauge-invariant function of the z^i, and f as in (13). We may think of the z^i as other moduli of string/M-theory compactifications, orthogonal to S, or the scalar fields of the minimal supersymmetric standard model (MSSM). It is easy to see that, under mild and plausible assumptions, the minimization of the potential V with respect to the z^i and S can be fully decoupled. If there is a field configuration $\langle z^i \rangle$ such that $\langle G_i \rangle = 0$ and $q^i \langle z^i \rangle = 0 \ \forall i$, then $\langle s \rangle$, as determined in the previous section, and $\langle z^i \rangle$, extremize the full potential V with no mass mixing between the z^i and s. A locally stable minimum can be obtained for a wide range of the parameters in G_0, with the same $\langle V \rangle$ as before. If, instead, there is a minimum of V such that $\langle V_D \rangle = 0$, then we can decouple a massive $N = 1$ vector multiplet and discuss a simpler, but less interesting effective theory.

The previous model can be further generalized by including additional chiral multiplets $C^\alpha \sim (C^\alpha, \psi^\alpha)$, which could stand for some or all the MSSM squarks and leptons, in the approximation of small field fluctuations about $\langle C^\alpha \rangle = 0$, but relaxing the factorization of K and W. For example, we could add to K a $\Delta K = \sum_\alpha |C^\alpha|^2 (S + \bar{S})^{n_\alpha}$ ($n_\alpha \in \mathbb{Z}$), and to W a ΔW polynomial in the C^α and transforming with the same phase as e^{-kS}, e.g., $\Delta W = d_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma$ with $q^\alpha + q^\beta + q^\gamma = -kq$. Also in this case, for suitable values of the new parameters, there is a local minimum of the full potential V with $\langle C^\alpha \rangle = \langle G_\alpha \rangle = 0 \ \forall \alpha$, positive squared masses for all the new scalar fields C^α, and all the remaining features as in the previous model.

For a consistent effective theory, all gauge and gravitational anomalies associated with our gauged $U(1)$ must vanish: in particular, the cubic ($\mathcal{A}_{U(1)^3}$), the gravitational ($\mathcal{A}_{U(1)}$) and the mixed-gauge anomaly ($\mathcal{A}_{U(1)\mathcal{G}^2}$) if the full gauge group is $U(1) \times \mathcal{G}$. The fermionic contributions to the cubic and gravitational anomalies are:

$$\mathrm{Tr}\, Q^3 = 3\left(-\frac{\xi}{2} \right)^3 + \sum_{i,\alpha} \left(q^{i,\alpha} + \frac{\xi}{2} \right)^3, \tag{17}$$

$$\mathrm{Tr}\, Q = -21\left(-\frac{\xi}{2} \right) + \sum_{i,\alpha} \left(q^{i,\alpha} + \frac{\xi}{2} \right), \tag{18}$$

where the contributions from λ and χ cancel each other and have been omitted. The remaining ones come from ψ_μ (see [17]) and possible $\psi^{i,\alpha}$, respectively. These contributions must cancel the Green-Schwarz (GS) contributions [18] coming from the variation of σ and proportional to q. All the resulting conditions are model dependent, in particular: all of them depend on the matter content; the GS contribution to $\mathcal{A}_{U(1)}$ depends on higher derivative terms (R^2); $\mathcal{A}_{U(1)\mathcal{G}^2}$ depends also on the details of $\mathcal{G}$. However, there are, in principle, strong combined constraints on the possible matter content and on the parameters k and q.

The mechanism discussed in this Letter should be relevant for the study of superstring and M-theory vacua, with the anomaly constraints automatically satisfied and the possibility of determining k and q. $N = 1$ supergravities obtained from compactifications with fluxes generically allow for some shift symmetry to be gauged.

In the heterotic theory, the shift symmetry of the universal axion [19], dual to $B_{\mu\nu}$, is gauged via the GS mechanism, and fluxes can be used to generate a superpotential W_0 [20,21]

$$\int_{X_6} (H + idJ) \wedge \Omega, \tag{19}$$

which can stabilize all geometrical moduli with vanishing F terms and positive masses. A modification of W (or, equivalently, of K) as in (12) would then stabilize also the dilaton S on a dS vacuum, breaking the local symmetries only spontaneously.

Also in type-IIA compactifications with fluxes, the superpotential [21,22]

$$\int_{X_6} \mathbf{G} e^{iJ} - i(H - idJ) \wedge \Omega, \tag{20}$$

can produce the stabilization of all bulk moduli with vanishing F terms, with the exception of at least one massless axion [14,22], associated with a shift symmetry that can eventually be gauged. In this case the role of S is played by a linear combination of the dilaton and the complex structure moduli Ω, and its dependence cannot be factorized from the other moduli anymore. Whether in this case an analogous modification of the superpotential would allow the lifting of the vacuum energy and the stabilization of all moduli remains an open problem.

Finally, it would be interesting to understand better how the needed superpotential modifications actually originate from string/M-theory, and what the corresponding constraints on the various parameters are.

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