Spectroscopy of Magnetic Excitations in Magnetic Superconductors Using Vortex Motion

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In magnetic superconductors a moving vortex lattice is accompanied by an ac magnetic field which leads to the generation of spin waves. At resonance conditions the dynamics of vortices in magnetic superconductors changes drastically, resulting in strong peaks in the dc *I-V* characteristics at voltages at which the washboard frequency of the vortex lattice matches the spin wave frequency $\omega_s(\mathbf{g})$, where $\mathbf{g}$ are the reciprocal vortex lattice vectors. We show that if the washboard frequency lies above the magnetic gap, measurement of the *I-V* characteristics provides a new method to obtain information on the spectrum of magnetic excitations in borocarbides and cuprate layered magnetic superconductors.

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The coexistence of magnetism and superconductivity was observed in many crystals, such as $R\mathrm{Mo_6S_8}$, $R\mathrm{Rh_4B_4}$, $R\mathrm{Ba_2Cu_3O_{7-\delta}}$, and $(R, A)\mathrm{CuO_{4-\delta}}$ ($A = \mathrm{Sr, Ce}$) with the temperatures of magnetic ordering T_M much smaller than the superconducting critical temperature T_c; and also in borocarbides $RT_2\mathrm{B_2C}$ and ruthenocuprate $\mathrm{RuSr_2GdCu_2O_8}$ with T_M of the same order as T_c. Here R is the rare-earth-metal element, while $T = $ Ni, Ru, Pd, Pt. In such crystals f electrons of ions R give rise to localized magnetic moments, while conducting electrons exhibit Cooper pairing. In all these crystals, except $\mathrm{HoMo_6S_8}$ and $\mathrm{Er}R\mathrm{h_4B_4}$, magnetic moments order antiferromagnetically below T_M. Such magnetic ordering coexists with superconductivity without strong interference because spin density varies on the scale much smaller than the superconducting correlation length and the net magnetic moment vanishes; for review see Refs. [1,2].

In this Letter we consider interplay between magnetic and superconducting *excitations* in magnetic superconductors, particularly interaction between a moving vortex lattice and spin waves via the ac magnetic field induced by moving vortices. The energy transfer from vortices to the magnetic system leads to dissipation which is additional to that caused by quasiparticles. This results in strong current peaks in the dc *I-V* characteristics at voltages at which the washboard frequency of vortex lattice [3] matches the spin wave frequency $\omega_s(\mathbf{k})$ and $\mathbf{k}$ matches a reciprocal vortex lattice vector $\mathbf{g}$. We propose a new technique to study the magnetic excitation spectrum of magnetic superconductors, based on this phenomenon. Our technique provides information similar to that obtained by the inelastic neutron scattering but large crystals are not needed, and it can be used when the inelastic neutron scattering is ineffective, as in the case of Sm compounds due to a large cross section for neutron capture.

First we consider slightly anisotropic superconductors, i.e., all systems mentioned above except $\mathrm{SmLa_{1-x}Sr_xCuO_{4-\delta}}$ and $\mathrm{RuSr_2GdCu_2O_8}$ crystals, and probably also $\mathrm{Sm_{2-x}Ce_xCuO_{4-\delta}}$. The latter are layered superconductors with intrinsic Josephson junctions [4–6].

We assume, for simplicity, a uniaxial crystal structure with the principal axis along z. The dc magnetic field is applied along the z axis (see Fig. 1) and we assume that the magnetic induction $\mathbf{B}(\mathbf{r})$, $\mathbf{r} = x, y$, inside the superconductor corresponds to the ideal Abrikosov vortex lattice (for simplicity we assume the square one takes place; such a lattice is realized in clean borocarbide crystals in field $\mathbf{B} \parallel c$ in some field intervals [1]). The sublattice magnetization in the case of antiferromagnetic ordering is assumed to be oriented in the (x, y) plane. The dc transport current with the density $\mathbf{j}$ is along the y axis, which, due to the Lorenz force, causes motion of the vortex lattice with the velocity $\mathbf{v}$ along the x axis.

We use the quasistatic approach assuming that the space structure of the magnetic field is the same as in the static vortex lattice, but the field moves in the same way as a vortex lattice does. Thus all quantities describing the moving vortex lattice, i.e., the magnetic field and supercurrents, have the dependence on the coordinates and time in the combination $(\mathbf{r} - \mathbf{v}t)$. In the field interval $B \ll H_{c2}$ the magnetic field should be found from the London equations [2,7]

$$\mathrm{curl}\,\mathbf{B} = \frac{4\pi}{c}\mathbf{j}_s + 4\pi\,\mathrm{curl}\,\mathbf{M}, \tag{1}$$

$$\mathbf{j}_s = \frac{c\Phi_0}{8\pi^2\lambda_\perp^2}\left(\nabla\phi - \frac{2\pi}{\Phi_0}\mathbf{A}\right), \qquad \mathbf{B} = \mathrm{curl}\,\mathbf{A}, \tag{2}$$

$$\mathrm{curl}\,\nabla\phi = \sum_n 2\pi\delta(\mathbf{r} - \mathbf{r}_n), \tag{3}$$

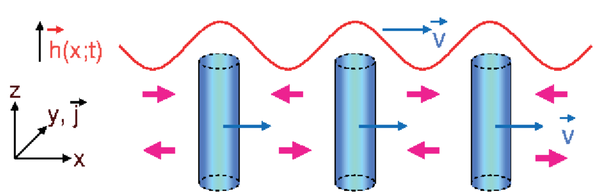

FIG. 1 (color). Vortex lattice moving with the velocity $\mathbf{v}$ induces a spatially periodic ac magnetic field $h(x, t)$ which excites the system of magnetic moments shown by purple arrows. This additional dissipation results in current peaks in the *I-V* characteristics in magnetic superconductors.

where $\mathbf{j}_s$ is the supercurrent, $\mathbf{A}$ is the vector potential, ϕ is the phase of the superconducting order parameter, $\mathbf{M}$ is the local magnetization, Φ_0 is the flux quantum and $\lambda_\perp = \lambda_x = \lambda_y$ is the London penetration length for currents in the (x, y) plane in the absence of magnetic moments. Further, $\mathbf{r}_n(t) = \mathbf{r}_n(0) + \mathbf{v}t$ are the coordinates of vortices and $\mathbf{r}_n(0)$ form a regular vortex lattice. From Eqs. (1)–(3) we obtain

$$\text{curl curl}(\mathbf{B} - 4\pi\mathbf{M}) + \frac{1}{\lambda_\perp^2}\mathbf{B} = \frac{\Phi_0}{\lambda_\perp^2}\sum_n \delta(\mathbf{r} - \mathbf{r}_n). \quad (4)$$

To relate the Fourier components of $M_z(\mathbf{r}, t) \equiv M$ and $B_z(\mathbf{r}, t) \equiv B$ we use the linear response approximation in which supercurrents induce the "external" magnetic field,

$$H(\mathbf{k}, \omega) = B(\mathbf{k}, \omega) - 4\pi M(\mathbf{k}, \omega), \quad (5)$$

acting on the magnetic moments, where $M(\mathbf{k}, \omega) = \chi(\mathbf{k}, \omega)H(\mathbf{k}, \omega)$ and $\chi(\mathbf{k}, \omega) \equiv \chi_{zz}(\mathbf{k}, \omega)$ is the susceptibility of the magnetic system. This approach is valid for the magnetization harmonics $g_x \neq 0$, satisfying the condition

$$|M(\mathbf{g}, g_x v)|^2/(\mu n_M)^2 \ll 1, \quad (6)$$

where n_M is the density of magnetic ions of magnetic moment μ. For an antiferromagnet with two sublattices the magnetic susceptibility is given [8] by

$$\chi(\mathbf{k}, \omega) = \frac{\omega_M \omega_s(\mathbf{k})}{\omega_s^2(\mathbf{k}) - \omega^2 - i\omega\nu_s}. \quad (7)$$

Here $\omega_M = \mu^2 n_M/(2\hbar)$ at $\mu B \ll k_B T_M$, $\omega_s(\mathbf{k})$ is the magnetically active spin wave dispersion renormalized by the superconductivity [2], while ν_s is the relaxation rate of spin waves due to the interaction with phonons. Using Eqs. (4) and (5), we obtain for the Fourier components $[\mathbf{k} = \mathbf{g} \equiv 2\pi(B_0/\Phi_0)^{1/2}(n, m, 0); \omega = g_x v]$ of the magnetic field

$$\left[1 + \frac{\lambda_\perp^2 \mathbf{k}^2}{1 + 4\pi\chi(\mathbf{k}, \omega)}\right]\frac{B(\mathbf{k}, \omega)}{(2\pi)^4} = \sum_{\mathbf{g}} B_0 \delta(\mathbf{k} - \mathbf{g})\delta(\omega - g_x v), \quad (8)$$

where B_0 is the average induction and n, m are integers. From Eq. (8) we see that magnetic moments renormalize the London penetration length [2] as $\Lambda_\perp(\mathbf{k}, \omega) = \lambda_\perp[1 + 4\pi\chi(\mathbf{k}, \omega)]^{-1/2}$. Solving Eq. (8) we obtain the Fourier components of the external field H as

$$H(\mathbf{k}, \omega) = (2\pi)^4 B_0 \frac{\delta(\mathbf{k} - \mathbf{g})\delta(\omega - g_x v)}{1 + 4\pi\chi(\mathbf{g}, \omega) + \lambda_\perp^2 g^2}. \quad (9)$$

Thus, the moving vortex lattice induces a spatially periodic ac external magnetic field $h(\mathbf{r}, t) = H(\mathbf{r}, t) - B_0$ along the z axis characterized by momenta $\mathbf{g}$ and washboard frequencies $\omega = v g_x$. At $\chi = 0$ for $\lambda_\perp = 1300$ Å, typical for borocarbides, the amplitude of the main harmonic, $n = 1$, $m = 0$ is about 20 G. The moving vortex lattice induces also an electric field $\mathbf{E} = [\mathbf{v} \times \mathbf{B}]/c$ (as well as an ac component) along the current direction.

When the alternating magnetic field is not parallel to the sublattice magnetization, it excites spin waves with momenta $\mathbf{g}$ and frequencies $\omega_s(\mathbf{g}) = \mathbf{g} \cdot \mathbf{v}$. This condition is satisfied if v exceeds some critical velocity determined by the spin wave velocity, as in the case of Cherenkov radiation. The spectrum of magnetic excitations is determined by the direct exchange of magnetic ions, by their RKKY interaction via the conducting electrons and by the magnetic anisotropy. The dispersion in a two-sublattice antiferromagnet is linear, $\omega_s(\mathbf{k}) = \mathbf{v}_s \cdot \mathbf{k}$, when the magnetic anisotropy is absent. Here $v_s = Ja/\hbar$ is the spin wave velocity, J is the exchange energy, and a is the magnetic correlation length. For such a spectrum, generation of spin waves by a moving vortex lattice occurs if $v \geq v_s$, as in the case of sound generation by a moving vortex lattice due to the ac electric field [9]. The magnetic anisotropy induces a gap, Δ, into the spin wave dispersion, $\omega_s(\mathbf{k}) = \sqrt{\Delta^2/\hbar^2 + v_s^2 k^2}$. Then the condition for spin wave generation of momentum k is $v^2 > v_s^2 + (\Delta/\hbar k)^2$.

Assuming that sublattice magnetizations are almost perpendicular to the applied magnetic field, we obtain for the power per unit volume transmitted from the vortex lattice to the magnetic system the expression [8]

$$\mathcal{P}_M = -\left\langle \mathbf{M}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} \right\rangle = \sum_{\mathbf{g}} 2 g_x v |H(\mathbf{g}, g_x v)|^2 \,\mathrm{Im}[\chi(\mathbf{g}, g_x v)], \quad (10)$$

where angular brackets denote time and space average.

To find the velocity of the vortex lattice at a given transport current density j, we equate the power per unit volume performed by the battery, jE, to the sum of the power dissipated by quasiparticles, ηv^2, and that transmitted to the magnetic system, $\mathcal{P}_M$. Here η is the viscous drag coefficient due to quasiparticles in normal vortex cores. It is given by the Bardeen-Stephen expression $\eta = B_0 H_{c2}^* \sigma_n / c^2$, where σ_n is the normal state conductivity, $H_{c2}^* = \Phi_0/(2\pi\xi_\perp^2)$ is the orbital upper critical field and $\xi_\perp$ is the superconducting correlation length in the direction perpendicular to the applied magnetic field. Taking into account that $E = vB_0/c$ and $\omega = v g_x = cEg_x/B_0$, we find v and finally j-E (i.e., I-V) characteristics in the intervals of E, where inequality Eq. (6) is fulfilled:

$$j(E) = \frac{c^2\eta}{B_0^2}E + \sum_{\mathbf{g}\neq 0} \frac{2 g_x c B_0 \,\mathrm{Im}[\chi(\mathbf{g}, cEg_x/B_0)]}{|1 + 4\pi\chi(\mathbf{g}, cEg_x/B_0) + \mathbf{g}^2\lambda_\perp^2|^2}. \quad (11)$$

From this equation we see that the current density as a function of E has peaks corresponding to resonances between the ac magnetic field and spin waves, i.e., when $\omega(n, m) = 2\pi v(B_0/\Phi_0)^{1/2} n = (2\pi nc/H_{c2}^*\sigma_n\Phi_0^{1/2}) j B^{1/2}$.

Let us discuss the behavior of $j(E)$ near resonances. We introduce the frequency deviation $\Delta\omega = \omega_s(\mathbf{g}) - \omega$ such that $\nu_s \ll \Delta\omega \ll \omega_s(n, m)$. Then $\chi(\mathbf{g}, \omega) \approx \omega_M/(2\Delta\omega)$ and $\mathrm{Im}[\chi(\mathbf{g}, \omega)] \approx \omega_M \nu_s/(2\Delta\omega)^2$. In the interval of frequency deviations $\Delta\omega$ where $\lambda_\perp^2 g^2 \gg 4\pi\chi(\mathbf{g}, \omega)$ we estimate

$$\frac{M(\mathbf{g}, \omega)}{\mu n_M} \approx \frac{\mu \Phi_0}{16\pi^2(n^2 + m^2)\lambda_\perp^2 \hbar \Delta\omega}. \quad (12)$$

Because of the condition Eq. (6) our approach is valid for $\hbar\Delta\omega > \mu\Phi_0/(4\pi\lambda_\perp)^2$. In this interval, we obtain an inequality on the ratio of the additional current caused by spin waves over the current background:

$$\frac{\Delta j(n, m)}{j} < \frac{16\pi^2 \hbar n_M \nu_s B_0}{\omega \eta \Phi_0} \frac{n^2}{(n^2 + m^2)^2}. \quad (13)$$

In magnetic insulators ν_s is typically of order $10^6\ \mathrm{s}^{-1}$. One can anticipate the same value in magnetic superconducting crystals, as conducting electrons are gapped. For $HoNi_2B_2C$, taking $H_{c2}^* \approx 10$ T, $n_M = 10^{22}\ \mathrm{cm}^{-3}$, $\sigma_n = 10^5(\Omega\,\mathrm{cm})^{-1}$ at $\omega = 10^{10}\ \mathrm{s}^{-1}$ we derive $\Delta j(n, m)/j < 0.8n^2/(n^2 + m^2)^2$. Thus, the peak $n = 1$, $m = 0$ is observable even in the frequency interval where our linear response approach is valid. Here the magnetic system deviates only slightly from equilibrium as energy is transformed further to phonon bath.

Based on Eq. (11) we see that measurements of the I-V characteristics at different magnetic fields and currents may provide information on the spin wave dispersion $\omega_s(\mathbf{g})$. The washboard frequency $\omega \sim j\sqrt{B_0}$ and the reciprocal vortex lattice vectors $\mathbf{g} \sim \sqrt{B_0}$ may be changed independently by varying B_0 and $\mathbf{j}$, but an important question is what are limitations on the variations of the magnetic field and the current density. Momentum $k \sim 2\pi(B_0/\Phi_0)^{1/2}$ is of order $10^6\ \mathrm{cm}^{-1}$ in fields $B_0 \leq 1$ T and increases as one approaches H_{c2}, but then harmonic amplitudes $h(\mathbf{g}, \omega)$ drop. Limitations on frequency are due to limitations on the current density, which should be lower than the depairing current density, and also should not lead to excessive heating. From Eq. (11), to reach frequency ω one needs current density $j(\omega) \geq \sigma_n \omega H_{c2}^*/c g_x$ and the electric field $E(\omega) = \omega B_0/c g_x$. At $B_0 = 1$ T we obtain $j(\omega) \approx 10^7 n^{-1}(\hbar\omega/1\ \mathrm{K})\ \mathrm{A/cm^2}$ for the lowest harmonics, while for higher harmonics higher ω may be reached. The depairing current density for borocarbides is of order $10^7\ \mathrm{A/cm^2}$ and thus spin waves with energies $\hbar\omega \lesssim 1$ K may be probed without strong suppression of superconductivity by the transport current. For the dissipation power per unit volume, $\mathcal{P}_{\mathrm{dis}} = jE \geq \sigma_n \omega^2 \Phi_0 H_{c2}^*/4\pi^2 c^2$, we estimate $\mathcal{P}_{\mathrm{dis}} \sim 10^8 n^{-2}(\hbar\omega/1\ \mathrm{K})^2\ \mathrm{W/cm^3}$. To diminish heating the pulse technique may be used, as in I-V measurements by Kunchur [10].

To date, neither the strength of magnetic anisotropy nor the structure of excitations are known in borocarbides. As only the low energy part of the spin wave spectrum may be probed by I-V measurements, we cannot predict yet whether resonance conditions for the lowest harmonics will be fulfilled in borocarbides. However, we can anticipate that higher harmonics will be effective in the case of weak pinning.

Next we discuss highly anisotropic layered crystals like $SmLa_{1-x}Sr_xCuO_{4-\delta}$. This material is especially interesting because its T^* structure leads to a two-dimensional character of the magnetic system. Here magnetic Sm_2O_2 and nonmagnetic $La_{2-x}Sr_xO_{2-\delta}$ layers alternate in the barriers between the superconducting CuO_2 layers. The Josephson nature of the interlayer coupling in this crystal has been confirmed by observation of the double Josephson plasma resonance stemming from two layers in a unit cell [4]. The specific heat measurements show that magnetic ordering is absent down to a temperature of 0.3 K and a magnetic gap, if any, lies below 0.3 K. They may also be indicative of competing interactions that might be described by the two-dimensional J_1-J_2 Heisenberg model with $J_2/J_1 > 0.4$ [11]. Such a model has very complex dynamics and contains a variety of transitions down to zero temperature, making it an ideal testing ground for the theory of quantum phase transitions.

If the magnetic field is applied perpendicular to the layers (along the c axis), it induces pancake vortices which do not form a regular lattice in magnetic fields above 20 G as they order along the c axis only due to weak Josephson and magnetic interactions [12]. This makes excitation of spin waves ineffective by moving the vortex lattice induced by a perpendicular magnetic field. When a magnetic field is applied parallel to the layers (in the ab plane, along the y axis), the situation is drastically different, because now Josephson vortices [13–16] are induced. In high fields they form a lattice which is quite regular in the x direction (parallel to the layers). Josephson vortices do not have normal cores and so only thermally induced quasiparticles (or those near the nodes in the case of d-wave pairing) cause dissipation. A weak interlayer tunneling transport current, which leads to vortex motion in the x direction, cannot destroy superconductivity and produces much less heating than in the case of isotropic or weakly anisotropic superconductors.

The distribution of the magnetic field $B(\mathbf{r})$ inside intrinsic Josephson junctions is described by coupled finite-difference differential equations for the phase difference φ_n and for the magnetic field B_n inside the junction n between layers n and $n + 1$ [14,16]. Accounting for the magnetization M_n of ions inside intrinsic Josephson junction n we obtain equations for the dimensionless variables φ_n, $b_n = B_n 2\pi\lambda_{ab}\lambda_c/\Phi_0$, $m_n = M_n 2\pi\lambda_{ab}\lambda_c/\Phi_0$, and $h_n = b_n - 4\pi m_n$:

$$\begin{aligned} &\frac{\partial^2\varphi_n}{\partial\tau^2} + \nu_c \frac{\partial\varphi_n}{\partial\tau} + \sin\varphi_n - \frac{\partial h_n}{\partial u} = 0, \\ &\nabla_n^2 h_n - \frac{b_n}{l^2} + \frac{\partial\varphi_n}{\partial u} + \nu_{ab}\frac{\partial}{\partial\tau}\left(\frac{\partial\varphi_n}{\partial u} - \frac{b_n}{l^2}\right) = 0, \end{aligned} \quad (14)$$

where $u = x/\lambda_J$, $\tau = t\omega_p$, $\lambda_J = \gamma s$, s is the interlayer distance, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio, λ_c and λ_{ab} are the London penetration lengths for currents along the c axis and in the ab plane, respectively, $\omega_p = c/(\lambda_c\sqrt{\epsilon_c})$ is the Josephson frequency, ϵ_c is the dielectric function along the c axis, $\nu_c = 4\pi\sigma_c/(\omega_p\epsilon_c)$, $\nu_{ab} = 4\pi\sigma_{ab}/(\gamma^2\epsilon_c\omega_p)$, σ_c and σ_{ab} are quasiparticle conductivities along the c axis and in the ab plane, respectively. Using a linear response approximation, $m_{\mathbf{k}\omega} = \chi b_{\mathbf{k}\omega}/(1 + 4\pi\chi_{\mathbf{k}\omega})$, where $\chi \equiv \chi_{yy}$, we see that $h_{\mathbf{k}\omega} = b_{\mathbf{k}\omega}/(1 + 4\pi\chi_{\mathbf{k}\omega})$ satisfies the same equations as $b_{\mathbf{k}\omega}$ at $\chi = 0$, but with the renormalized parameter $\tilde{l}^{-2} = (1 + 4\pi\chi_{\mathbf{k}\omega})l^{-2}$. For $SmLa_{1-x}Sr_xCuO_{4-\delta}$ we estimate $\tilde{l}^{-2} \ll 1$ because $\omega_M \approx 1.8 \times 10^8\ \mathrm{s}^{-1}$, $l^2 \approx 2 \times 10^4$ at $\mu = 0.8\mu_B$, $n_M = 5 \times 10^{21}\ \mathrm{cm}^{-3}$, and $\lambda_{ab} \approx 2000$ Å.

In the following we consider large enough fields $B > B_J \equiv \Phi_0/(2\pi s\lambda_J)$. Then the Josephson vortices fill all intrinsic junctions, overlap strongly and form a regular triangular lattice [13–16]. (An illustration is given in Ref. [13].) For $SmLa_{1-x}Sr_xCuO_{4-\delta}$ we have $\gamma \approx 500$, $\omega_p \approx 10^{12}\ \mathrm{s}^{-1}$, and $B_J \approx 0.5$ T. In a Josephson system the washboard frequency is the Josephson frequency $\omega = \omega_J = 2eV/\hbar$, where V is the voltage between neighboring layers. For a triangular lattice at frequencies and the magnetic fields satisfying the conditions $l^2 \gg (1 + 4\pi\chi)$ and $|2\tilde{\omega} - \tilde{b}| \gtrsim 1$, where $\tilde{\omega} = \omega/\omega_p$ and $\tilde{b} = B_0/B_J$, the solution of Eq. (14) has the form

$$\varphi_n(u, \tau) \approx \tilde{\omega}\tau - \tilde{b}u + \pi n + \frac{4\sin(\tilde{\omega}\tau - \tilde{b}u + \pi n)}{4\tilde{\omega}^2 - \tilde{b}^2},$$

$$h_n(u, \tau) \approx -h_0 \cos(\tilde{\omega}\tau - \tilde{b}u + \pi n), \qquad h_0 \approx \frac{\tilde{b}}{4\tilde{\omega}^2 - \tilde{b}^2},$$

where we neglected ν_c and ν_{ab}. We estimate $h \equiv h_0\Phi_0/(2\pi\lambda_{ab}^2\gamma) \approx 0.16$ G at $\omega = 0.1\omega_p$ and $B = B_J$. Near the Eck resonance, $2\tilde{\omega} \approx \tilde{b}$, the amplitude of the magnetic field h is larger. For the reciprocal lattice vector we have $\mathbf{g} = (2\pi sB/\Phi_0, 0, \pi/s)$. So $g_x = 1/\lambda_J \approx 10^4\ \mathrm{cm}^{-1}$ at $B = B_J$.

Assuming that sublattice magnetization is almost perpendicular to the applied magnetic field or that magnetic ordering is absent, we obtain for the I-V characteristics

$$j(V) = \sigma_{\mathrm{eff}}\frac{V}{s} + \frac{esh^2}{\hbar}\,\mathrm{Im}\left[\chi_{yy}\left(\mathbf{g}, \frac{2eV}{\hbar}\right)\right], \qquad (15)$$

where $\sigma_{\mathrm{eff}} = \sigma_c + 2\sigma_{ab}B_J^2/(\gamma B)^2$ describes dissipation due to quasiparticles. At resonance $\omega_J = \omega_s(\mathbf{g})$, we estimate $\Delta j/j \approx 2\pi^2 c^2 s^2 h^2 \omega_M/(\omega_J \sigma_{\mathrm{eff}} \nu_s \Phi_0^2)$. Estimating $\sigma_c \simeq 10^{-3}(\Omega\,\mathrm{cm})^{-1}$ and $\sigma_{ab} \simeq 4 \times 10^4(\Omega\ \mathrm{cm})^{-1}$ as in $Bi_2Sr_2CaCu_2O_8$ and taking $s \simeq 12$ Å as in $Sm_{2-x}Ce_xCuO_{4-\delta}$, we obtain $\Delta j/j \approx 4$ and $|M(\mathbf{g}, \omega_J)|/(\mu n_M) \approx 0.3$ at $\omega = 10^{12}\ \mathrm{s}^{-1}$ and $B = B_J$ and bigger values near the Eck resonance. Certainly, such frequencies are sufficient to probe almost the complete spectrum in $SmLa_{1-x}Sr_xCuO_{4-\delta}$.

In conclusion, we propose to probe low-frequency magnetic excitations in magnetic superconductors by measuring I-V characteristics in the mixed state with a moving vortex lattice. Coupling of such a lattice to magnetic moments is due to an ac magnetic field which is inherent to vortex motion. The energy interval of spin waves which can be probed in isotropic and moderately anisotropic superconductors is limited by the depairing current and heating. If spin wave energies exist in this interval, they affect the vortex motion strongly and should be easily seen in the I-V characteristics as current peaks at corresponding voltages. Such an effect may be observed in borocarbides if they support spin waves with energies below 1 K. For highly anisotropic layered superconductors in parallel magnetic fields, higher spin wave energies may be probed by use of moving Josephson vortices. This is sufficient to study almost the complete spin wave spectrum in $SmLa_{1-x}Sr_xCuO_{4-\delta}$ with exotic magnetic ordering, otherwise inaccessible by neutron scattering techniques.

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