Transmission Resonances of Metallic Compound Gratings with Subwavelength Slits

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Transmission metallic gratings with subwavelength slits are known to produce enhanced transmitted intensity for certain resonant wavelengths. One of the mechanisms that produce these resonances is the excitation of waveguide modes inside the slits. We show that by adding slits to the period, the transmission maxima are widened and, simultaneously, this generates phase resonances that appear as sharp dips in the transmission response. These resonances are characterized by a significant enhancement of the interior field.

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In the last few years, considerable attention has been paid to the study of gratings with subwavelength slits due to the experimental evidence of extraordinary transmission in 2D arrays of holes in a metallic film [1,2]. Even though there is not complete agreement within the scientific community on the physical origin of this phenomenon [3], in the case of 1D structures two different mechanisms can be identified as responsible for this phenomenon: surface plasmon excitations and coupling to waveguide modes of the slits [4,5]. These are two of the four mechanisms known to produce anomalies in the response of metallic gratings. The appearance or disappearance of a diffracted order can also produce a sudden change in the diffracted efficiency. These anomalies are known as Rayleigh anomalies [6]. There is a fourth kind of resonances that might rise in a periodic grating when its period comprises several cavities or slits. For a particular wavelength, the field distribution inside the different cavities/slits takes a particular form so that its phase in adjacent cavities can be opposite to each other and its amplitude maximizes the inner field. These are known as phase resonances [7–9], and were first reported in structures comprising a finite number of cavities on a perfect conductor [10,11]. The addition of cavities/slits to the period of the grating introduces new degrees of freedom regarding the possible near field configurations. The condition of pseudoperiodicity of the fields imposes that all periods of the grating are equivalent. In the case of a simple grating—formed by a single groove or slit in the period—this implies that, with the exception of a phase factor, all the grooves have the same field. On the other hand, a compound grating—formed by several grooves/slits in the period—allows for different phase configurations inside each period, and this can lead to resonances and to sudden variations of the diffracted efficiency.

In Fig. 1 we schematize the transmission gratings under study; only one period is shown. Grating (a) is the simple transmission grating, with a single slit of width a in the period d. The number of slits in the period is J. Each period of grating (b) comprises two slits ($J = 2$) of equal width, separated by a thin wire of width c. Cases (c), (d), and (e) correspond to $J = 3$, 4, and 5, respectively. The number of thin wires in the period is $J - 1$. We consider that the grating is illuminated at normal incidence by a p-polarized plane wave of wavelength λ. The reflected and the transmitted magnetic fields are represented by Rayleigh expansions. Then the field in the incident and in the transmission medium are given by

$$H_{z\,\mathrm{inc}}(x, y) = \exp[i(\alpha_0 x - \beta_0 y)] + \sum_n R_n \exp[i(\alpha_n x + \beta_n y)], \quad (1)$$

and

$$H_{z\,\mathrm{trans}}(x, y) = \sum_n T_n \exp[i(\alpha_n x - \beta_n y)], \quad (2)$$

respectively, where $\alpha_n = (2\pi/\lambda)\sin\theta_0 + n(2\pi/d)$, $\beta_n^2 = (2\pi/\lambda)^2 - \alpha_n^2$, θ_0 is the angle of incidence, and R_n and T_n are the reflected and transmitted Rayleigh amplitudes,

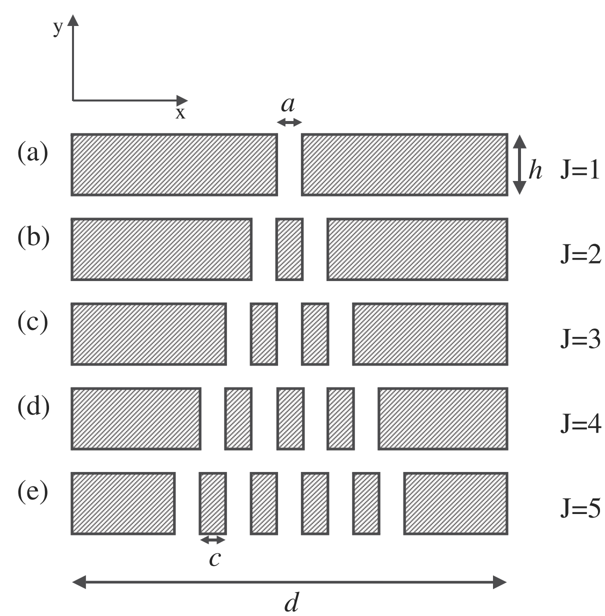

FIG. 1. Scheme of the simple and compound gratings.

respectively. Inside the slits the fields are expanded in terms of eigenfunctions that take into account the surface impedance boundary condition (SIBC) on the lateral walls of each slit [12]:

$$H_{z\,\mathrm{slits}}(x,y) = \sum_{m=0}^{\infty}\left\{\cos[u_m(x-x_j)] + \frac{\eta}{u_m}\sin[u_m(x-x_j)]\right\} \times \{a_{mj}\cos[v_m y] + b_{mj}\sin[v_m y]\}, \quad (3)$$

where $u_m^2 + v_m^2 = k^2$, $\eta = -ik/\sqrt{\mu_m \epsilon_m}$, and the eigenvalues u_m are found by solving the following equation, which results from the imposition of the boundary conditions at the walls of the slits:

$$\tan u_m a = \frac{2\eta u_m}{u_m^2 - \eta^2}. \quad (4)$$

The x_j are the positions of the left wall of each slit (the subscript j denotes the slit), ϵ_m and μ_m are the permittivity and the permeability of the metal, respectively, and a_{mj} and b_{mj} are complex amplitudes. The fields are matched on the horizontal boundaries by imposing the continuity of the tangential components in the open sections, and by applying the SIBC in the metallic regions. This method leads to a system of coupled equations that are projected in convenient bases to get a matrix system for the unknown reflected and transmitted amplitudes.

In Fig. 2 we show curves of transmitted intensity vs wavelength, for a sequence of gratings with an increasing number of slits (J) in the period. In the lower curve, for $J = 1$, we observe a peak at $\lambda/d \approx 1.4$ and another peak at $\lambda/d \approx 3$. As explained by other authors [5], these peaks are associated with waveguide mode resonances. The number of propagating eigenmodes increases with the depth of the grating. For a perfectly conducting structure, waveguide mode resonances are expected to occur at $\lambda = 2h/n$, with n integer. In the highly (but finitely) conducting case, we found that a better estimate of the location of these resonances can be obtained by imposing the condition

$$\mathrm{Re}\{v_m h\} = n\pi. \quad (5)$$

Since we consider slits that are narrow compared with the wavelength, the only propagating mode is the first eigenmode corresponding to $m = 1$ in Eq. (5). The same two peaks that appear in the lower curve are present in the next curve, corresponding to a period comprising two slits ($J = 2$). Both peaks are widened and slightly shifted. A significant change in the behavior is found for three slits in the period ($J = 3$): a sharp dip splits each peak into two. This behavior is also found in the $J = 4$ curve, and in this case the dip has slightly shifted to longer wavelengths. Finally, for five slits in the period (upper curve), one more dip appears in each waveguide resonance peak.

The physical origin of these dips can be explained in terms of phase resonances. It is well known that in a perfectly periodic grating the treatment of the diffraction problem can be reduced to just a single period, due to the pseudoperiodic condition. Then, the fields in all slits or cavities of a simple grating are essentially equal. However, when we add slits to the period, i.e., when we make a compound grating, new degrees of freedom open up: the distribution of field phases in the different slits that comprise the period can have different configurations. Since we are considering narrow slits, the field inside each slit is almost constant in the x direction. Besides, under normal incidence, the possible phase configurations must be symmetrical. Then, the number of possible phase configurations is finite and depends on the number of slits. For instance, for three slits in the period, there are only two possible configurations: (i) all the slits have equal phase $(+\,+\,+)$, and (ii) the external slits have equal phase, different from the central one. In general, two requirements are needed for a phase resonance to occur: (i) at least one nonzero phase difference is found between the field phases in adjacent slits—what is not allowed in simple periodic structures—and (ii) a particular distribution of the field amplitude, naturally generated by the incidence conditions, is obtained [11]. Phase resonances are also characterized

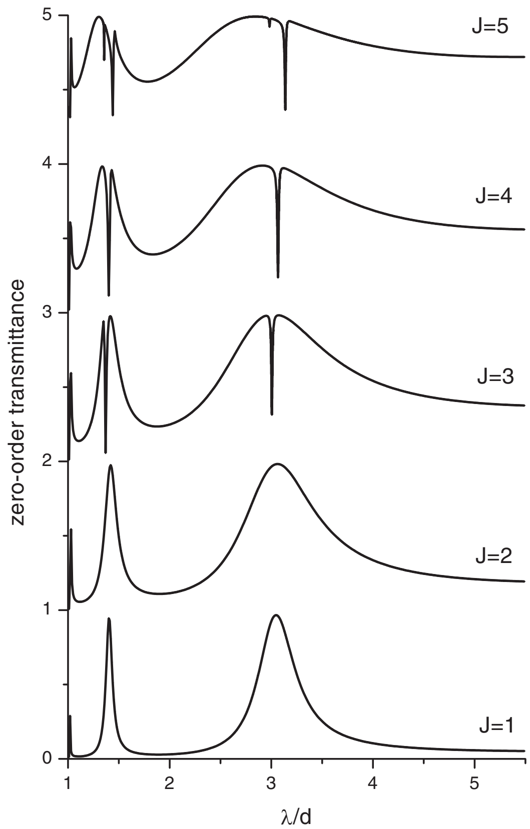

FIG. 2. Zero-order transmittance as a function of the wavelength for a normally incident, p-polarized plane wave impinging on a metallic grating with $a/d = c/d = 0.08$ and $h/d = 1.14$. The refraction index of the metal is $\nu = 0.15 + i24.9$. The different curves correspond to different numbers of slits in the period: $J = 1$ to 5.

by a strong intensification of the field inside the slits/cavities, and at the same time, by a significant power absorption. In particular, when the phases in adjacent slits are opposite to each other, π resonances can be excited [11]. These two conditions are usually fulfilled near the waveguide mode resonant wavelength. For one or two slits in the period, different phases in adjacent slits cannot occur, and therefore no phase resonance is present. For $J = 3$, the π mode can be excited, and this is the case of the dips that appear in each one of the waveguide resonance peaks (at $\lambda/d \approx 1.4$ and 3). In Fig. 3 we plot the absolute value of the phase difference ($|\Delta\phi|_{\rm ce}$) between the fields at the central and the external slits (evaluated at the top surface of the structure, at the horizontal center of each slit), as a function of λ/d. We only show curves for $J = 3$ and $J = 5$, since for $J = 1$ there is only a single slit in the period and for $J = 2$ under normal incidence, the phase difference must be zero, as expected on symmetry considerations. In the lower curve ($J = 3$) $|\Delta\phi|_{\rm ce}$ is nearly zero for most of the range of wavelengths considered, except for two distinct wavelengths at which it takes the π value, and these wavelengths correspond to the dips in Fig. 2 ($J = 3$ curve). In Fig. 4 we show contour plots of the phase of the magnetic field at those resonant wavelengths. It can be observed that the phase distribution of the field at both wavelengths are of the type $(+ - +)$; i.e., the phases in adjacent slits are opposite. Besides, significant intensifications of the field inside the slits are obtained, the intensification factors being at around 20 and 40 for the first and second dip, respectively, (not shown). For other wavelengths, the intensification does not exceed 7. It is also clear from Fig. 4 that the longer wavelength dip corresponds to the first waveguide mode of the slit ($h \approx \lambda/2$) and the shorter one corresponds to the second mode ($h \approx \lambda$). For $J = 4$, as for $J = 3$, there is only one possibility of having different phases in adjacent slits, due to the symmetry imposed by the normal incidence condition, and the phase difference curve vs wavelength is very similar to the lower curve in Fig. 3 (not shown). We have checked that the dip observed in the $J = 4$ curve in Fig. 2 corresponds to a phase configuration $(+ - - +)$. However, for $J = 5$ we can have more combinations of phase distributions, and this can be observed in the upper curve in Fig. 2: an additional dip appears in each peak. The deeper dips correspond to a phase mode of the type $(+ - - - +)$, and in the new ones the central slit phase is opposite to the phase of its adjacent slits and the external ones have a $\pi/2$ phase difference with their adjacent slits. Also, the amplitude distribution of the field in each mode is different. The phase difference between the central and the middle slits, and between the middle and the external slits is plot in the upper curve in Fig. 3. There are two wavelengths at which

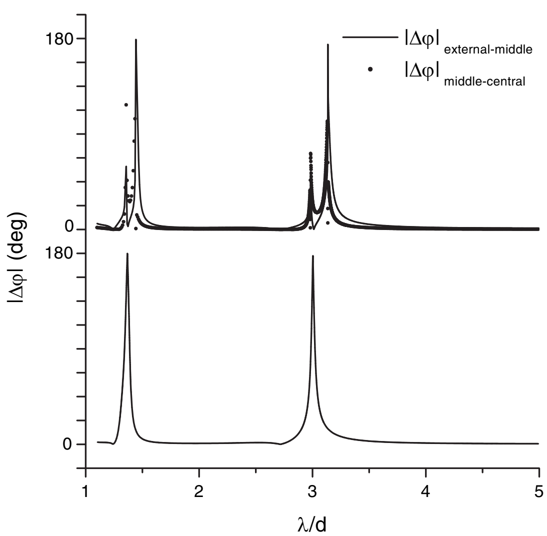

FIG. 3. Phase difference of the magnetic field at adjacent slits as a function of the wavelength for the same structure considered in Fig. 2, for $J = 3$ and $J = 5$.

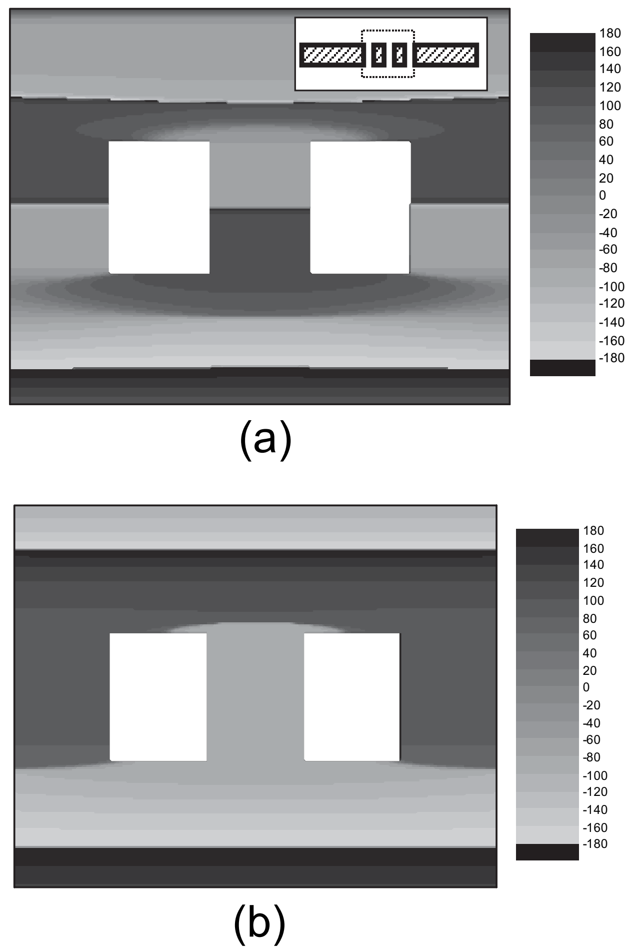

FIG. 4. Phase of the magnetic field for a normally incident p-polarized plane wave impinging on a metallic grating with $a/d = 0.08$, $h/d = 1.14$, and $J = 3$. The refraction index of the metal is $\nu = 0.15 + i24.9$. (a) $\lambda/d = 1.372$, (b) $\lambda/d = 3.004\,96$. The inset in part (a) shows the region where the field is calculated.

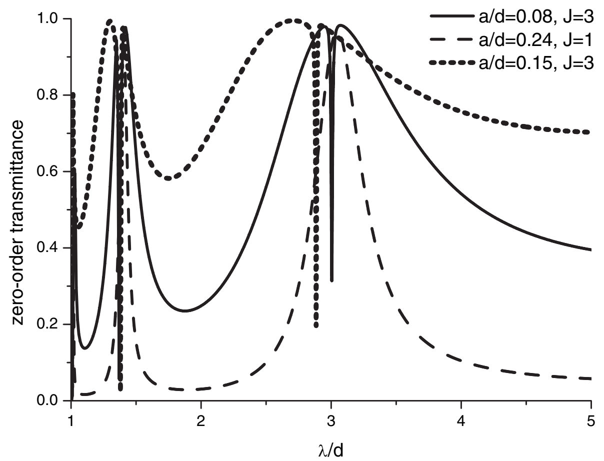

FIG. 5. Comparison between the $J = 3$ curve in Fig. 2 (solid line) and the transmission response of a grating with a single slit of width $a/d = 0.24$, which is 3 times that corresponding to the solid curve (dashed line). The dotted curve corresponds to $J = 3$ and $a/d = 0.15$.

$|\Delta\phi|_{\text{external}-\text{middle}}$ is π and $|\Delta\phi|_{\text{middle}-\text{central}}$ is 0, at which the deeper dips of the upper curve in Fig. 2 are found. However, there are two new distinct wavelengths at which there is a nonzero phase difference between adjacent grooves, which gives rise to the new dips in the $J = 5$ case. From the sequence of Fig. 2, it is also clear that as the number of slits increases, the waveguide resonance peaks widen. This fact is interesting from the point of view of the enhanced transmission, since we can get an enhanced response for a wider frequency range. However, this widening is obtained at the expense of having dips in between. On the other hand, the dips are interesting by themselves, since they provide a way of getting very intense fields inside the slits, which can be exploited for the obtention of nonlinear effects. In the whole range of wavelengths considered, the transmission for s polarization is practically null.

One would think that the widening of the peaks could be due to the fact that the ratio between the empty and the filled part of the grating (occupation ratio) increases when slits are added to the structure. To explore this possibility, we plot in Fig. 5 the transmittance curves for two different gratings with equal occupation ratio. The solid curve corresponds to a compound grating with three slits (the $J = 3$ curve in Fig. 2) and the dashed one corresponds to a simple grating with a slit 3 times as wide as the slits of the previous one. It can be noticed that the dip only appears for the compound grating. Besides, in such case the peaks are wider than in the simple grating, suggesting that the widening is not only due to an increase in the occupation ratio, but also to a structural effect.

The effect of varying the width of the slits is also illustrated in Fig. 5, where the solid curve is to be compared with the dotted one, which corresponds to the same parameters but with $a/d = 0.15$. In this case, the overall transmittance increases due to a larger occupation ratio, but the dips are still present, although slightly shifted. This behavior was confirmed for several widths (not shown), suggesting that the phase resonances responsible for the dips are not highly dependent on this parameter.

For non-normal incidence, new phase configurations that were forbidden by the normal incidence symmetry are allowed. For instance, we have observed that a dip is found for a structure with two slits per period ($J = 2$) for $\theta_0 = 40°$, and this dip corresponds to the π mode. Notice that no phase resonances occur for $J = 2$ under normal incidence (Fig. 2).

In conclusion, we have shown that compound transmission gratings can exhibit a transmission response very different from that of simple gratings. While for simple gratings with subwavelength slits transmission maxima are observed for the slit waveguide mode resonant wavelengths, for compound gratings these peaks are significantly widened. Besides, we have shown that phase resonances, characterized by a particular distribution of the phase of the field inside the slits, take place for wavelengths contained in each peak and that the mechanism of phase resonance is manifested as sharp dips in the transmittance and also by a significant enhancement of the interior field. The capability of exciting phase resonances in compound gratings opens up new possibilities for practical applications, such as polarization sensitive aperture shapes for field enhancement devices.

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