Continuous Quantum Measurement with Independent Detector Cross Correlations

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We investigate the advantages of using two independent, linear detectors for continuous quantum measurement. For single-shot measurement, the detection process may be quantum limited if the detectors are twins. For weak continuous measurement, cross correlations allow a violation of the Korotkov-Averin bound for the detector's signal-to-noise ratio. The joint weak measurement of noncommuting observables is also investigated, and we find the cross correlation changes sign as a function of frequency, reflecting a crossover from incoherent relaxation to coherent, out of phase oscillations. Our results are applied to a double quantum-dot charge qubit, simultaneously measured by two quantum point contacts.

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There has recently been intensive research, both experimental and theoretical, into the development of quantum detectors in the solid state. Mesoscopic structures, such as the quantum point contact, single electron transistor, and SQUID have been used for fast qubit readout. Contrary to the historical assumption that the quantum measurement occurs instantaneously, in the modern theory of quantum detectors the continuous nature of the measurement process is essential to the understanding and optimization of how quantum information is collected. The ultimate goal for quantum computation is the development of "single-shot" detectors, where in one run the qubit's state is unambiguously determined. An important figure of merit is the detector's efficiency, defined as the product of the typical time taken to measure the state of the qubit, with the measurement-induced dephasing rate. In a quantum limited detector the efficiency is minimized, and physically corresponds to the situation when all information about the qubit state encoded in the detector degrees of freedom may be deduced from the measured output.

Single-shot detectors are difficult to realize because of the fast time resolution required. Another approach is that of *weak measurement*, where the detector is continuously measuring the state as the qubit undergoes Hamiltonian evolution. Detector backaction renders the state of the qubit invisible in the average output of the detector, but a signature of quantum coherent oscillations is uncovered in the spectral density of the detector. These measurements are easier to preform because both detector and qubit averaging are permitted, and the experiments only require a bandwidth resolution of the qubit energy splitting. One important result in weak measurement theory is the Korotkov-Averin bound. It states that as the weak measurement is taking place, the detector backaction quickly destroys the quantum oscillations, so the maximum detector signal-to-noise ratio is fundamentally limited at 4. This result was derived in Ref. [1], confirmed in Ref. [2], generalized in Ref. [3], and measured in Ref. [4].

In this Letter, the theoretical advantages of considering the cross-correlated output of independent quantum detectors are investigated. It is clear that cross correlations bestow an experimental advantage [5] in quantum measurement, because the procedure filters out any noise not shared by the two detectors. Thus, extraneous noise produced by sources such as charge traps in one detector will be removed. This technique is also used in quantum noise measurements for this same advantage [6]. We demonstrate that although the two detectors cannot improve the efficiency of the detection process, the cross-correlated output can violate the Korotkov-Averin bound: The background detector noise may be eliminated completely, so the signal-to-noise ratio is divergent. An important application of cross-correlated detectors is the simultaneous weak measurement of noncommuting observables. The fact that this question is ill-defined in the theory of projective measurements gives additional impetus to investigate this fundamental issue. We find that the cross correlator changes sign as a function of frequency, reflecting a crossover from incoherent relaxation at low frequency, to out of phase, coherent oscillations at high frequency. As a solid-state implementation of our results, Fig. 1 depicts two quantum point contacts capacitively coupled to the same double quantum dot representing a charge qubit. It should

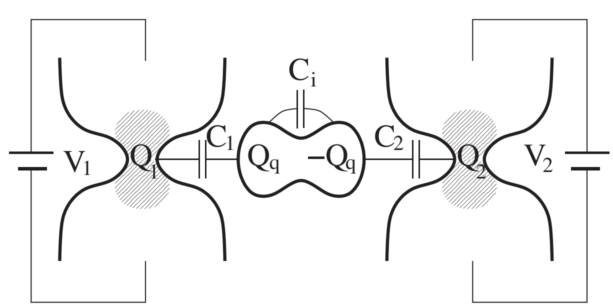

FIG. 1. Cross-correlated quantum measurement setup: Two quantum point contacts are measuring the same double quantum-dot qubit. As the quantum measurement is taking place, the current outputs of both detectors can be averaged or cross correlated with each other.

be stressed that such two-detector structures have already been fabricated [7].

Detector assumptions and linear response.—We employ the linear response approach to quantum measurement because of its elegant simplicity and general applicability to a wide range of detectors [1,8,9]. The quantum operator to be measured is σ_z. The Hamiltonian is

$$H = -(\epsilon\sigma_z + \Delta\sigma_x)/2 + H_1 + H_2 + Q_1\sigma_z/2 + Q_2\sigma_z/2, \tag{1}$$

where $Q_{1,2}$ are the bare input variables of detector 1 and 2, $H_{1,2}$ are their Hamiltonians, and ϵ and Δ are, respectively, the energy asymmetry and tunnel-coupling of the qubit. The small coupling constants between qubit and detectors are incorporated into the definition of $Q_{1,2}$. We assume that the detector is much faster than all qubit time scales, so the relevant detector correlation functions are the stationary zero frequency correlators:

$$\langle I_i(t+\tau)I_j(t)\rangle = S_I^{(i)}\delta_{ij}\delta(\tau), \tag{2a}$$

$$\langle Q_i(t+\tau)Q_j(t)\rangle = S_Q^{(i)}\delta_{ij}\delta(\tau), \tag{2b}$$

$$\langle Q_i(t+\tau)I_j(t)\rangle = (\mathrm{Re}S_{QI}^{(i)} + i\,\mathrm{Im}S_{QI}^{(i)})\delta_{ij}\delta(\tau), \tag{2c}$$

$$\langle I_i(t+\tau)Q_j(t)\rangle = (\mathrm{Re}S_{QI}^{(i)} - i\,\mathrm{Im}S_{QI}^{(i)})\delta_{ij}\delta(\tau), \tag{2d}$$

where $I_{1,2}$ are the bare output variables of the detectors. The time δ functions have a small shift $\delta(\tau - 0)$ to resolve the ambiguity in the correlators (2c) and (2d). Physically, this shift reflects the finite response time of the detector. Linear response theory tells us that the response coefficients $\lambda_{1,2}$ are given by $\lambda_i = -2\,\mathrm{Im}S_{QI}^{(i)}/\hbar$, so the output of the detectors (with the background average subtracted) is $\mathcal{O}_i = I_i + \lambda_i\sigma_z/2$. As the detector is turned on, it gradually collects information about the operator σ_z. The state of the qubit may be determined after the integrated difference in qubit signal exceeds the detector noise. In the simplest case of $\Delta = 0$, the standard expressions for the dephasing rate Γ and measurement time T_M are [10]

$$\Gamma = S_Q/(2\hbar^2), \qquad T_M = 4S_I/\lambda^2. \tag{3}$$

Let us next observe

$$\hbar^2\lambda^2 = 4(\mathrm{Im}S_{QI})^2 \leq 4|S_{QI}|^2 \leq 4S_QS_I, \tag{4}$$

where we have used the Cauchy-Schwartz inequality. For a lone detector the above relations imply $\Gamma T_M \geq 1/2$, where equality is reached for quantum limited detectors. The two conditions needed to reach this limit are

$$\mathrm{Re}S_{QI} = 0, \tag{5a}$$

$$|S_{QI}|^2 = S_QS_I. \tag{5b}$$

In the context of mesoscopic scattering detectors, condition (b) is related to the energy dependence of the transmission of the scatterer, while condition (a) is related to the symmetry of the scatterer [1]. Pilgram and one of the authors derived Eq. (5) for arbitrary detectors described by scattering matrices [8]. Clerk, Girvin, and Stone interpreted these conditions as "no lost information" about the qubit state, either through (a) phase or (b) energy averaging of the detector degrees of freedom [9].

Can we do better with two detectors?—By adding an additional detector to the qubit, the measurement time may be reduced because the signals may be averaged, $\mathcal{O} = (\mathcal{O}_1 + \mathcal{O}_2)/2$. On the other hand, the new detector dephases the qubit more quickly. For statistically independent detectors, the measurement-induced dephasing rate is simply the sum of the individual dephasing rates, so the two-detector efficiency is

$$\Gamma T_M = 2(S_I^{(1)} + S_I^{(2)})(S_Q^{(1)} + S_Q^{(2)})/\hbar^2(\lambda_1 + \lambda_2)^2 \geq 1/2, \tag{6}$$

where equality is reached for twin detectors that are themselves quantum limited. This condition may also be interpreted as no lost information about the state between the two detectors. Rather than averaging the signals, we could instead cross correlate them. However, this also brings no advantage because the new signal obtained by multiplying the output from the two detectors, $\mathcal{O}_1(t_1)\mathcal{O}_2(t_2)$, has its own noise. If we could average over many trials the noise could be eliminated, but for single-shot measurement the efficiency is still intrinsically limited.

Violation of the Korotkov-Averin bound.—Consider next Korotkov and Averin's bound on the signal-to-noise ratio for a weakly measured qubit [1]. It states that the ratio of the measured qubit signal to detector noise, $\mathcal{R}$, is fundamentally limited by 4. This bound can be overcome with quantum nondemolition measurements by increasing the signal [11]. In this Letter, we are concerned with reducing the noise. To see how this bound emerges, we briefly derive this inequality for one detector. The Hamiltonian is given by Eq. (1) with $Q_2 = 0$. The time averaged output of the detector (with background average subtracted) is $\langle\mathcal{O}\rangle = (\lambda/2)(1/T)\int_0^T dt\langle\sigma_z(t)\rangle$. For a weakly measured qubit, the statistical average over σ_z is taken with respect to the stationary, mixed, density matrix of the qubit that is proportional to the identity matrix, $\rho = (1/2)\mathbb{1}$. The qubit therefore makes *no* contribution to the average output current. The detector's spectral density is $S(\omega) = S_I + (\lambda^2/4)S_{zz}(\omega)$, where

$$S_{ij}(\omega) = 2\int_0^\infty dt\cos(\omega t)\langle\sigma_i(0)\sigma_j(t)\rangle. \tag{7}$$

The qubit dynamics may be found by expanding the evolution operator to second order in the coupling constant, and averaging over the δ-correlated Q fluctuations to obtain equations of motion, with dephasing rate Γ. In the special case of $\epsilon = 0$, the noise spectrum in the vicinity of $\omega = \Delta/\hbar \equiv \Omega$ is [1]

$$S(\omega) = S_I + \frac{\lambda^2\Gamma}{2}\frac{\Omega^2}{(\omega^2 - \Omega^2)^2 + \omega^2\Gamma^2}. \quad (8)$$

At the qubit frequency, $\omega = \Omega$, the noise spectrum has a maximum "signal" of $S_{\max} = \lambda^2/(2\Gamma) = \hbar^2\lambda^2/S_Q$. Again, we use the linear response relation (4) to bound the signal-to-noise ratio of the detector as

$$\mathcal{R} = S_{\max}/S_I \le 4. \quad (9)$$

This is the Korotkov-Averin bound.

Consider now the cross correlation of the outputs from two independent detectors, both measuring the same qubit operator σ_z. The qubit dynamics is the same, except that $\Gamma = \Gamma_1 + \Gamma_2$. The spectral density of the cross correlation $S_{1,2}(\omega)$ contains four terms,

$$S_{1,2}(\omega) = \int_0^\infty dt \cos(\omega t)[2\langle I_1(0)I_2(t)\rangle + \lambda_1\langle\sigma_z(0)I_2(t)\rangle + \lambda_2\langle I_1(0)\sigma_z(t)\rangle + (\lambda_1\lambda_2/2)\langle\sigma_z(0)\sigma_z(t)\rangle]. \quad (10)$$

According to Eq. (2a) the bare detector noise of the two detectors are uncorrelated, the averaged qubit dynamics is uncorrelated with the bare detector noise, so only the qubit signal (7) contributes to the correlation function (10). The remaining question is the detector configuration that maximizes the signal at $\omega = \Omega$. The signal is given by $S_{\max} = \lambda_1\lambda_2/[2(\Gamma_1 + \Gamma_2)]$, and we may use the relations (4) to bound the cross-correlated signal in relation to the noise power of the individual detectors as

$$S_{\max} \le 2\sqrt{S_I^{(1)}S_I^{(2)}}, \quad (11)$$

where equality is reached for $S_Q^{(1)} = S_Q^{(2)}$. For twin detectors, (11) is half of the single detector signal because of the doubled dephasing rate [12].

We have successfully removed the background noise, and can now see the naked destruction of the qubit [13]. The signal-to-noise ratio $\mathcal{R}$ is divergent, violating the Korotkov-Averin bound. Why did cross correlations help here, but not in the efficiency? The reason is that the efficiency is a measure of the information acquired on one observable versus detector influence on the complementary observables, and is thus protected by the uncertainty principle. In contrast, while the signal-to-noise ratio $\mathcal{R}$ is a useful detector diagnostic, there is no fundamental limitation on its measurement.

Weak measurement of noncommuting observables.—Once we have two detectors involved, there is no reason why they both have to measure the same observable (or one that commutes with it). We now consider an experiment where one detector weakly measures σ_z, the other weakly measures σ_x, and the outputs are cross correlated. The measured correlator is $S_{1,2}(\omega) = (\lambda_1\lambda_2/4)S_{zx}(\omega)$. This experiment could be implemented with a split Cooper-pair box [14], where a SQUID is weakly measuring the persistent current, and a quantum point contact is weakly measuring the electrical charge. In standard projective measurement theory, the question of a simultaneous measurement of noncommuting observables cannot even be posed.

The coupling part of the Hamiltonian is altered to be $H_c = (1/2)Q_1\sigma_z + (1/2)Q_2\sigma_x$. We parameterize any traceless qubit operator as $\sigma(t) = \sum_i x_i(t)\sigma_{x_i}$ and the density matrix $\rho = [\mathbb{1} + \sigma(t)]/2$. Variables (x, y, z) also represent coordinates on the Bloch sphere. After averaging over the white noise of Q_1 and Q_2, the equations of motion for x_i are

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -\Gamma_z & -\epsilon/\hbar & 0 \\ \epsilon/\hbar & -\Gamma_x - \Gamma_z & -\Delta/\hbar \\ 0 & \Delta/\hbar & -\Gamma_x \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad (12)$$

where $\Gamma_z = S_Q^{(2)}/(2\hbar^2)$ and $\Gamma_x = S_Q^{(1)}/(2\hbar^2)$. Diagonalization of the transition matrix in the case $\Gamma_x = 0$ gives the usual expressions for the dephasing and relaxation rates. This setup is far away from an efficient measurement because one detector is destroying the signal the other is trying to measure. However, in the case of weak measurement, this situation displays interesting behavior. The cross correlation $S_{1,2}(\Omega)$ attains its maximum signal at the symmetric point $\epsilon = \Delta$, $\Gamma_x = \Gamma_z = \Gamma$, so the qubit frequency is $\Omega = \sqrt{2}\Delta/\hbar$. The master equation may be solved in the weak dephasing limit ($\Gamma \ll \Omega$), giving the correlation (for positive frequencies)

$$S_{xz}(\omega) = S_{zx}(\omega) = \frac{\Gamma}{\Gamma^2 + \omega^2} - \frac{3\Gamma}{9\Gamma^2 + 4(\omega - \Omega)^2}. \quad (13)$$

The first term has a peak at zero frequency, while the second term has a dip at $\omega = \Omega$, with width $3\Gamma/2$, and signal $-1/3\Gamma$. Bounding this signal in relation to the noise in the individual twin detectors gives $|S_{1,2}(\Omega)| \le (2/3)S_I$. The interesting feature of this correlator is that it changes sign as a function of frequency. The low frequency part describes the incoherent relaxation to the stationary state, while the high frequency part describes the out of phase, coherent oscillations of the z and x degrees of freedom. The measured correlator S_{zx}, as well as S_{xx}, S_{zz} are plotted as a function of frequency in Figs. 2(b)–2(d) for different values of ϵ. These correlators all describe different aspects of the time domain destruction of the quantum state by the weak measurement, visualized in Fig. 2(a). We note that the cross correlator changes sign for $\epsilon = -\Delta$.

Implementation.—We now consider two quantum point contacts (QPCs), measuring a double quantum-dot qubit. The QPC obeys conditions (5) perfectly. Under the additional condition that the applied bias is larger than the temperature and Δ (so thermal equilibrium effects can be neglected), the QPC is an ideal detector [1,8,9]. The bare input detector variable Q is identified with the electrical charge in the point contact, while the bare output variable I is identified with the noisy current (shot noise).

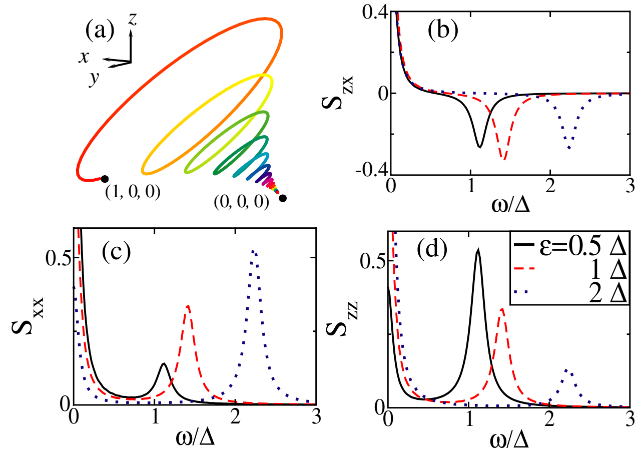

FIG. 2 (color online). (a) Time domain destruction of the quantum state by the weak measurement process for $\epsilon = \Delta$. The elapsed time is parameterized by color, and (x, y, z) denote coordinates on the Bloch sphere. (b) The measured cross correlator $S_{zx}(\omega)$ changes sign from positive at low frequency (describing incoherent relaxation) to negative at the qubit oscillation frequency (describing out of phase, coherent oscillations). (c),(d) The correlators S_{xx}, S_{zz} have both a peak at zero frequency and at qubit oscillation frequency. We take $\Gamma = \Gamma_x = \Gamma_z = 0.07\Delta/\hbar$. S_{ij} are plotted in units of Γ^{-1}.

The conductance of the QPC is sensitive to the electron's position on the double dot. A measurement of the quantum state occurs when the integrated current difference exceeds the shot noise power. In the geometry shown in Fig. 1, one detector measures σ_z, while the other detector measures $-\sigma_z$, so the qubit signal will be anticorrelated. The charges on the two detectors are not independent, but rather must be the opposite of each other to have charge neutrality in the system. This electrical screening generates correlations between the potentials of the two QPCs, so the detectors will in general be statistically dependent on each other. This situation is in marked contrast with the single detector case [8], where screening simply renormalized the coupling constant. However, in realistic detectors there will always be other gates to control the quantum double dot, creating a larger capacitance matrix than the minimal one shown in Fig. 1. In this extended geometry, a charge fluctuation in one detector will be screened by the surrounding metallic gates (not by the other detector), justifying the independent detector model [13]. We mention that in the experiment already done by Buehler *et al.* [5], the detectors seem to be completely independent.

Conclusions.—We considered the advantages that two independent quantum detectors measuring the same qubit can bring to the quantum measurement problem. The quantum limit on the efficiency could be reached with twin detectors. For weak continuous measurement, the cross-correlated signal removes the noise background, and allows a violation of the Korotkov-Averin bound on the signal-to-noise ratio. The simultaneous weak measurement of noncommuting operators was investigated, and revealed a crossover from positive to negative correlation as a function of frequency. Although we have focused on mesoscopic qubits, our results easily extend to other systems where similar bounds have been derived, such as single spins and nanomechanical oscillators [15].

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[12] The response functions may be quite different, so the cross-correlated signal $S_{\max} \leq 2S_I^{(2)}\lambda_1/\lambda_2 = 2S_I^{(1)}\lambda_2/\lambda_1$ may be much larger than the noise in one detector, provided it is much smaller than the noise in the other detector.
[13] Weak direct coupling between the detectors may be accounted for by including a term $H_{1,2} = \alpha Q_1 Q_2$, in the Hamiltonian where α is a relative coupling constant between the two detectors. The additional contribution to the cross correlator is $\delta S_{1,2} = \alpha\lambda_1 \mathrm{Re} S_{QI}^{(2)} + \alpha\lambda_2 \mathrm{Re} S_{QI}^{(1)}$ and vanishes if the detector is efficient [(5a)], providing a useful test of detector efficiency.
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