Self-Consistent Generation of Superthermal Electrons by Beam-Plasma Interaction

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It has been known since the early days of plasma physics research that superthermal electrons are generated during beam-plasma laboratory experiments. Superthermal electrons (the κ distribution) are also ubiquitously observed in space. To explain such a feature, various particle acceleration mechanisms have been proposed. However, self-consistent acceleration of electrons in the context of plasma kinetic theory has not been demonstrated to date. This Letter reports such a demonstration. It is shown that the collisionality, defined via the "plasma parameter" $g = 1/\hat{n}\lambda_D^3$, plays a pivotal role. It is found that a small but moderately finite value of g is necessary for the superthermal tail to be generated, implying that purely collisionless ($g = 0$) Vlasov theory cannot produce a superthermal population.

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Since the early days of plasma physics research, it was known from laboratory experiments [1] that a small population of electrons which possess much higher energy than the original beam kinetic energy is generated. In space, energetic particles are ubiquitously observed [2,3], which is often modeled by the κ distribution [2,4]. It is widely believed that their origin lies in the acceleration by wave turbulence, i.e., second-order Fermi acceleration. Various authors have addressed this problem, but invariably the discussion is either qualitative or based upon non-self-consistent approaches. Early qualitative theories are best represented by Ref. [5]. Among the later works, Hasegawa *et al.* [6] obtained the analytical κ solution in the presence of a high-intensity radiation field. Ma and Summers [7] replied upon stationary Whistler turbulence. Numerical solutions of the particle diffusion equation in Refs. [8,9], with model diffusion coefficients are also similar in this regard. The combined approach of the strong-turbulence (i.e., Zhakarov) equation for the waves and the weak-turbulence diffusion equation for the particles [10] were also suggested. Collier [11], on the other hand, employed the Lévy flight probability concept to derive a κ distribution.

As briefly surveyed above, a variety of physical processes may lead to κ-like distributions. However, conspicuously lacking is a concrete demonstration of a *self-consistent* generation of superthermal particles within the context of plasma kinetic theory. The reason seems to be that the particle diffusion equation is relatively easy to handle, provided the diffusion coefficient is simply modeled. However, if one is to solve the wave intensity from the nonlinear wave kinetic equation instead of modeling it, then the matter becomes notoriously difficult. At present, no one has managed to derive the nonlinear wave kinetic equation in full generality from first principles, let alone to solve it. The only available theory pertains to the Langmuir/ion-sound turbulence problem in unmagnetized plasmas [12]. In this Letter we demonstrate, for the first time to our knowledge, the self-consistent generation of superthermal electrons within the context of plasma kinetic theory. We restrict ourselves to the temporal beam-plasma relaxation and Langmuir/ion-sound turbulence problem. Strictly speaking, the laboratory generation of energetic electrons is more relevant to the spatial beam relaxation problem [13], but the two are related to each other.

In the present weak-turbulence theory, the "plasma parameter" $g = 1/\hat{n}\lambda_D^3$ [where $\lambda_D = (T_e/4\pi\hat{n}e^2)^{1/2}$ is the Debye length, $\hat{n}$ is the density, T_e is the electron temperature, and e is the unit charge] turns out to play a pivotal role. It is found that a small but moderately finite value of g is necessary for the superthermal tail to be generated, implying that purely collisionless ($g = 0$) Vlasov theory cannot produce superthermal population. Particle-in-cell simulation [14] deals with pseudoparticles averaged over the cell, rather than true individual particles. The present weak-turbulence simulation has its limitations, but it describes individual particle effects in a most faithful manner. Thus, such a theory occupies a unique place among the arsenal of plasma physics research tools. We start from the electron particle kinetic equation,

$$\frac{\partial F_e}{\partial t} = \frac{\pi e^2}{m_e^2}\frac{\partial}{\partial \mathbf{v}}\cdot\sum_{\sigma=\pm 1}\int d\mathbf{k}\,\delta_L\left(\hat{\mathbf{k}}\frac{m_e}{4\pi^2}(\hat{\mathbf{k}}\cdot\mathbf{v})F_e + \hat{\mathbf{k}}\,\hat{\mathbf{k}}\,I_{\mathbf{k}}^{\sigma L}\cdot\frac{\partial F_e}{\partial \mathbf{v}}\right), \tag{1}$$

where $\delta_L = \delta(\sigma\omega_{\mathbf{k}}^L - \mathbf{k}\cdot\mathbf{v})$, and for later purposes, $\delta_S = \delta(\sigma\omega_{\mathbf{k}}^S - \mathbf{k}\cdot\mathbf{v})$; $\hat{\mathbf{k}} = \mathbf{k}/|k|$ is the unit wave vector; F_e is the electron distribution function (normalized to unity, $\int d\mathbf{v}F_e = 1$); and m_e is the electron mass. Dispersion relations for Langmuir and ion-sound modes, $\omega_{\mathbf{k}}^L$ and $\omega_{\mathbf{k}}^S$, are defined, respectively, by

$$\omega_{\mathbf{k}}^{L}/\omega_{pe} = 1 + 3\kappa^2/2,$$
$$\omega_{\mathbf{k}}^{S}/\omega_{pe} = [(1+3\tau)/M]^{-1/2}\kappa(1+\kappa^2)^{-1/2}, \tag{2}$$

where $\kappa = k\lambda_D$, $M = m_i/m_e$, and $\tau = T_i/T_e$. Here, m_i is the ion (proton) mass, T_i is the ion temperature, and $\omega_{pe} = (4\pi\hat{n}e^2/m_e)^{1/2}$ is the plasma frequency. The spectral intensities for Langmuir and ion-sound waves, $I_{\mathbf{k}}^{\sigma\alpha}$ ($\alpha = L, S$), are defined via $\langle \delta E^2\rangle_{\omega,\mathbf{k}}^{\alpha} = \sum_{\sigma=\pm1} I_{\mathbf{k}}^{\sigma\alpha}\delta(\omega - \sigma\omega_{\mathbf{k}}^{\alpha})$, where δE represents the fluctuating electric field. For a collisional plasma in which collective modes are dominant, the appropriate equation for the particles is Eq. (1). The Balescu-Lénard equation is valid only when the system is close to thermal equilibrium [15].

The wave kinetic equation for mode α (=L, S) is given in terms of the normalized wave intensity $N_{\mathbf{k}}^{\sigma\alpha} = I_{\mathbf{k}}^{\sigma\alpha}/\mu_{\mathbf{k}}^{\alpha}$ by

$$\frac{\partial N_{\mathbf{k}}^{\sigma\alpha}}{\partial t} = \int d\mathbf{v}\,\delta_\alpha(P_{\mathbf{k}}^{\alpha} + \gamma_{\mathbf{k}}^{\alpha}N_{\mathbf{k}}^{\sigma\alpha}) + \sum_{\sigma',\sigma''=\pm1}\int d\mathbf{k}'\,V_{\mathbf{k},\mathbf{k}'}^{\alpha}[\sigma\omega_{\mathbf{k}}^{L}N_{\mathbf{k}'}^{\sigma' L}N_{\mathbf{k}-\mathbf{k}'}^{\sigma''\beta} - (\sigma'\omega_{\mathbf{k}'}^{L}N_{\mathbf{k}-\mathbf{k}'}^{\sigma''\beta} + \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}N_{\mathbf{k}'}^{\sigma' L})N_{\mathbf{k}}^{\sigma\alpha}]$$
$$- \sum_{\sigma'=\pm1}\int d\mathbf{v}\int d\mathbf{k}'\,\Delta_\alpha[S_{\mathbf{k},\mathbf{k}'}^{\alpha}(\sigma'\omega_{\mathbf{k}'}^{L}N_{\mathbf{k}}^{\sigma\alpha} - \sigma\omega_{\mathbf{k}}^{L}N_{\mathbf{k}'}^{\sigma'\alpha}) - U_{\mathbf{k},\mathbf{k}'}^{L}N_{\mathbf{k}'}^{\sigma'\alpha}N_{\mathbf{k}}^{\sigma\alpha}], \tag{3}$$

where $\beta = S$ if $\alpha = L$, and $\beta = L$ if $\alpha = S$; $\Delta_L = \delta[\sigma\omega_{\mathbf{k}}^{L} - \sigma'\omega_{\mathbf{k}'}^{L} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$ and $\Delta_S = \delta[\sigma\omega_{\mathbf{k}}^{S} - \sigma'\omega_{\mathbf{k}'}^{S} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$. The first two terms on the right-hand side of Eq. (3) represent spontaneous ($P_{\mathbf{k}}^{\alpha}$) and induced ($\gamma_{\mathbf{k}}^{\alpha}$) emission processes, where $P_{\mathbf{k}}^{L} = P_k F_e$, $P_{\mathbf{k}}^{S} = \mu_{\mathbf{k}}^{S}P_k(F_e + F_i)$, $\gamma_{\mathbf{k}}^{L} = \gamma_{\mathbf{k}}(\hat{\mathbf{k}}\cdot\partial_{\mathbf{v}})F_e$, and $\gamma_{\mathbf{k}}^{S} = \mu_{\mathbf{k}}^{S}\gamma_{\mathbf{k}}(\hat{\mathbf{k}}\cdot\partial_{\mathbf{v}})(F_e + F_i/M)$. Here, $P_k = \hat{n}e^2\omega_{pe}^2/k^2$, and $\gamma_{\mathbf{k}} = \pi\omega_{pe}^2\sigma\omega_{\mathbf{k}}^{L}/k$. The quantity $\sigma = \pm1$ represents the forward/backward propagation (with respect to the beam propagation direction). The decay processes are depicted by $V_{\mathbf{k},\mathbf{k}'}^{\alpha}$,

$$V_{\mathbf{k},\mathbf{k}'}^{L} = V_{\mathbf{k}}\mu_{\mathbf{k}-\mathbf{k}'}^{S}(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')^2\delta(\sigma\omega_{\mathbf{k}}^{L} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S}),$$
$$V_{\mathbf{k},\mathbf{k}'}^{S} = (V_{\mathbf{k}}/2)\{\mu_{\mathbf{k}}^{S}[\hat{\mathbf{k}}'\cdot(\mathbf{k}-\mathbf{k}')]^2/k^2\}$$
$$\times\,\delta(\sigma\omega_{\mathbf{k}}^{S} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}), \tag{4}$$

where $V_{\mathbf{k}} = (\pi/2)(e^2/T_e^2)\sigma\omega_{\mathbf{k}}^{L}/(\mathbf{k}-\mathbf{k}')^2$. The quantities $\mu_{\mathbf{k}}^{\alpha}$ are, respectively, given by $\mu_{\mathbf{k}}^{L} = 1$ and $\mu_{\mathbf{k}}^{S} = |k|^3\lambda_D^3(m_e/m_i)^{1/2}(1 + 3T_i/T_e)^{1/2}$. Finally, the last term in Eq. (3) depicts spontaneous and induced scattering processes, where

$$S_{\mathbf{k},\mathbf{k}'}^{L} = S_{\mathbf{k}}(F_e + F_i),$$
$$S_{\mathbf{k},\mathbf{k}'}^{S} = \mu_{\mathbf{k}}^{S}\mu_{\mathbf{k}'}^{S}(S_{\mathbf{k}}/k^2k'^2\lambda_{De}^4)W_{\mathbf{k},\mathbf{k}'}(F_e + F_i),$$
$$U_{\mathbf{k},\mathbf{k}'}^{L} = U_{\mathbf{k}}(\mathbf{k}-\mathbf{k}')\cdot\partial_{\mathbf{v}}F_i, \tag{5}$$
$$U_{\mathbf{k},\mathbf{k}'}^{S} = \mu_{\mathbf{k}}^{S}\mu_{\mathbf{k}'}^{S}(U_{\mathbf{k}}/k^2k'^2\lambda_D^4)[W_{\mathbf{k},\mathbf{k}'} + \sigma\sigma'(k'/k)]$$
$$\times\,(\mathbf{k}-\mathbf{k}')\cdot\partial_{\mathbf{v}}F_i.$$

In the above, $S_{\mathbf{k}} = \hat{n}e^4\sigma\omega_{\mathbf{k}}^{L}(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')^2/m_e^2\omega_{pe}^4$, and $U_{\mathbf{k}} = \pi e^2\sigma\omega_{\mathbf{k}}^{L}(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')^2/m_em_i\omega_{pe}^2$. Here, an additional quantity $W_{\mathbf{k},\mathbf{k}'}$ is defined by

$$W_{\mathbf{k},\mathbf{k}'} = (1+\xi^2)^2/[\xi^4(\mathbf{k}-\mathbf{k}')^4\lambda_D^4|\epsilon|^2],$$
$$\epsilon = 1 + \frac{2(\mathbf{k}\cdot\mathbf{k}' - \sigma\sigma' kk')}{\xi^2(\mathbf{k}-\mathbf{k}')^4\lambda_D^2} + \frac{i(\pi/2M)^{1/2}\xi}{(\mathbf{k}-\mathbf{k}')^2\lambda_{De}^2} \tag{6}$$
$$\times\,[e^{-(\xi^2/2M)} + (M/\tau)^{2/3}e^{-(\xi^2/2\tau)}],$$

where $\xi = (k - \sigma\sigma' k')/|\mathbf{k}-\mathbf{k}'|$.

In what follows, we take a one-dimensional (1D) reduction. The initial electron distribution function is given by, $F_e(v,0) = (1 - \hat{n}_b/\hat{n})(\pi v_e)^{-1}e^{-v^2/v_e^2} + (\hat{n}_b/\hat{n})(\pi v_b)^{-1}\times e^{-(v-V_0)^2/v_b^2}$. The ions are treated as a stationary background with $F_i = (\pi v_i)^{-1}e^{-v^2/v_i^2}$. The ion thermal speed is defined by $v_i = (2T_i/m_i)^{1/2}$. The dimensionless input parameters are $\hat{n}_b/\hat{n}$, V_0/v_e, v_b/v_e, τ; and $g = 1/\hat{n}\lambda_D^3$. Here $v_e = (2T_e/m_e)^{1/2}$ and $v_b = (2T_b/m_e)^{1/2}$ are the bulk electron and beam thermal speeds, respectively. We choose $\hat{n}_b/\hat{n} = 10^{-2}$, $V_0/v_e = 4$, $v_b/v_e = 1$, $1/\tau = 7$, and vary g. For interplanetary space, $g \approx 5\times10^{-3}$, while for glow discharge experiment, $g \approx \times10^{-2}$. In the chromosphere, $g \approx 5\times10^{-4}$, and for thermonuclear devices, g can be as low as $\sim10^{-8}$. Of course, the mass ratio is $M = 1836$. The ranges of normalized velocity and wave number are $-16 < u = v/v_e < 16$ and $10^{-4} < q = kv_e/\omega_{pe} < 1$, respectively. We choose 201 grids for u and 101 grids for q. In plotting the results for backward waves ($\sigma = -1$), we invoke the symmetry property and display the intensities in the negative range $-1 < q < -10^{-4}$. The numerical scheme is the standard leapfrog explicit method with time increment $\Delta t = 0.01\omega_{pe}^{-1}$.

Numerical solutions of 1D equations have been attempted in the past. However, they are incomplete in that not all the terms on the right-hand side of Eq. (3) were included. Reference [16] is the first to partially solve the weak-turbulence equation, but it did not include decay terms, and the S wave equation was not solved at all. Later, Refs. [17,18] solved more complete equations, but again, they ignored spontaneous fluctuations and neglected the S mode scattering term. By the same token, Ref. [19] is only partially complete in that scattering terms were completely ignored altogether. Finally, our recent work [15] also suffers from the similar shortcoming in that we failed to include the S mode scattering term. Needless to say, one must include all the terms in order to properly characterize the nonlinear dynamics of the system. We found that failure to do so prevents the generation of the superthermal tail. To demonstrate this, we show in Fig. 1, the normalized L and S mode spectral intensities, $I_q^{\alpha} = (2\pi)^2 g I_k^{\alpha}/(m_e v_e^2)$ in log-scale vs q. The top panels correspond to the full

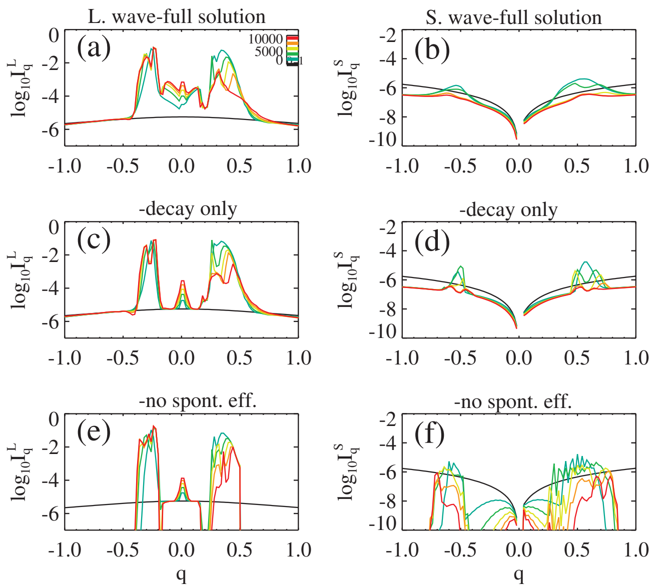

FIG. 1 (color). Evolution of the Langmuir (left) and ion-sound (right) mode spectral wave energy density for $g = 5 \times 10^{-3}$, up to $\omega_{pe} t = 10^4$. Panels (a) and (b) are full solutions; (c) and (d) are when only decay terms are retained for nonlinear interactions; (e) and (f) correspond to when all the spontaneous processes are ignored.

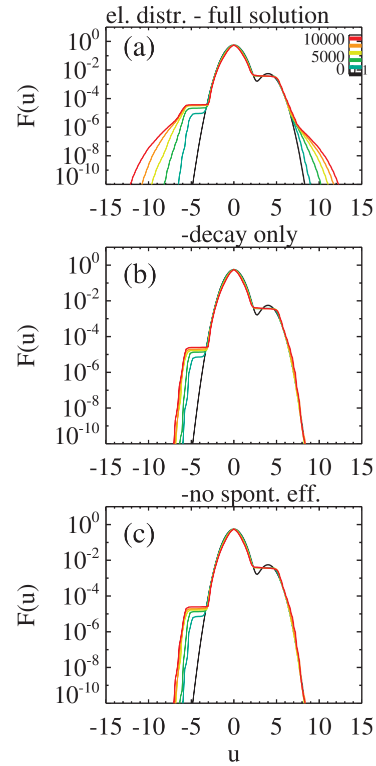

FIG. 2 (color). Electron distribution (a) when all the terms in Eq. (3) are included, (b) when only the decay terms are retained, and (c) when spontaneous processes are ignored.

solution. In the middle and bottom panels, we present results of two common approximations. The middle panels correspond to the approximation where induced and spontaneous scattering terms are arbitrarily ignored [i.e., (c) and (d)] [19]. The bottom panel is when all the spontaneous effects are ignored [(e),(f)] [17]. Note that incomplete theories are quite adequate if the purpose is an approximate description of the wave dynamics.

However, the generation of a superthermal tail critically depends on inclusion of all the terms. Particularly, we find that spontaneous scattering terms play crucial roles, as they are responsible for the generation of turbulence spectra in the gap region between the primary and backscattered L, and the condensate modes. Our recent work [15] only includes the spontaneous scattering term for the L mode. We find that the same term due to the S mode contributes equally, such that ignoring S mode scattering leads to a sizable discrepancy. Shown in Fig. 2 is the normalized electron distributions $F(u)$ vs u for three cases considered in Fig. 1. Figure 2 shows that approximate theories fail to produce the superthermal tail population. Figures 1 and 2 were generated for $g = 5 \times 10^{-3}$ and for normalized time up to $\omega_{pe} t = 10^4$. However, for low values of g, we find that the tail production is suppressed such that in the limit $g \to 0$, the tail is almost completely absent even if we include all the terms in Eq. (3). To show this, we display in Fig. 3 $F(u)$ for g ranging from $g = 10^{-6}$ to $g = 5 \times 10^{-3}$, computed at maximum time $\omega_{pe} t = 2 \times 10^4$. For the low collisionality regime, no tail formation can be seen. However, when g approaches $g = 5 \times 10^{-3}$, a significant heating of the electrons can be seen.

Space observation of energetic electron distributions are often modeled by the κ distribution; the 1D version of which is given in normalized form by

$$F_\kappa(u) = \frac{\Gamma(\kappa + 1)}{(\pi\kappa)^{1/2}\Gamma(\kappa + 1/2)} \frac{1}{(1 + u^2/\kappa)^{\kappa+1}}. \quad (7)$$

Figure 4 plots $F(u)$ at $\omega_{pe} t = 2 \times 10^4$, for $g = 5 \times 10^{-3}$, vs u. Superposed are κ distribution ([4]) with index $\kappa = 3.5$ and the Gaussian model ($\kappa \to \infty$). Observe the rather excellent fit of the real solution with κ distribution.

The discussion thus far is pertinent to the particle acceleration in an unbounded uniform plasma. In highly inho-

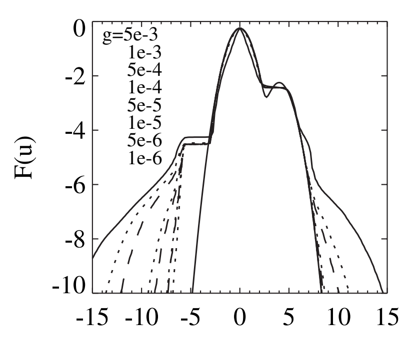

FIG. 3. Electron distribution at $\omega_{pe} t = 2 \times 10^4$ versus u for a range of g. Significant tail formation takes place only for a sufficiently high value of g.

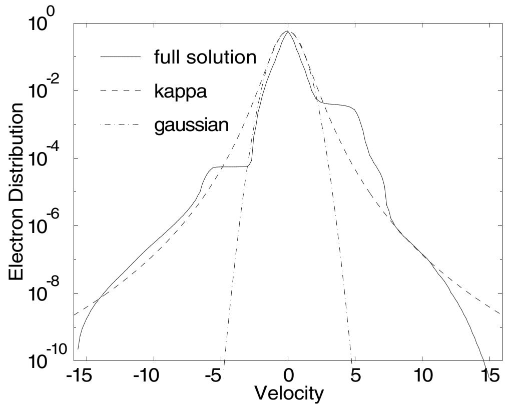

FIG. 4. Comparison of $F(u)$ at $\omega_{pe}t = 2 \times 10^4$ computed for $g = 5 \times 10^{-3}$ with κ distribution with index $\kappa = 3.5$ and the Gaussian.

mogeneous plasmas, superthermal particles can be readily generated in a variety of situations. For instance, particle acceleration is shown to occur at collisionless shocks [20,21], via parametric instability driven by a large-amplitude ion-acoustic-like wave [22], and during the magnetic reconnection process [23], to mention just a few. Our analysis is not directly applicable for these situations.

Finally, we note that an alternative approach to understanding the superthermal κ distribution has been put forth recently. The novel idea does not involve turbulent acceleration at all, but instead relies on an alternative thermodynamical concept called the ''nonextensive'' entropy [24–26]. In this approach, κ-like distributions are natural thermodynamic equilibrium solutions, in contrast to the Maxwell-Boltzmann distribution in the case of the customary, or ''extensive,'' statistics. The present Letter takes a more traditional view regarding the issue of superthermal distribution. That is, we have implicitly confined ourselves to the conventional Boltzmann-Gibbs statistics, upon which the present day plasma physics is largely based.

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