Current-Drive Efficiency in a Degenerate Plasma

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In a degenerate plasma, the rates of electron processes are much smaller than the classical model would predict, affecting the efficiencies of current generation by external noninductive means, such as by electromagnetic radiation or intense ion beams. For electron-based mechanisms, the current-drive efficiency is higher than the classical prediction by more than a factor of 6 in a degenerate hydrogen plasma, mainly because the electron-electron collisions do not quickly slow down fast electrons. Moreover, electrons much faster than thermal speeds are more readily excited without exciting thermal electrons. In ion-based mechanisms of current drive, the efficiency is likewise enhanced due to the degeneracy effects, since the electron stopping power on slow ion beams is significantly reduced.

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The use of magnetic fields for confining and controlling plasma is essential in many schemes for achieving thermonuclear fusion. These fields may be sustained by the driving of steady-state electric currents through radio frequency fields. However, whereas the efficient generation of electric current in low-density plasma has occupied the attention of the magnetic fusion community for several decades, relatively little attention has been paid to exploring how the ideas for steady-state current drive might be carried over to high-density plasma.

In dense matter, or in moderately dense matter, the plasma can be Fermi-degenerate, strongly affecting electron collisions. In addition to natural regimes such as stellar interiors, Fermi-degenerate plasma arises when a pellet of hydrogen is compressed to many times the solid density for the purpose of inertial confinement fusion [1]. These plasma conditions are achieved by intense laser compression using laser power over nanosecond and picosecond time scales. In these plasmas, whether driven by intense lasers or ion beams, there might be either intentional or incidental generation of steady-state current. Since the collision frequencies in these plasmas are high, any currents generated by the laser radiation would be steady-state on the time scale of the laser pulse, and very intense steady-state magnetic fields might result.

The compression by intense laser power might achieve densities of 10^{24}–10^{28} cm^{-3}. The Fermi energy can be written as 36.4 eV $\times\, n_{24}^{2/3}$, where n_{24} is the density normalized by 10^{24} cm^{-3}. Over much of the compression, the plasma temperature can be much less than the Fermi energy. The plasma can be strongly or weakly coupled, but the processes that we consider here that affect the noninductive current-drive effects depend on the degeneracy and not on the plasma parameter.

Of interest is the efficiency of noninductive current generation in such matter. By noninductive, we mean that the current is not generated by a dc electric field with curl, which would require a time-varying magnetic field. Rather, the current is generated by wave-particle interactions or collisions, which selectively accelerate certain resonant electrons or ions. Our focus here is on the current-drive efficiency, assuming that means of selective acceleration are found.

In Fermi-degenerate matter, electron-electron (e-e) collisions and ion-electron (i-e) collisions [2,3] are much smaller than the classical prediction, while ion-ion (i-i) collisions remain classical. This leads to interesting consequences, including higher electrical and heat conductivity [4,5] than would be predicted from the classical model. The large reduction of i-e coupling also makes possible hot-ion regimes suitable for aneutronic fusion [6,7]. The large reduction in e-e [8] and i-e collisions should also affect the generation of current in Fermi-degenerate plasma. However, as opposed to the electrical conductivity of plasma, which is the result of the integrated effect of uniform force exerted on all electrons, methods of noninductive current drive are inherently kinetic effects, where resonance conditions selectively accelerate only certain ions or electrons [9]. In fact, as we show, degeneracy effects in plasma significantly enhance the efficiency of noninductive current generation. In carrying over to degenerate plasma the mechanisms predicted for classical plasma, it will be necessary to consider separately electron-based and ion-based methods of current drive.

The most efficient electron-based methods of current drive in classical plasma employ radio frequency waves either to directly push electrons in the direction of the (electron) current [10] or to select resonantly electrons already moving in the direction of the (electron) current and to push them perpendicular to that direction [11].

Consider, for either classical or degenerate plasma, the current generated from pushing an electron from some velocity space location v_A to v_B, which over time generates $\delta j(t; v_A, v_B) = -e(v_B(t) - v_A(t))$, where $v_A \equiv v_A(t=0)$ and $v_B \equiv v_B(t=0)$. The functions $v_A(t)$ and $v_B(t)$, the probable velocities after time t given initial positions, reflect the slowing-down physics. This current decays over time, and the accumulated current due to the initial energy expenditure in the push can be written as

$$\int_0^\infty \delta j(t; v_A, v_B) dt = \mathbf{S} \cdot \frac{\partial}{\partial \mathbf{v}} \chi(\mathbf{v}), \quad (1)$$

where we assumed an incremental push, along direction $\mathbf{S}$, $v_B \to v_A \equiv \mathbf{v}$, and used $\chi(\mathbf{v})$ as the Green's function for the current, which now contains the details of the collisional slowing-down process [9]. For the classical case, for example, for resonant velocities much greater than the electron thermal velocity, we have $\chi(\mathbf{v}) = (e/\Gamma) v_{\|} v^3/(5 + Z)$, where $\Gamma = 4\pi n_e e^4 \log\Lambda/m_e^2$, where $v_{\|}$ is the velocity in the direction of the current, and where Z is the ion charge state. The current-drive efficiency, defined as current generated J over steady-state power dissipated P_D, is then given as

$$\frac{J}{P_D} = \frac{\mathbf{S} \cdot (\partial \chi(\mathbf{v})/\partial \mathbf{v})}{\mathbf{S} \cdot (\partial (m v^2/2)/\partial \mathbf{v})}. \quad (2)$$

These same considerations apply for a degenerate plasma, where the electrons pushed live in a Fermi sea as shown in Fig. 1, so that electrons must respect the exclusion principle in the presence of occupied sites, both when pushed and when decelerated by collisions. As discussed above, we do not concern ourselves here with the mechanism of the push, which could occur through a variety of means, including the resonant wakefield of lasers. Here we consider the electron scattering in pitch angle by ions only, since the e-e collision frequency is negligible compared to e-i collision frequency if most of the electrons are degenerate.

Assume that initially site **A** in energy shell **F** is occupied by an electron and site **B** in energy shell **G** is occupied by a hole. Let f be the electron occupation number of shell **F** and, similarly, g of shell **G**. Consider now an electron pushed from site **A** into site **B**. Given an electron at site **A**, the ensemble average current on shell **F** is given by $\langle j \rangle = -e(1 - f) v_A$. Similarly, the ensemble average current on shell **G**, given that there is a hole at site **B**, is given as $\langle j \rangle = e g v_B$. The initial current is then $j^i = -e(1 - f) v_A + e g v_B$. Since the current after the push is $j^f = e f v_A - e(1 - g) v_B$, the current produced is

$$\delta j = -e[v_B(1 - g) - v_A f - v_A(1 - f) + v_B g], \quad (3)$$

or $\delta j = -e(v_B - v_A)$ as in the classical case. Similarly, if an electron at site **B** is scattered in pitch angle, for example, to site **C**, the current changes initially by $-e v_C + e v_B$. The ensemble-averaged current on, say, shell **G**, given an electron at site **B**, then decays by

$$d\langle j \rangle/dt = -e(1 - g) v_B \nu = -e \nu_{ei} v_B, \quad (4)$$

where ν is the pitch-angle scattering rate, which is simply the classical rate ν_{ei} reduced by the occupation availability $(1 - g)$. Each electron in partially occupied shells collides less frequently with ions than classically, but it effectively carries less charge. Thus, the classical current decay rate is recovered for electron-ion collisions.

The most important consequence of this calculation is that J/P_D is inversely proportional to Z rather than $(5 + Z)$ as in the classical case [9]. The factor of 5 comes from the electron-electron collisions, which are negligible in a degenerate plasma. Thus, for $Z = 1$, the efficiency is enhanced over the classical case by a factor of 6.

In addition to the lack of e-e collisions, several other effects increase even further the noninductive current-drive efficiency in the Fermi-degenerate plasma. First, the electrons participating in any current are all near the Fermi surface. This electron velocity will be proportional to the Fermi velocity $v_F = \hbar(3\pi^2 n_e)^{1/3}/m_e$. Since, from Eq. (2), the efficiency is proportional to v^2, this means that the effective collision frequency is small compared to what would be predicted for classical electrons near the thermal velocity. Second, because of the exclusion principle, many electrons will be found at these higher energies; in the classical plasma, there are exponentially few electrons much faster than the thermal velocity. Third, since it is impossible to push electrons below the Fermi surface, it should be possible to exercise more precise selectivity in pushing the fastest electrons, which are those near the Fermi surface. In classical plasma, although fast electrons may be selected by means of resonance conditions, often slower, more inefficient, electrons are also resonant. In the Fermi-degenerate plasma, that there is no option to push these slow electrons is advantageous. Fourth, in connection with this increased selectivity, note that it may be possible to achieve in Fermi-degenerate plasma what could not be achieved in a classical plasma [12], namely, to push in the direction of the electron current electrons with small $v_{\|}$ but with high energy, which, as seen from Eq. (2), optimizes the efficiency. Although pushing electrons nearly tangent to their constant energy surface is certainly advantageous energetically, in classical plasma there are few electrons that can be exploited this way even if they could be selected precisely [9]. Degenerate plasma, however, assures not only the selectivity but, due to its high electron density, also assures copious resonant electrons. Thus, the electrons pushed tend to be near the Fermi surface rather than at several times the thermal velocity. Since the efficiency scales as the energy of the pushed electrons, the efficiency will be further enhanced in the highly degenerate plasma by approximately the ratio of Fermi energy to thermal energy.

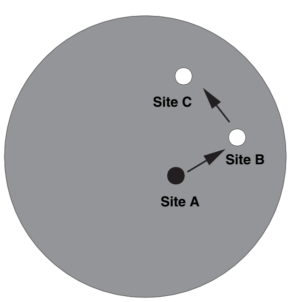

FIG. 1. Pushing an electron in a partially occupied Fermi sea.

Note that, for a degenerate plasma, the number of ion scatterers scales with n, while the electron velocities scale with the Fermi velocity, or $n^{1/3}$. Hence, the efficiency scales inversely proportional to $n^{1/3}$. Thus, the absolute efficiency is a mildly decreasing function of density.

Note that the single electron approach described above recovers the electrical conductivity in a degenerate plasma [4]. For electric field E in, say, the x direction, the steady-state Boltzmann equation can be written as $(eE/m_e) \times (\partial f_e/\partial v_x) = -C(f_e)$. Using Chapman-Enskog expansion and assuming a Lorentz collision model, we can obtain $f_1 = [eE/m_e \nu_{ei}(v)](\partial f_{e0}/\partial v_x) = \chi(v) f_{e0}$, where $\nu_{ei} = \Gamma Z/v^3$ and $\chi(v) = [eE/m_e \nu_{ei}(v)](m v_x^2/T_e)(1-f)$ and f_{e0} (f) is the equilibrium Fermi distribution (occupation fraction). The current is given as $J = \int d^3 v e v_x f_1(v)$, which is Lorentz-Spitzer conductivity. The flux $\mathbf{S}$ is given as $\mathbf{S} = eE/m_e(1+\chi) f_{e0}\hat{\mathbf{x}}$. Note that, by the single-particle approach, $\chi(v)$ is proportional to e/ν, where the charge e is reduced to $(1-f)e$, and the collision frequency is reduced to $\nu(1-f)$, so that $\chi(v)$ is independent of occupation fraction. The power dissipated is then given by $P_D = \int d^3 v \mathbf{S} \cdot \nabla_v (m v^2/2)$. Using $\mathbf{S} = (eE/m_e) f_{e0}$ in Eq. (2), we recover the current derivable through the Chapman-Enskog method.

Consider now ion-based methods of current drive, which rely on directed momentum of a minority ion population B, which has a different ion charge state Z from the majority ion population A (which is, say, hydrogen). For charge neutrality, we have $n_B Z + n_A = n_e$. Since current in a neutralized plasma is Lorentz invariant, the current generation can be considered in the frame of reference in which the ion current vanishes. Suppose in this frame minority ions have velocity V_B and majority ions have velocity V_A. Electrons, whether classical or degenerate, collide more often with the beam of the higher ion charge state and so will drift in that direction, generating current opposite to the direction of the higher ion charge state ion drift. In classical plasma, it is the basis for a number of ion-based methods of generating steady-state current, including neutral beam current drive [13] and minority-species current drive [14].

The minority ions slow down by $dV_B/dt = -(\nu_{Be} + \nu_{BA})V_B$, where ν_{Be} represents collisions of minority ions with electrons and ν_{BA} represents collisions of minority ions with majority ions. The electron current relies only on the fact that the ion-electron collision rates are proportional to the ion charge state, so that, in steady state, the electron drift v_e must obey the force balance $n_A(V_A - v_e) + n_B Z^2 (V_B - v_e) = 0$. Using $J = e(-n_e v_e)$ and charge neutrality, we obtain $J = e n_B V_B (1-Z)Z$. A rough estimate of the power necessary to sustain the minority beam that sustains this current is $P_D = (\nu_{Be} + \nu_{BA}) n_B M V_B^2/2$. The current-drive efficiency can then be put in the form $J/P_D = eZ(1-Z)/V_B(\nu_{Be} + \nu_{BA})$. For the case of classical slowing down on both ions and electrons, a more accurate current-drive efficiency can be had by including kinetic effects of the slowed down beam distribution for different electron temperature [15]. However, a few observations can be made at once. First, for a classical plasma, for a minority ion beam, $\nu_{BA} \sim V_B^{-3}$, whereas $\nu_{Be} \sim$ constant. Thus, the efficiency as a function of minority injection velocity V_B is low at low injection velocities because of too many collisions with majority ions, while at high injection velocities, where electron slowing down dominates, the efficiency goes inversely with the electron velocity. The maximum efficiency then occurs where the slowing-down rates are approximately equal. The most advantageous minority injection energy is about $40 A_B T_e$, where A_B is the beam atomic number [16].

The application of these considerations changes considerably in degenerate plasma. The ion-electron collision frequency in degenerate plasma can be written as

$$\nu_{Be} = \frac{8}{3\pi} \frac{m^2 Z^2 e^4}{\mu \hbar^3} C(\chi), \tag{5}$$

where μ is the ion mass, m is the electron mass, $\chi^2 = e^2/\pi\hbar v_F$, v_F is the Fermi velocity, and $C(\chi) \cong 0.5 \times [\log(1 + 1/\chi^2) - 1/(1+\chi^2)]$ [17], valid for $v \ll v_F$ and $r_s \ll 1$, where v is the ion velocity and $r_s = (me^2/\hbar)(3/4\pi n_e)^{1/3}$ [2,3,17]. Because slow electrons are degenerate, the collisions occur between the ions and the fastest electrons rather than, as in a weakly coupled hot plasma, between the ions and the thermal electrons. For large Fermi energies, $C(\chi)$ scales logarithmically with the density. The collisional cross section decreases as $1/v_F^4$. This strong dependence of the cross section on v_F just suffices to cancel the effect of the greater electron density, the greater energy loss per collision, so that the stopping frequency is independent of the electron density. While the ion-electron and electron-electron collision frequencies are both considerably smaller than that given by the classical formula, because the ions remain classical, the ion-ion collision rate is the same as the classical prediction.

Since the ion-electron collision frequency ν_{Be} is so very much reduced from the classical value, it can be seen that the ion-beam current-drive efficiency in a degenerate plasma will optimize for injection velocities much larger than for a classical plasma. For ν_{Be} independent of V_B, it follows that the efficiency is maximized for $V_B \sim \nu_{Be}^{-1/3}$, for both classical electron collisions and Fermi-degenerate electron collisions, and this maximum efficiency then scales with $\nu_{Be}^{-2/3}$. The current-drive efficiency for ion-based methods should, thus, increase by approximately $(\nu_{Be}^C/\nu_{Be})^{2/3} \sim 2.36 \times (E_f/T_e) n_{24}^{2/3}$. This estimation is valid when $T_e \ll E_f = 36.4 \times n_{24}^{2/3}$ eV.

The ion-electron collision frequency is constant as a function of ion velocity in Fermi-degenerate plasma if the ion velocity is small compared to the Fermi velocity. As the ion velocity approaches the Fermi velocity, the coupling to the free electrons is largest, and the collision frequency becomes larger [18]. For ion velocities much greater than the Fermi velocity, the coupling again decreases. Thus, the ion-electron slowing-down rate is not a monotonically decreasing function of ion velocity, as in classical plasma. This maximum of the collision frequency as a function of the ion velocity might be exploited by matching the velocity of one beam to $V_B \cong v_F$, generating current even with only one species of ions.

One way to producing counterstreaming ion beams is by heating ions of the minority species that are traveling in the direction of the current, even if the heating is transverse to the direction of the current [14]. The heated ions, being more energetic, collide less with the majority-species ions and, thus, retain their directed momentum for longer. The current-drive efficiency is given as in Eq. (2) with $\chi(v) = eZ_B(1 - Z_B/Z_A)(v_{\|}/\nu)$, where Z_B and Z_A are the charge states of the minority and majority ions, respectively, M_B and M_A the mass of the minority (majority) ion, and $\nu = \nu_{Be} + \nu_{BA}$. The ion-electron collision frequency ν_{Be} is given in Eq. (5), and the ion-ion collision frequency is given as $\nu_{BA} = (1 + M_B/M_A)(4\pi Z^2 e^4 \log(\Lambda) n_A / M_B^2 V^3)$. For ions pushed in the direction perpendicular to the current, the efficiency can be simplified to $J/P_D = (Z_B - Z_B^2)(3e/M_B \nu_{Be})(V_z V_B / V_S^3)[1/(1 + V_B^3/V_S^3)^2]$, where V_z is the component of the ion velocity V_B in the direction of the current, and we define $V_S = v_F[m_e(1 + M_B/M_A)/M_B C(\chi)]^{1/3}$. Assuming $V_z = V_B$, the above equation attains a maximum $J/P_D = (4/3)(e/M)(1/\nu_{ie})(1/2^{2/3} V_S)$ when $V_B = (1/2)^{1/3} V_S$. This maximum is larger than in the classical case by the factor $(\nu_{Be}^C/\nu_{Be})^{2/3}$ as in the case for injecting this current directly. In classical plasmas, the optimum ion velocity in minority-species current drive is less than the electron thermal velocity by a factor of several. However, in a degenerate plasma, it is considerably less than the Fermi velocity. Thus, compared to injecting ions, it will be less efficient. Also note that, for ion temperatures much smaller than the Fermi temperature, it will not be possible to accelerate very many minority ions to take advantage of the small collision rate at high ion energies.

To generate extremely large magnetic fields with laser power, consider a torus-shaped current, with major radius R and minor radius r, and suppose that energy E is absorbed by electrons over time τ. To reach steady state, τ is chosen greater than the L/R time. For degenerate plasma, neglecting corrections to the conductivity (which would somewhat lengthen this time) due to the concomitant external wave heating [19], we have $L/R \approx 44 \times n_{24} r_{10}^2$ ps, where r_{10} is the minor radius normalized to 10 microns. The current generated by pushing electrons in the direction of the current can be written as $I_{10} = 168\Theta(v_z/v_F) \times (E_{100}/\tau_{\rm ps} R_{40} n_{24}^{1/3})$, where I_{10} is the current normalized by 10 MA, R_{40} is the major radius normalized by 40 μm, $\tau_{\rm ps}$ is the duration of the energy input in picoseconds, and E_{100} is the energy input normalized to 100 kJ. The function $\Theta(v_z/v_F) = v_z/v_F + v_F/3v_z$ is maximized for small v_z, where v_z is the velocity component of the pushed electron in the direction of current. The magnetic field generated is then $B = 2I_{10}/r_{10}$GG. For example, for an L/R time of 0.5 ns, pick $n_{24} \approx 10$ and $r_{10} = 1$. Then for $R_{40} = E_{100} = 1$, and for $v_z = v_F$, we have $I_{10} = 0.15$, and $B = 300$ MG. For $v_z = 0.1 v_F$, a GG is generated. Suppose now National Ignition Facility (NIF) parameters [1]. Since NIF delivers 2 MJ over 5 ns, there remains the possibility to compress this gigagauss field using the remainder of the NIF pulse energy. Once the magnetic field is impressed in it, if the plasma is compressed within the (new) L/R time of the plasma by a factor of, say, 10 in minor radius, a 100 gigagauss field might be generated.

In summary, we obtained the current-drive efficiency in a degenerate plasma for several methods of driving current. For electron-based methods such as wave heating of electrons, we showed that, since e-e collisions can be negligible, the efficiency is, further, higher than the classical prediction by at least a factor of $(5 + Z)/Z$. The electron-based methods also facilitated selecting resonant electrons with advantageously high velocity. This high efficiency may permit the generation of extreme magnetic fields in very dense plasma. Although the considerations here show that, in principle, extreme magnetic fields might be generated with high efficiency in very dense plasma, what remains to be proposed is a specific practical means through which this possibility might be realized.

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