B-Meson Decay Constant from Unquenched Lattice QCD

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We present determinations of the B-meson decay constant f_B and of the ratio f_{B_s}/f_B using the MILC Collaboration unquenched gauge configurations, which include three flavors of light sea quarks. The mass of one of the sea quarks is kept around the *strange* quark mass, and we explore a range in masses for the two lighter sea quarks down to $m_s/8$. The heavy b quark is simulated using nonrelativistic QCD, and both the valence and sea light quarks are represented by the highly improved (AsqTad) staggered quark action. The good chiral properties of the latter action allow for a more accurate chiral extrapolation to physical up and down quarks than has been possible in the past. We find $f_B = 216(9)(19)(4)(6)$ MeV and $f_{B_s}/f_B = 1.20(3)(1)$.

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Accurate determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the standard model and tests of its consistency and unitarity constitute an important part of current research in experimental and theoretical particle physics. Experimental studies of neutral B_d-$\bar{B}_d$ mixing, carried out as part of this program, are now well established, and the mass difference ΔM_d is known with high precision [1]. Uncertainty in our present knowledge of the CKM matrix element $|V_{td}|$ is, hence, dominated by theoretical uncertainties, the most important of which are errors in $f_B\sqrt{B_B}$, where f_B is the B-meson decay constant and B_B its bag parameter. Lattice QCD allows for first principles calculation of the hadronic matrix elements that lead to f_B and $f_B\sqrt{B_B}$, and in recent years the onus of reducing theoretical errors in determinations of $|V_{td}|$ has been on the lattice QCD community. In this Letter, we address and significantly improve upon two of the errors that have plagued f_B calculations on the lattice in the past, namely, uncertainties due to lack of correct vacuum polarization in the simulations and errors due to chiral extrapolations to physical *up* and *down* quarks. The generation of unquenched gauge configurations by the MILC Collaboration [2], which include effects of vacuum polarization from the *strange* plus two much lighter quarks, has led to successful and realistic full QCD calculations of a variety of quantities involving both heavy and light quarks [3–10]. The generation of these realistic configurations on current computers was made possible by the development of the improved staggered formalism for light quarks [11]. One drawback of the staggered quark action is that each flavor comes in four different types, called "tastes." To simulate just one taste of sea quark per flavor, a fourth root of the quark determinant is used. This raises some theoretical issues [4] on which encouraging progress has been made, but tests continue. Another crucial development has been to use the improved staggered light quark action also for the valence light quarks inside heavy-light mesons [12]. Small valence masses mean a much milder chiral extrapolation for f_B and f_D than in the past, reducing errors from this source of uncertainty to a few percent.

In this study, we work mainly with four of the "coarse" MILC ensembles with lattice spacing a around 0.12 fm. We have also accumulated results on two of MILC's "fine" lattices with $a \sim 0.087$ fm. On the fine lattices, we use staggered valence light propagators created by the Fermilab Collaboration. The heavy b quark is simulated using the same nonrelativistic effective action, NRQCD, used for a recent successful study of the Υ system [10] on the same configurations. The lattice spacing is determined from the Υ $2S$-$1S$ splitting. On two ensembles where this was not measured directly, we have used the MILC Collaboration's heavy quark potential variable r_1, fixing its physical value from the Υ $2S$-$1S$ splitting [3,10]. The bare s and b quark masses have been fixed by the kaon and Υ masses, respectively [4,10], and, based on studies of light quark masses in Ref. [6], we take as the physical chiral limit the point $m_s/m_q = 27.4$.

The basic quantity that needs to be calculated in decay constant determinations is the matrix element of the heavy-light axial vector current between the B-meson state and the hadronic vacuum. Taking, as is customary, the temporal component of the axial current, in Euclidean space and in the B rest frame, one has

$$\langle 0|A_0|B\rangle = M_B f_B. \tag{1}$$

In the past couple of years, we have made considerable progress in reducing statistical errors in numerical determinations of this matrix element. In particular, better operators to represent the B meson have enabled good statistical errors at light valence quark masses.

Table I summarizes results for the quantity $\Phi_q \equiv f_{B_q}\sqrt{M_{B_q}}$, where B_q denotes a "B" meson with a light valence quark of mass m_q. In the third column, we show $a^{3/2}\Phi_q^{(0)}$, the result for Φ_q in lattice units when only the lowest order lattice version of A_0 is used, i.e., before including $1/M$ or radiative corrections. The next column shows $a^{3/2}\Phi_q$, our results after one-loop matching and inclusion of $1/M$ currents. All corrections to the heavy-light current at $\mathcal{O}(\Lambda_{\rm QCD}/M)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(a\alpha_s)$, $\mathcal{O}(\alpha_s/(aM))$, and $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/M)$ have been included. The dimension 4 current corrections that enter into the matching at this order have been discussed in Ref. [13]. The one-loop perturbative matching coefficients specific to the actions used in this study are given in Ref. [14]. One sees that the difference between $\Phi_q^{(0)}$ and Φ_q is small, about 2%–4% on the coarse lattices and ~7% on the fine lattices. The very small change on the coarse lattices may be partially accidental. There is cancellation between the $\mathcal{O}(\alpha_s)$ correction to the zeroth order current and the $1/M$ corrections. The coefficient of the $\mathcal{O}(\alpha_s)$ term switches sign as one goes from a bare b quark mass of $aM_0 = 2.8$ on the coarse lattices to $aM_0 = 1.95$ on the fine lattices, so that the cancellation does not occur on the latter. In the last column of Table I, we give results for Φ_q in $\mathrm{GeV}^{3/2}$. The first errors are statistical and the second come from lattice spacing uncertainties. The scales a^{-1} employed here are, in order of the most chiral to the least chiral ensembles, 1.623(32), 1.622(32), 1.596(30), and 1.605(29) GeV, respectively, on the four coarse lattices and 2.258(32) and 2.312(31) GeV, respectively, on the two fine lattices.

TABLE I. Simulation results for $\Phi_q \equiv f_{B_q}\sqrt{M_{B_q}}$. Sea (valence) quark masses are denoted by m_f (m_q) and $u_0 = [plaq]^{1/4}$ is the link variable used by the MILC Collaboration in their normalization of quark masses. See text for definitions of the last three columns. The second error in the last column comes from uncertainties in the scale $a^{-3/2}$.

u_0am_f	u_0am_q	$a^{3/2}\Phi_q^{(0)}$	$a^{3/2}\Phi_q$	$\Phi_q(\mathrm{GeV})^{3/2}$
Coarse				
0.005	0.005	0.2579(26)	0.2494(26)	0.516(5)(15)
	0.040	0.3024(15)	0.2926(17)	0.605(4)(18)
0.007	0.007	0.2571(27)	0.2512(26)	0.519(5)(15)
	0.040	0.2993(20)	0.2917(20)	0.603(4)(18)
0.010	0.005	0.2571(23)	0.2507(24)	0.506(5)(14)
	0.010	0.2622(28)	0.2562(38)	0.517(8)(15)
	0.020	0.2767(27)	0.2710(27)	0.547(5)(15)
	0.040	0.3000(32)	0.2917(38)	0.588(8)(17)
0.020	0.020	0.2751(22)	0.2658(23)	0.540(5)(15)
	0.040	0.2988(24)	0.2873(28)	0.586(6)(16)
Fine				
0.0062	0.0062	0.1550(17)	0.1443(22)	0.490(7)(10)
	0.031	0.1804(15)	0.1676(16)	0.569(5)(12)
0.0124	0.0124	0.1583(39)	0.1474(42)	0.519(15)(10)
	0.031	0.1718(45)	0.1584(54)	0.557(19)(11)

Table II shows results for the ratio $\xi_\Phi \equiv \Phi_s/\Phi_q$. This quantity, unlike Φ_q itself, is not affected directly by errors in the lattice spacing. Several other systematic errors inherent in f_B determinations, that will be discussed in more detail below, are also cancelled to a large extent in the ratio. For instance, one sees that going from ratios of $\Phi^{(0)}$ to ratios of Φ's that include $1/M$ and one-loop matching corrections produces almost no change at all. The data for ξ_Φ are plotted in Fig. 1 as a function of m_q/m_s. The full curve comes from a fit to formulas of staggered chiral perturbation theory (SχPT) [15–17] and represents the prediction for full QCD. The vertical line at small m_q corresponds to the physical chiral limit $m_q/m_s = 1/27.4$.

SχPT for heavy-light decay constants has been developed by Aubin and Bernard in Ref. [17]. For Φ_q, their formula reads

$$\Phi_q = c_0(1 + \Delta_q + \text{analytic terms}). \tag{2}$$

The term encompassing the chiral logarithms, $\Delta_q \equiv \delta f_{Bq}/(16\pi^2 f^2)$, is given in Ref. [17] and includes $\mathcal{O}(a^2)$ lattice artifact terms specific to the staggered light quark action that we employ. For the ratio ξ_Φ, we use the Ansatz

$$\xi_\Phi = 1 + (\Delta_s - \Delta_q) + \sum_k^{N_k} c_k(am_q - am_s)^k. \tag{3}$$

N_k was increased until $\xi_\Phi^{(\mathrm{phys.})}$, the fit result for ξ_Φ at $m_q/m_s = 1/27.4$, and its error had stabilized (in practice,

TABLE II. Simulation results for $\xi_\Phi \equiv \Phi_s/\Phi_q$ without and with $1/M$ plus one-loop corrections.

u_0am_f	u_0am_q	$\Phi_s^{(0)}/\Phi_q^{(0)}$	Φ_s/Φ_q
Coarse			
0.005	0.005	1.173(7)	1.173(9)
0.007	0.007	1.164(11)	1.162(11)
0.010	0.005	1.166(15)	1.163(16)
	0.010	1.144(17)	1.139(22)
	0.020	1.085(15)	1.076(17)
0.020	0.020	1.086(13)	1.081(15)
Fine			
0.0062	0.0062	1.164(17)	1.161(22)
0.0124	0.0124	1.092(19)	1.084(29)

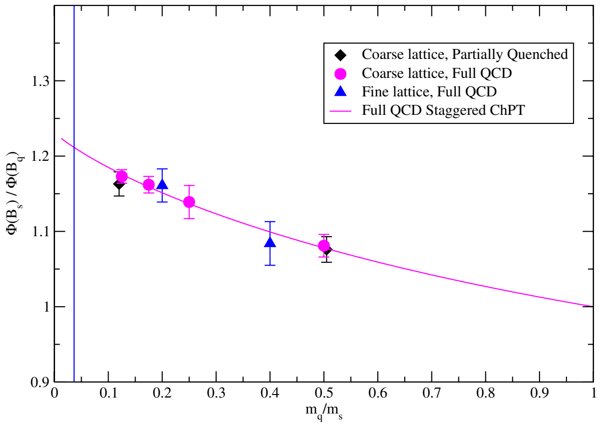

FIG. 1 (color online). The ratio $\xi_\Phi = \Phi_s/\Phi_q$ versus m_q/m_s. The full line through the data shows a fit to full QCD staggered χPT (see text). Errors are statistical errors only. The fine lattice points were not included in the fit. The vertical line at $m_q/m_s = 1/27.4$ denotes the physical chiral limit.

$N_k = 2$ was sufficient). Other Ansätze including, for instance, the direct ratio, $[1 + \Delta_s + c_1(2m_f + m_{sd}) + c_2 m_s]/[1 + \Delta_q + c_1(2m_f + m_{sd}) + c_2 m_q]$ (m_{sd} is the sea strange quark mass which, on the coarse lattices, is slightly larger than the true strange quark mass m_s we use for valence strange quarks) or simple linear fits without any chiral logarithms were also tried, as were fits with all the $\mathcal{O}(a^2)$ lattice artifact terms turned off. All these different chiral extrapolations lead to values for $\xi_\Phi^{(\text{phys.})}$ that differ at most by 3%. We fit simultaneously to the six coarse lattice points, 4 full QCD and 2 partially quenched (PQQCD) points, using full QCD and PQQCD SχPT formulas, respectively. Figure 1 shows just the full QCD curve.

The terms Δ_q involve the $BB^*\pi$ coupling $g_{B\pi}$, which is not known experimentally. We have carried out fits at several fixed values for $g_{B\pi}^2$ between $g_{B\pi}^2 = 0$ and $g_{B\pi}^2 = 0.75$. Good fits were obtained (χ^2/degrees of freedom ≈ 1 or less) for $g_{B\pi}^2 < 0.5$ with $\xi_\Phi^{(\text{phys.})}$ differing again by less than 3% in the range $\xi_\Phi^{(\text{phys.})} = 1.21$–$1.24$. We have also let $g_{B\pi}$ float as one of the fit parameters and find $g_{B\pi}^2 = 0.0(2)$ together with $\xi_\Phi^{(\text{phys.})} = 1.21(2)$. This fit result for $g_{B\pi}^2$ with the large uncertainty of $\Delta g_{B\pi}^2 = 0.2$ shows that our data are not able to determine $g_{B\pi}^2$ with any accuracy, the same message we get from the fixed $g_{B\pi}$ fits, where a range of $g_{B\pi}^2$ between zero and $\sim 2\Delta g_{B\pi}^2$ all give acceptable fits. Fortunately, within this range $\xi_\Phi^{(\text{phys.})}$ is not very sensitive to $g_{B\pi}^2$. We take as our central value for $\xi_\Phi^{(\text{phys.})}$ the result from the floating $g_{B\pi}$ fit, which we consider the least biased fit. This fit gives the curve shown in Fig. 1. We then take ± 0.03 as the error due to statistics and chiral extrapolation uncertainties and which also covers the spread we observe upon trying different Ansätze and different ways of handling $g_{B\pi}^2$. Remaining errors such as those due to discretization and relativistic corrections and higher order operator matchings not yet included will affect f_B and f_{B_s} in similar ways and largely cancel in the ratio. One expects their effects to come in at the level of the corresponding error in Φ_q times $a(m_s - m_q)$ or $(m_s - m_q)/\Lambda_{\text{QCD}}$. We have already seen that $1/M$ and one-loop matching corrections cancel almost completely in ξ_Φ. Furthermore, the two full QCD fine lattice points in Fig. 1 fall nicely on the full QCD SχPT curve fixed by the coarse lattice points, indicating that any residual discretization errors in ξ_Φ are smaller than the current statistical errors. Taking all these arguments into account, we estimate a $\sim$1% further uncertainty in ξ_Φ from these other sources. Our final result for $f_{B_s}/f_B = \xi_\Phi \sqrt{M_B/M_{B_s}}$ is then

$$f_{B_s}/f_B = 1.20(3)(1). \quad (4)$$

We emphasize that the reason the chiral extrapolation errors are small here is because the light quark action employed in this study allowed us to go down as low as $m_s/8$ and only a modest extrapolation to the physical chiral limit was required. This differs from the case with Wilson type light quarks, where simulations have typically been restricted to $m_q/m_s > 0.5$, i.e., to the region to the right of the heaviest data point in Fig. 1.

Figure 2 shows the data points for Φ_q itself for $m_q/m_s \leq 0.5$ together with a full QCD SχPT fit curve. For chiral extrapolation of Φ_q, we use directly Eq. (2) with analytic terms $c_1(2m_f + m_{sd}) + c_2 m_q$. We again carry out simultaneous fits to the coarse lattice full QCD and PQQCD points. Fits with the coupling $g_{B\pi}^2$ held fixed between 0.0 and 0.6 all lead to good fits with $\Phi^{(\text{phys.})}$ varying by 4%. Allowing this coupling to float gives $g_{B\pi}^2 = 0.1(5)$, which is consistent with the fixed $g_{B\pi}$ fit results, and $\Phi^{(\text{phys.})} = 0.496(20)\ \text{GeV}^{3/2}$ with again a 4% error. We take the 4% to be our best estimate for the combined error from statistics, chiral extrapolation, and determination of a^{-1}. The full QCD SχPT curve in Fig. 2 comes from the floating $g_{B\pi}^2$ fit.

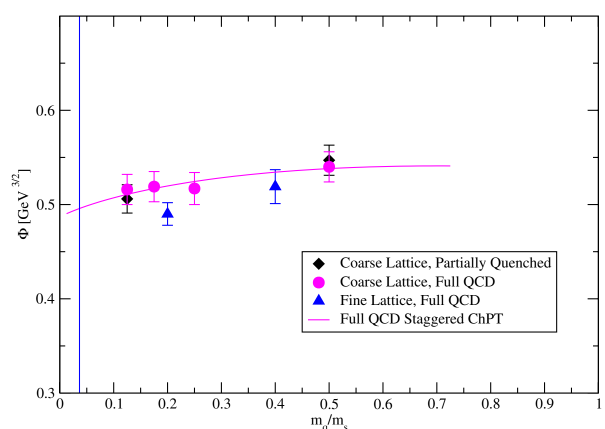

FIG. 2 (color online). Φ_q versus m_q/m_s. Errors include both statistical and scale uncertainty errors. The fine lattice points were not included in the fit.

We turn next to estimates of the other systematic errors in $\Phi^{(\text{phys.})}$.

A major source of systematic error in $\Phi^{(\text{phys.})}$ is higher order matching of the heavy-light current. Although the one-loop contributions turned out to be small (as described above), in fact much smaller than a naive estimate of $\mathcal{O}(\alpha_s) \sim 30\%$, we have no argument guaranteeing this to be true at higher orders. Hence, we allow for an $\mathcal{O}(\alpha_s^2) \approx 9\%$ systematic matching error. This will be the dominant systematic error in our decay constant determination. Another source of systematic error comes from discretization effects. The fine lattice points in Fig. 2 lie about 3%–5% lower than those from the coarse lattices. Since the statistical plus scale uncertainty errors on all our points range between 2%–3%, it is not obvious how much of this difference comes from discretization effects. It should also be noted that the difference between the coarse and fine lattice data would disappear if it were not for the one-loop matching corrections (recall the 2%–4% corrections on the coarse lattices versus the ~7% corrections on the fine lattices giving a 3%–5% difference in the radiative corrections on the two lattices). In other words, it is difficult to disentangle discretization errors from radiative corrections. One could quote a combined discretization and higher order matching error again at the ~9% level. We opt instead to keep the 9% as the pure (and dominating) $\mathcal{O}(\alpha_s^2)$ error and use a conventional naive estimate of $\mathcal{O}(a^2\alpha_s) \approx 2\%$ for discretization errors. As the last nontrivial systematic error, we estimate uncertainties from relativistic corrections and tuning of the b quark mass [10] to be at the ~3% level. Putting all this together, we obtain $\Phi^{(\text{phys.})} = 0.496(20)(45)(10)(15)\ \text{GeV}^{3/2}$. This leads to our result for the B-meson decay constant of

$$f_B = 0.216(9)(19)(4)(6)\ \text{GeV}. \tag{5}$$

The errors, from left to right, come from statistics plus scale plus chiral extrapolations, higher order matching, discretization, and relativistic corrections plus m_b tuning, respectively. Combining this result with our result for f_{B_s}/f_B, Eq. (4), one finds $f_{B_s} = 0.259(32)$ GeV. This is very consistent with the direct calculation of f_{B_s} published earlier in Ref. [5], where we quote a value of 0.260(29) GeV.

To summarize, we have completed a determination of the B-meson decay constant in full (unquenched) QCD. Our main results are given in Eqs. (4) and (5). The use of a highly improved light quark action has led to good control over the chiral extrapolation to physical up and down quarks. Better operators to represent the B meson have significantly reduced statistical errors. For the ratio f_{B_s}/f_B, these improvements translate into an accurate final result with errors at the ~3% level. For f_B itself, other systematic errors not yet addressed in the present study dominate, and the current total error is at the ~10% level. The main such error comes from higher order operator matching. More studies should also be carried out on the fine lattices and at other values of the lattice spacing, to reduce discretization uncertainties. Errors in the scale a^{-1} need to come down for all the ensembles. Improvements on all these fronts are underway. Calculations of the bag parameter B_B have also been initiated.

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[1] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[2] C. Bernard *et al.*, Phys. Rev. D **64**, 054506 (2001).
[3] C. Aubin *et al.*, Phys. Rev. D **70**, 094505 (2004); **70**, 114501 (2004).
[4] C. T. H. Davies *et al.*, Phys. Rev. Lett. **92**, 022001 (2004).
[5] M. Wingate *et al.*, Phys. Rev. Lett. **92**, 162001 (2004).
[6] C. Aubin *et al.*, Phys. Rev. D **70**, 031504(R) (2004).
[7] C. Aubin *et al.*, Phys. Rev. Lett. **94**, 011601 (2005).
[8] I. Allison *et al.*, Phys. Rev. Lett. **94**, 172001 (2005).
[9] C. Aubin *et al.*, Phys. Rev. Lett. **95**, 122002 (2005).
[10] A. Gray *et al.*, hep-lat/0507013.
[11] S. Naik, Nucl. Phys. **B316**, 238 (1989); G. P. Lepage, Phys. Rev. D **59**, 074502 (1999); J. Lagae and D. Sinclair, *ibid.* **59**, 014511 (1999); K. Originos *et al.*, *ibid.* **60**, 054503 (1999); C. Bernard *et al.*, *ibid.* **61**, 111502(R) (2000).
[12] M. Wingate *et al.*, Phys. Rev. D **67**, 054505 (2003).
[13] C. Morningstar and J. Shigemitsu, Phys. Rev. D **57**, 6741 (1998).
[14] E. Gulez *et al.*, Phys. Rev. D **69**, 074501 (2004).
[15] C. Bernard, Phys. Rev. D **65**, 054031 (2002); C. Aubin and C. Bernard, Phys. Rev. D **68**, 034014 (2003); **68**, 074011 (2003).
[16] W. Lee and S. Sharpe, Phys. Rev. D **60**, 114503 (1999).
[17] C. Aubin and C. Bernard, Nucl. Phys. B, Proc. Suppl. **140**, 491 (2005); C. Bernard (private communication).